Dark Universe and detection of Dark Matter

The Dark Side of the Universe: experimental evidences ...

COMA Cluster

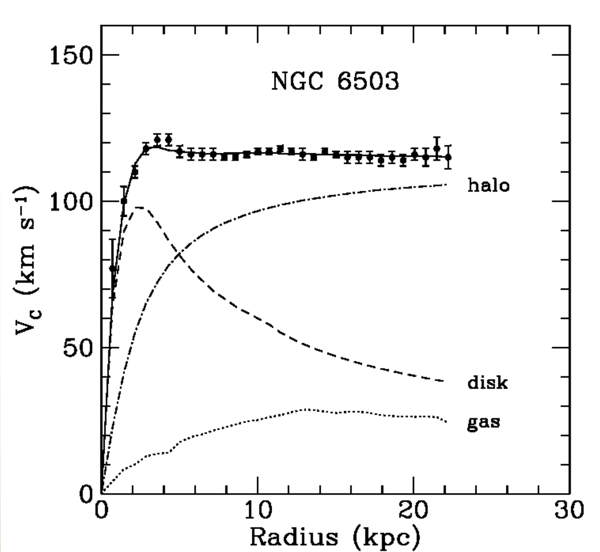
First evidence and confirmations:

1933 **F. Zwicky:** studying dispersion velocity of Coma galaxies

1936 **S. Smith:** studying the Virgo cluster

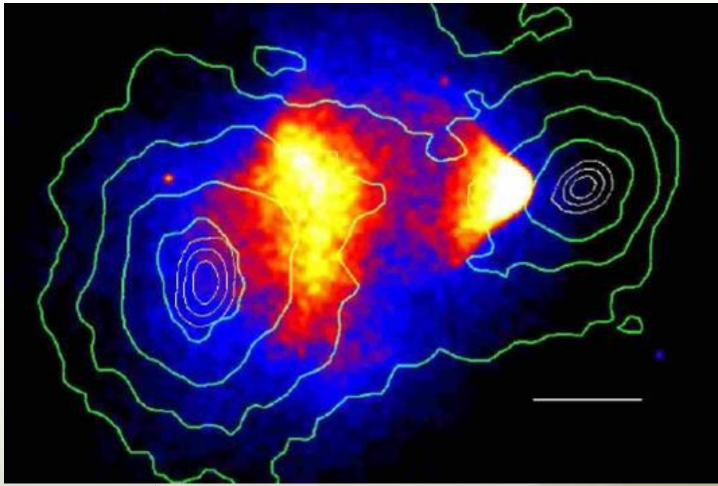
1974 **two groups:** systematical analysis of *mass density* vs *distance from center* in many galaxies

Other experimental evidences



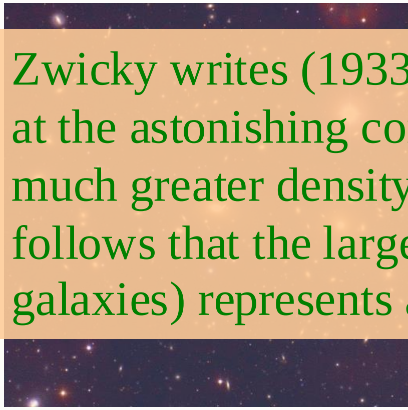
Rotational curve of a spiral galaxy

- ✓ from LMC motion around Galaxy
- ✓ from X-ray emitting gases surrounding elliptical galaxies
- ✓ from hot intergalactic plasma velocity distribution in clusters
- ✓ ...
- ✓ bullet cluster 1E0657-558



$M_{\text{visible Universe}} << M_{\text{gravitational effect}} \Rightarrow$ about 90% of the mass is DARK

The Dark Side of the Universe: experimental evidences ...



First evidence and confirmations:

Zwicky writes (1933): "If this [overdensity] is confirmed we would arrive at the astonishing conclusion that dark matter is present [in Coma] with a much greater density than luminous matter. From these considerations it follows that the large velocity dispersion in Coma (and in other clusters of galaxies) represents an unsolved problem."

1933 F. Zwicky: studying dispersion velocity of Coma galaxies

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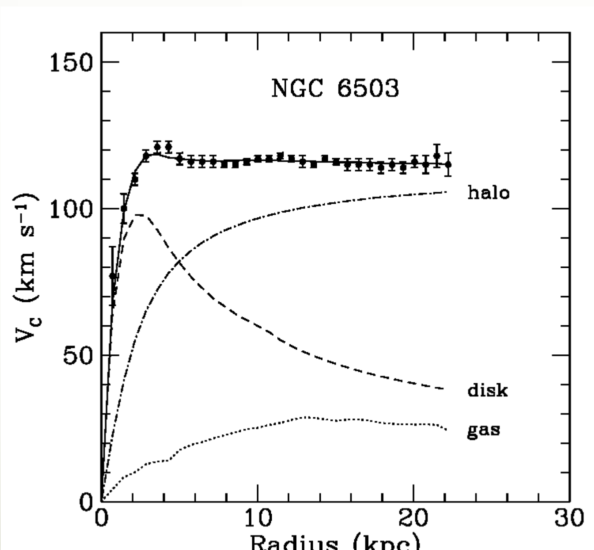
1974 two groups: systematical analysis of *mass density* vs *distance from center* in many galaxies

COMA Cluster

Other experimental evidences



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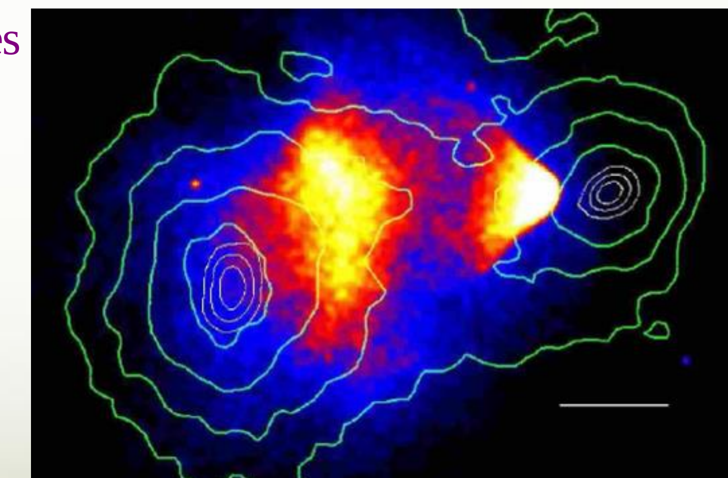

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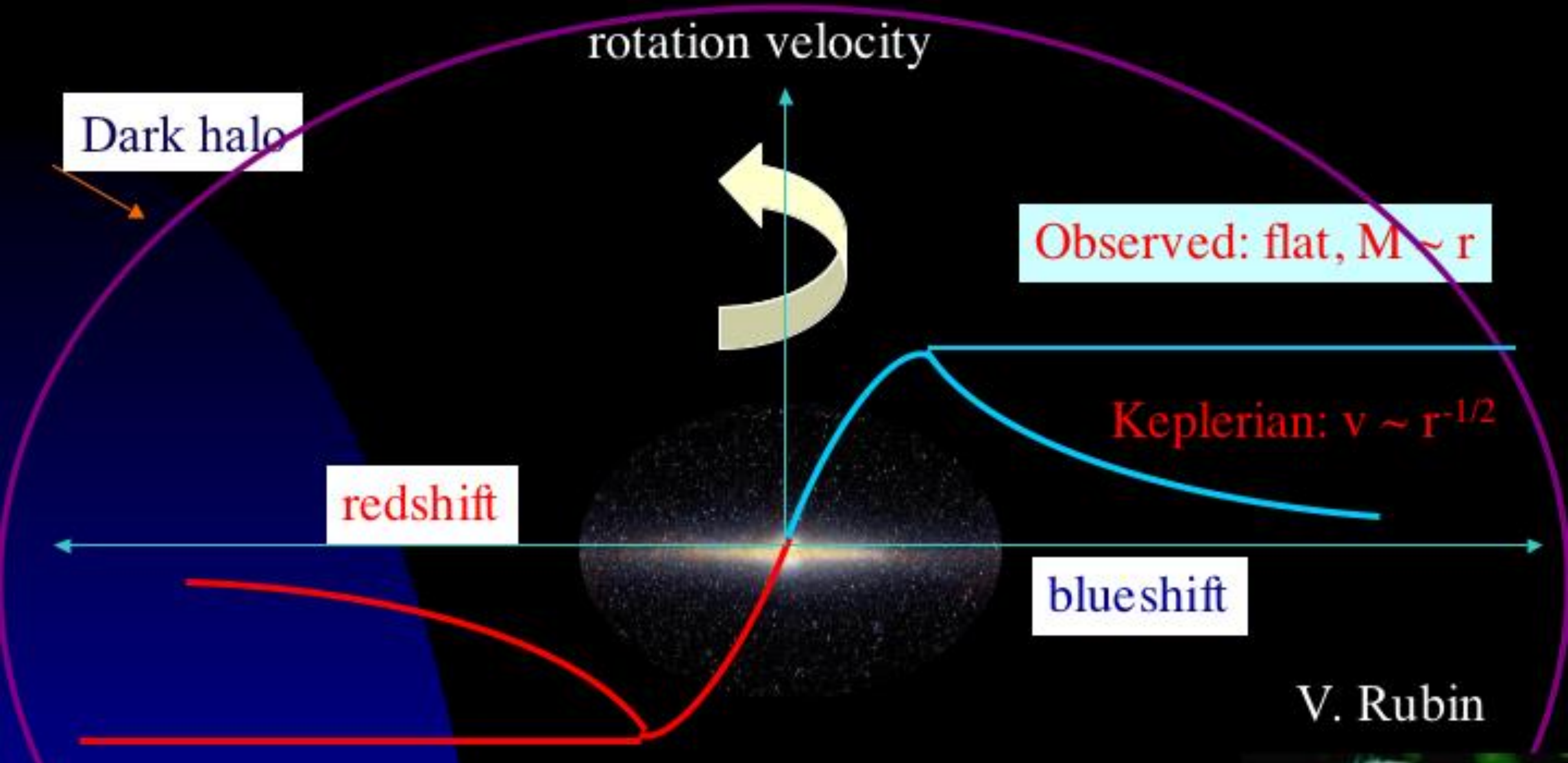
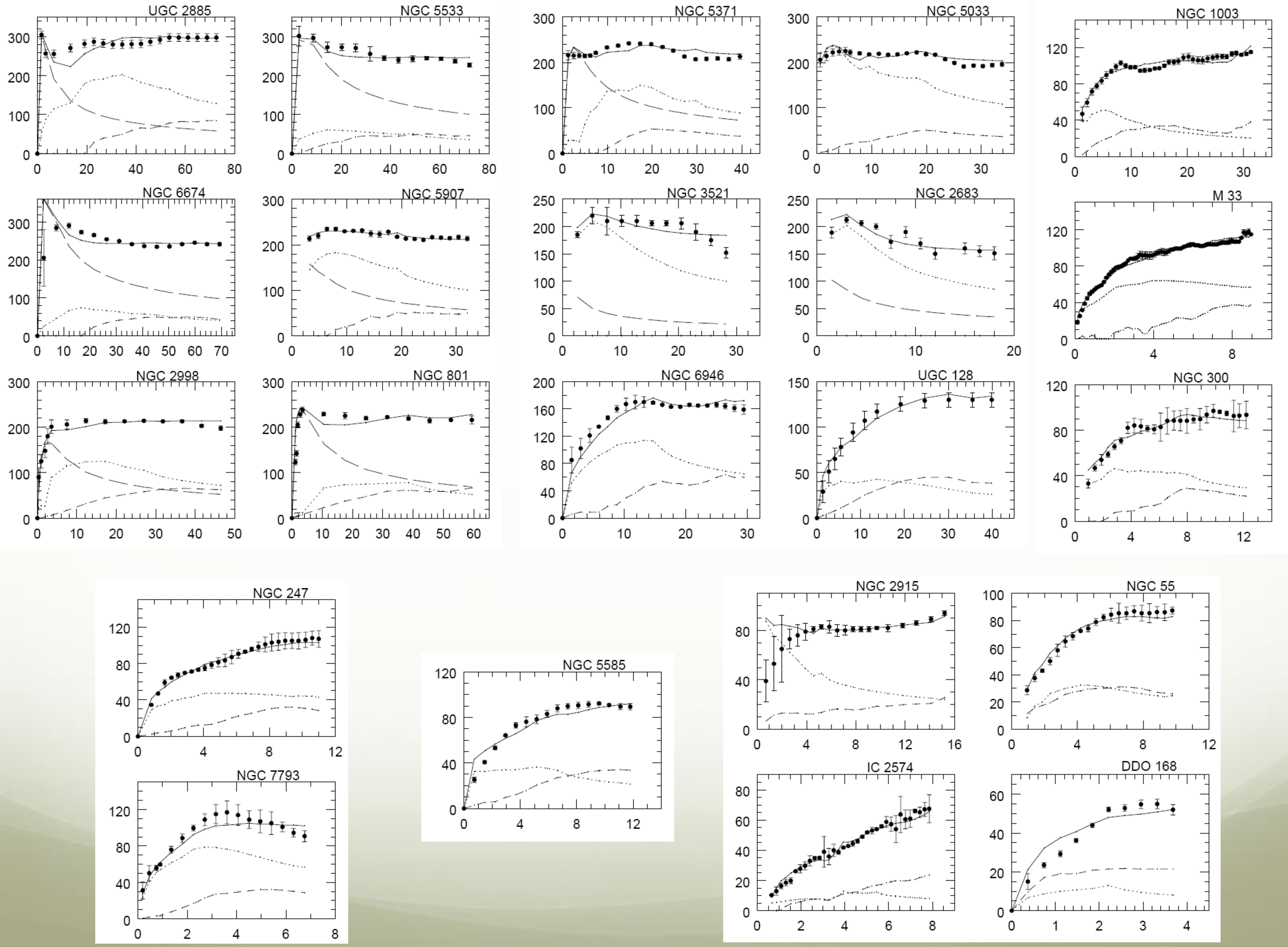


V. Rubin

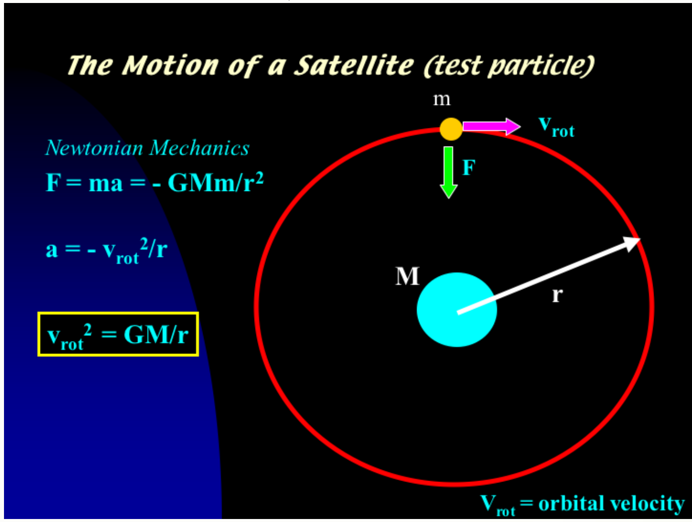
Typical rotation speed ~ 200 km/sec and visible disk size ~ 10 kpc

Halos extend ~10 times farther out

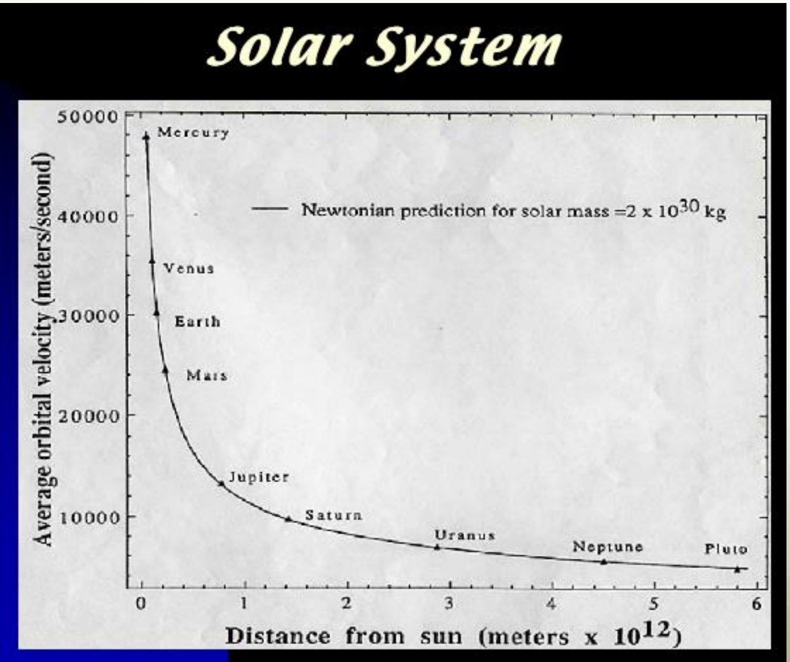

UGC 2885
NGC 5533
NGC 5371
NGC 5033
NGC 1003
NGC 6674
NGC 5907
NGC 3521
NGC 2683
M 33
NGC 2998
NGC 801
NGC 6946
UGC 128
NGC 300
NGC 247
NGC 7793
NGC 5585
NGC 2915
NGC 55
IC 2574
DDO 168

In particular, spherical symmetry; mass inside a sphere: $M(r) = \int_V \rho dV$

$$\frac{GM(r)m}{r^2} = m\frac{v^2}{r}; \quad \Rightarrow \quad v^2 = \frac{GM(r)}{r}$$



- Solar system or solar-system-like: $v \propto \frac{1}{\sqrt{r}}$
- Galaxy: v about flat: $M(r) \propto r; \quad \Rightarrow \quad \rho \propto \frac{1}{r^2}$



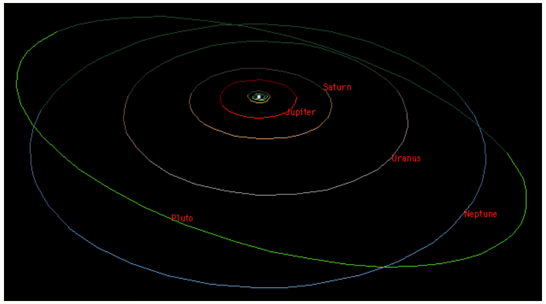
In fact: $\rho(r) = \rho_0 \frac{r_0^2}{r^2}; \quad \Rightarrow$

$$\Rightarrow \quad M(r) = 4\pi \int_0^r \rho_0 \frac{r_0^2}{r'^2} r'^2 dr' = 4\pi\rho_0 r_0^2 r$$

$$V = \frac{4}{3}\pi r^3$$

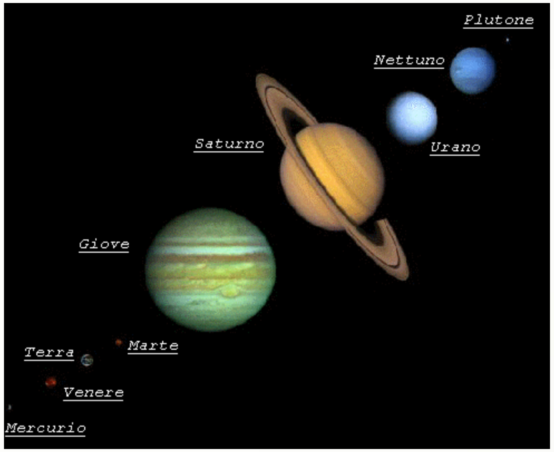
$$v = \sqrt{\frac{GM(r)}{r}} = \sqrt{\frac{G4\pi\rho_0 r_0^2 r}{r}} = \sqrt{4\pi G\rho_0 r_0^2} = const.$$

Esempio di come si sono individuati corpi oscuri a partire da osservazioni su altri oggetti luminosi



Nettuno

- **1821**: Bouvard il moto di Urano divergeva in maniera apprezzabile dalle previsioni teoriche; il fenomeno poteva essere spiegato solo teorizzando la presenza di un altro corpo di notevoli dimensioni nelle regioni più esterne del sistema solare.



- Adams (nel **1843**) e Le Verrier (nel **1846**) teorizzarono con buona approssimazione posizione e massa di questo presunto nuovo pianeta.
- **23 settembre 1846**: Galle e d'Arrest individuarono il pianeta a meno di un grado dalla posizione prevista da Le Verrier (ed a dodici gradi dalla posizione prevista da Adams). Disputa e polemiche di lungo corso

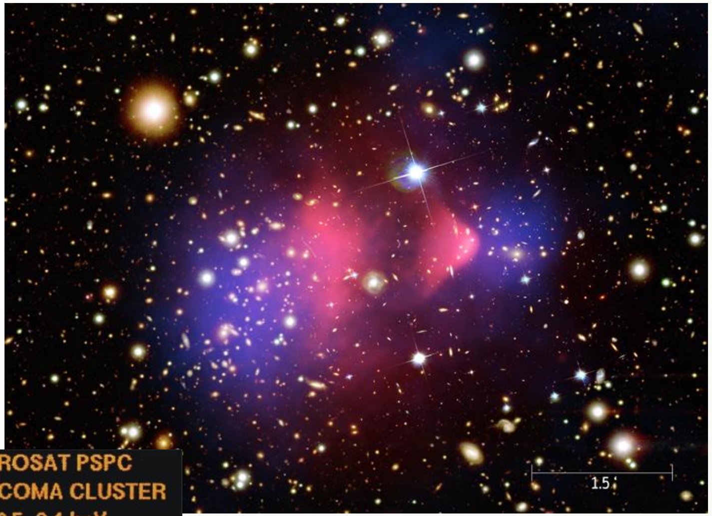
Plutone – caso di Plutone, scoperto studiando il moto di Urano e Nettuno ??

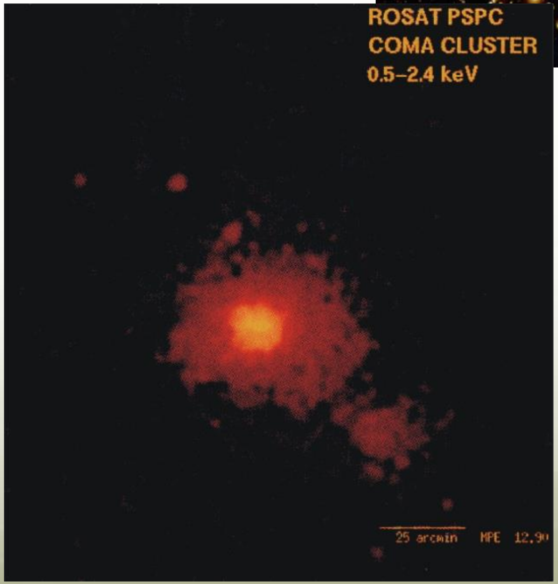
- **1915**: P. Lowell dedusse la presenza di un pianeta di massa uguale a 6.5 masse terrestri dalle residue perturbazioni dell'orbita di Nettuno.
- **1930**: Tombaugh trovò il nuovo pianeta assai vicino alla posizione definita da Lowell a cui diede il nome di Plutone. Dal momento però che la massa del pianeta era troppo piccola rispetto a quella prevista rimase per molto tempo il dubbio che si trattasse del pianeta di Lowell.
- Successivamente, 1989: l'accumularsi dei dati (*Voyager 2*) sulle posizioni di Urano e Nettuno permise di calcolare le nuove masse e di ridurre il numero delle perturbazioni di cui rendere conto, per cui il valore della massa di Plutone si ridusse a meno della massa della Terra. La scoperta poi di un satellite di Plutone permise una più corretta valutazione della massa riducendone ulteriormente il valore a 0.0026 M_T (masse terrestri).

Plutone e il suo satellite naturale Caronte fotografati dal Telescopio Spaziale Hubble. *Cortesia NASA/STScI.*

Some astrophysical probes for the existence of dark matter

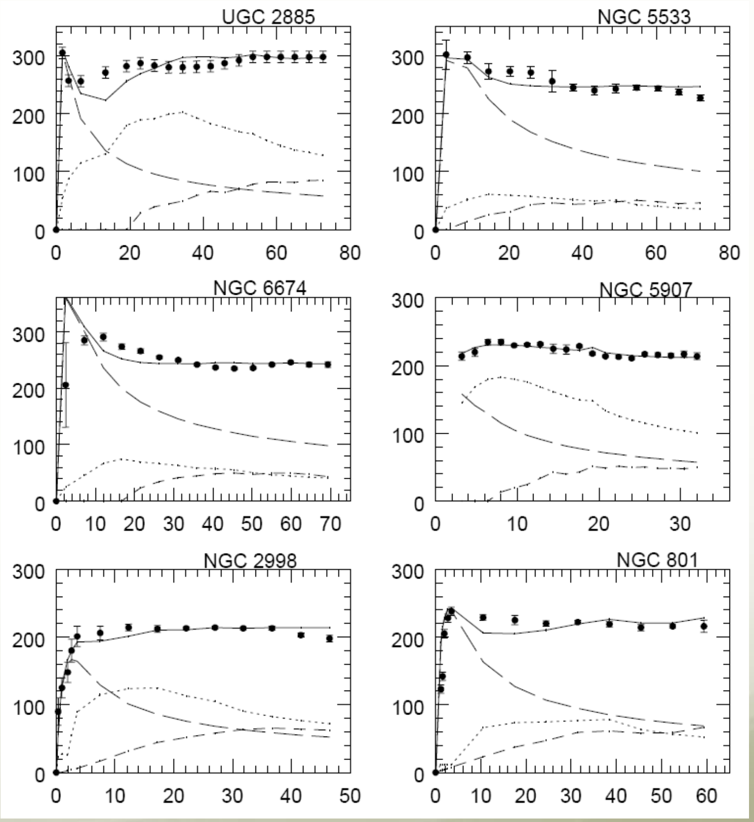
- Rotation curves of spiral galaxies
- Gas distribution in galaxy clusters
- The bullet cluster





- **Rotation curves** of spiral galaxies

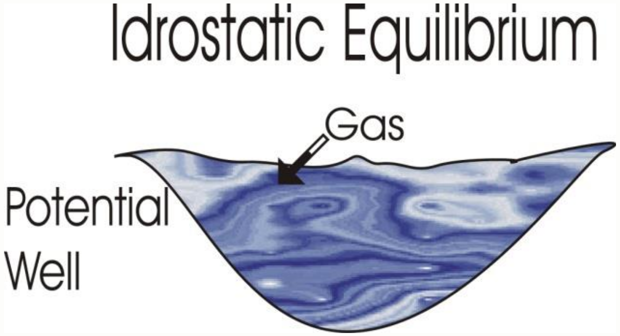
(see above)

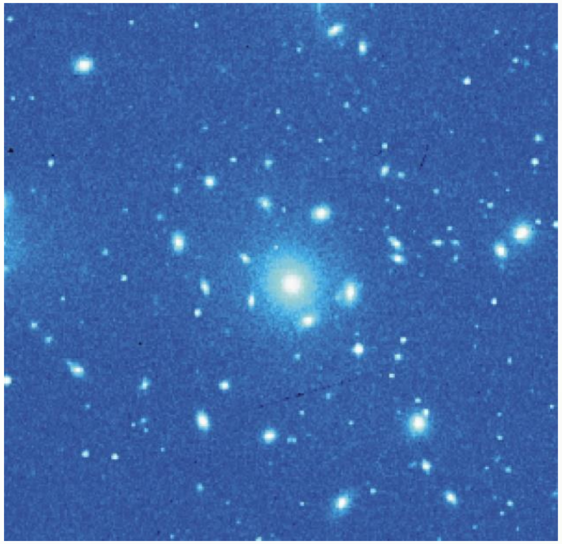


Cluster of Galaxies

Clusters can be studied in the optical

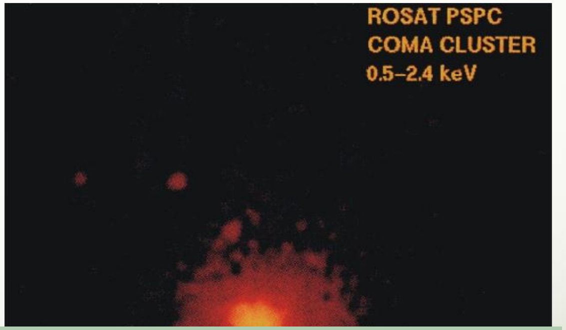
Why do we need to observe them in the X-ray?





Optical image of Coma cluster

Hot Gas ($>10^7$ K) $\Rightarrow$ X-Ray Emission for Thermal Bremsstrahlung



Surface Brightness Profile Gas Density Distribution

Spectral Analysis Temperature, Metal Enrichment

Results: the baryonic mass in galaxies and in the diffuse intra-cluster medium is only the **15%** of the total mass!

$\Rightarrow$ **Dark matter dominates also at cluster scales!**

P

P

($>10^{14} M_\odot$)

X-Ray image of Coma cluster

X-ray Galaxy clusters

Assuming that the gas is **spherically** distributed and in **hydrostatic equilibrium,** we can use it as a tracer of the whole matter distribution:

$$\frac{dP_{gas}}{dr} = -\rho_{gas}\frac{d\phi}{dr} = -\rho_{gas}\frac{GM(<r)}{r^2}$$

Assuming an **equation of state** for the gas:

$$M(<r) = -\frac{r}{G}\frac{k_B T}{\mu m_p}\left(\frac{d\ln\rho_{gas}}{d\ln r} + \frac{d\ln T}{d\ln r}\right)$$

The gas density profile can be directly obtained from the X-ray surface brightness, while the temperature can be derived from a spectral analysis

Results: the baryonic mass in galaxies and in the diffuse intra-cluster medium is only the **15%** of the total mass!

=> **Dark matter dominates also at cluster scales!**

- EoS of gas
- $p_{gas}\mathrm{V} = \mathrm{NkT} \;\; \rightarrow p_{gas} = \frac{N}{V}kT$
- $\rho_{gas} = \frac{N\mu m_p}{V}$
- Then: $p_{gas} = \frac{\rho_{gas}KT}{\mu m_p}$

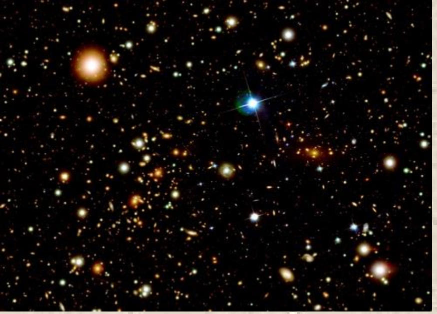
$$\frac{dp_{gas}}{dr} = \frac{p_{gas}}{r}\frac{\frac{dp_{gas}}{p_{gas}}}{\frac{dr}{r}}$$

- $\frac{dp_{gas}}{dr} = \frac{p_{gas}}{r}\frac{d\,\ln p_{gas}}{d\,\ln r} = \frac{p_{gas}}{r}\left(\frac{\partial\,\ln p_{gas}}{\partial\,\ln \rho_{gas}}\frac{\partial\,\ln \rho_{gas}}{\partial\,\ln r} + \frac{\partial\,\ln p_{gas}}{\partial\,\ln T}\frac{\partial\,\ln T}{\partial\,\ln r}\right)$
- $\frac{dp_{gas}}{dr} = \frac{p_{gas}}{r}\left(\frac{\partial\,\ln \rho_{gas}}{\partial\,\ln r} + \frac{\partial\,\ln T}{\partial\,\ln r}\right) = \frac{\rho_{gas}KT}{\mu m_p r}\left(\frac{\partial\,\ln \rho_{gas}}{\partial\,\ln r} + \frac{\partial\,\ln T}{\partial\,\ln r}\right)$

Starting from:

$$\frac{dP_{gas}}{dr} = -\rho_{gas}\frac{d\phi}{dr} = -\rho_{gas}\frac{GM(<r)}{r^2}$$

$$M(<r) = -\frac{r}{G}\frac{k_B T}{\mu m_p}\left(\frac{d\ln \rho_{gas}}{d\ln r} + \frac{d\ln T}{d\ln r}\right)$$



A direct empirical proof of the existence of dark matter

D. Clowe et al., astro-ph/0608407

the Bullet cluster
1E0657-558 **z=0.296**

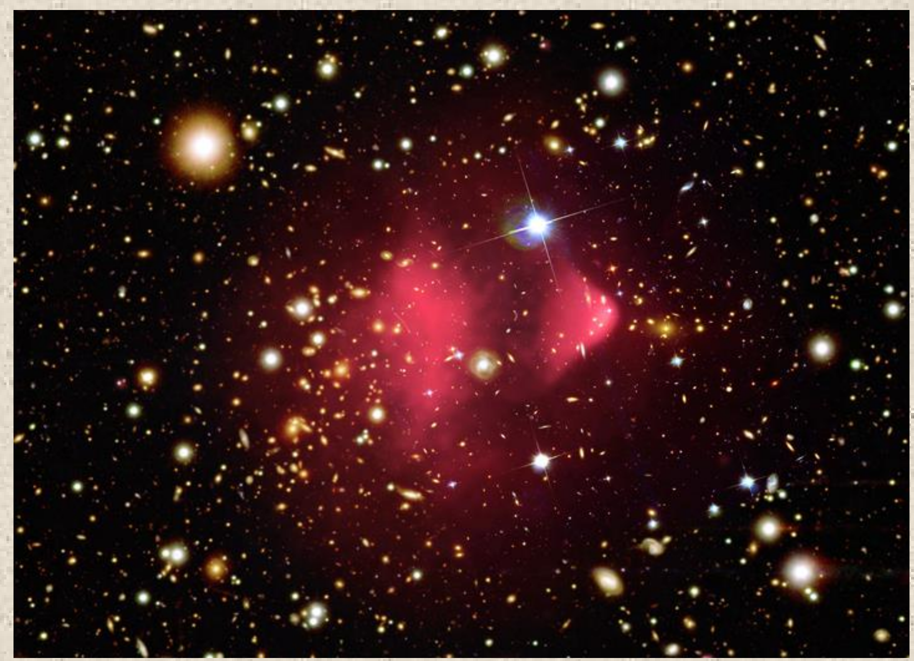
Clowe et al. 2006
Bradac et al. 2006

Collision of two clusters of galaxies

the galaxies' nuclei collided about 100Myr ago

The largest part of the matter in the system is not luminous

The Bullet cluster

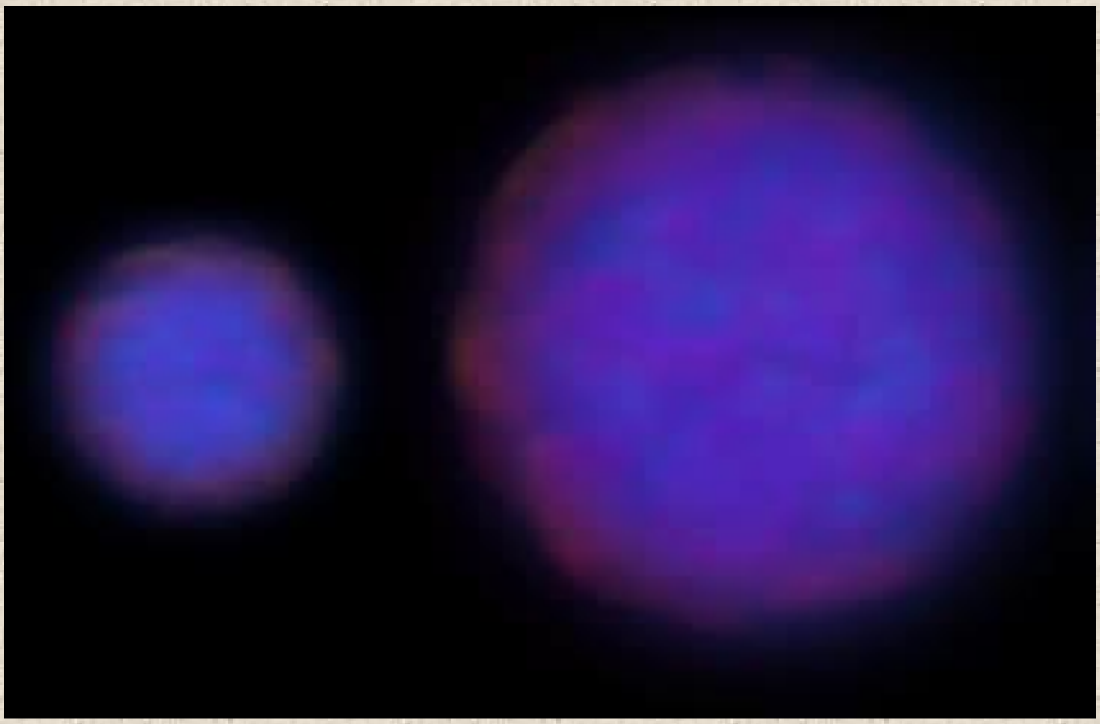


X-ray observations show the distribution of the hot gas inside the cluster

Weak lensing observations show the distribution of the total mass of the cluster

There is an evident displacement between the gas distribution and the total one => **evidence for dark matter**

The Bullet cluster

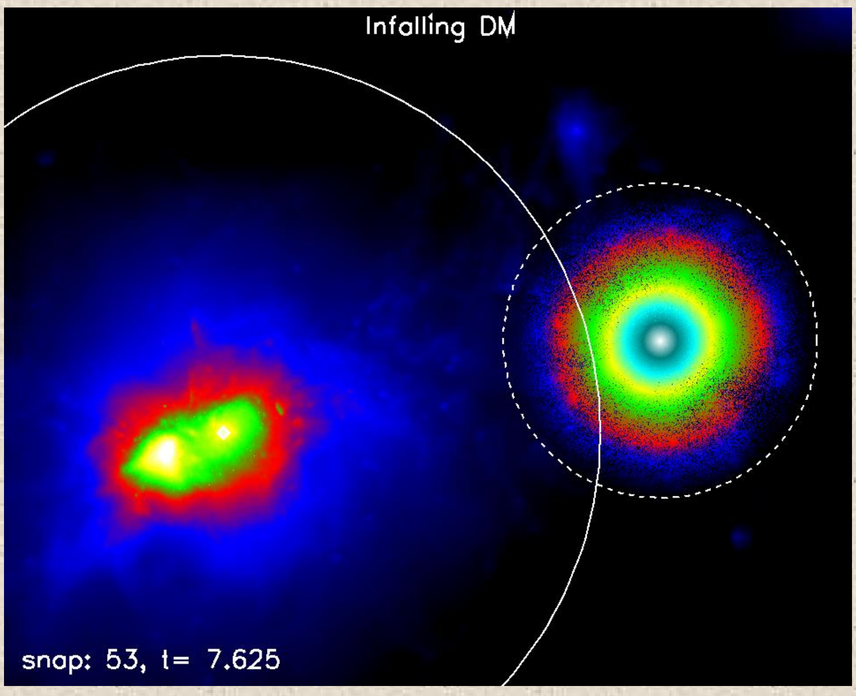


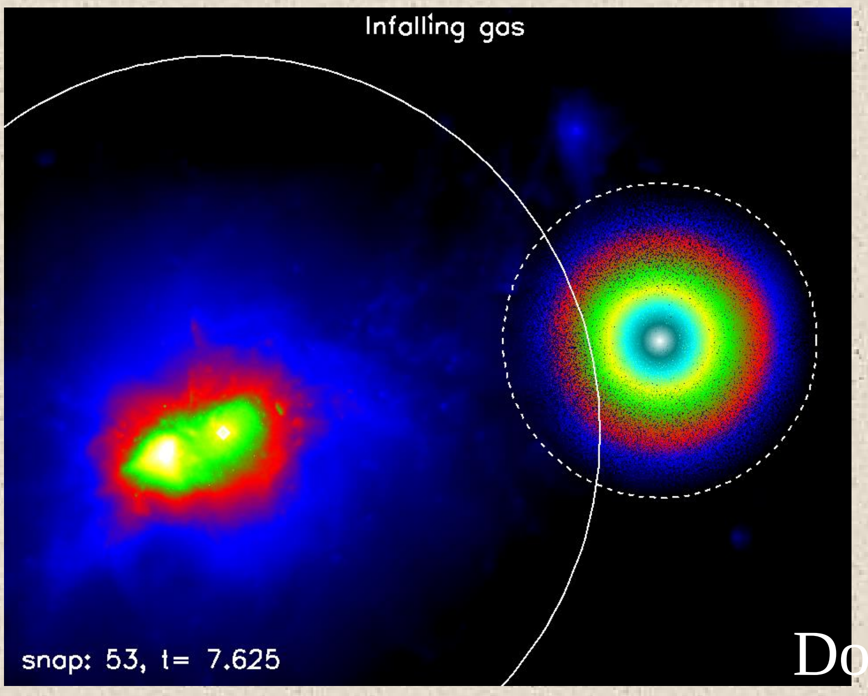
The Bullet cluster: the DM explaination?

A different dynamical behaviour:
collisionless vs. collisional

dark matter

baryonic matter





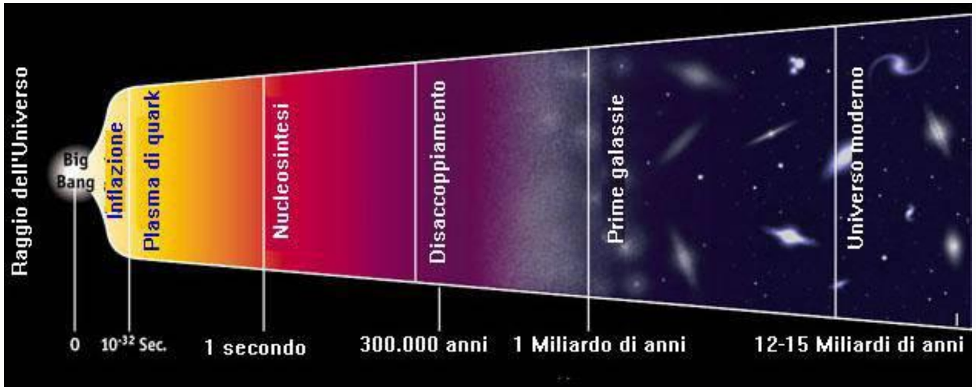
Dolag

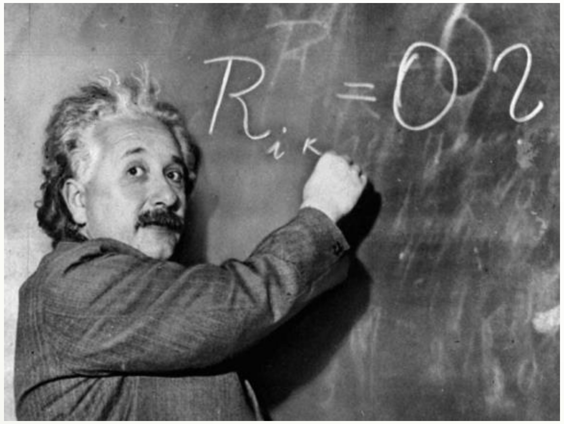
The Standard Big Bang Model

Big Bang, big initial "explosion"

The origin of all was an initial "singularity"

The basis of the Big Bang model are :





The theory of the General Relativity

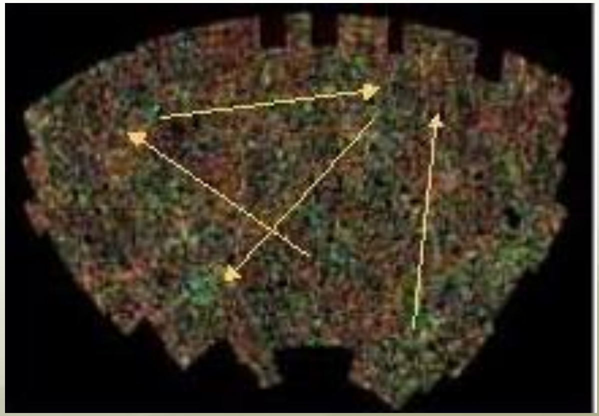
The gravity is a distortion of the space-time

"Matter tells space how to curve, and space tells matter how to move. How do gravity waves fit in this?" – J. Wheeler

The Cosmological Principle

The Matter in the Universe is homogeneously and isotropically distributed on large scale

Experimental evidences e.g. from the distribution of a large number of galaxies



- The Universe is assumed to be a **perfect fluid** with equation of state $p=w\rho c^2$

The Einstein field equations:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi G_{\rm N} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

It is common to assume that the matter content of the Universe is a perfect fluid, for which $T_{\mu\nu} = -p g_{\mu\nu} + (p+\rho) u_\mu u_\nu$

The expansion of the Universe

The metric:

$d = a(t)\chi$ $\quad\quad$ $\dot{d} = \left(\frac{\dot{a}}{a}\right) d + a(t) v_{pec}$ $\quad$ Hubble–Lemaitre law

The observed homogeneity and isotropy enable us to describe the overall geometry and evolution of the Universe in terms of two cosmological parameters accounting for the spatial curvature and the overall expansion (or contraction) of the Universe. These two quantities appear in the most general expression for a space-time metric known as the Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

- $k \to \alpha^2 k$
- $r \to \frac{1}{\alpha} r$
- $a(t) \to \alpha a(t)$

By rescaling the radial coordinate, we can choose the curvature constant *k* to take only the discrete values +1,−1, or 0 corresponding to closed, open, or spatially flat geometries. In this case, it is often more convenient to re-express the metric as:

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + S_k^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$S_k(\chi) = \begin{cases} \sin\chi & k=+1 \\ \chi & k=0 \\ \sinh\chi & k=-1 \end{cases}$$

The Friedmann equations

- Combining the Einstein field equation and the metric, Friedmann obtained his equations, describing the evolution of the space-time [e.g. the scale factor a(t)]:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

The geometry of the Universe

- The **geometry** of the universe is given by the metric curvature ***k***, which receives contributions from all energy components in the universe: matter, radiation, vacuum, etc.
- We need to estimate the **total energy density** of the Universe.
- $H = \left(\frac{\dot{a}}{a}\right)$; whose present value is the Hubble constant
- Usually, they are expressed in terms of a critical value for the energy density:

today

$$\rho_{crit} = \frac{3H^2}{8\pi G} \approx 10^{-29} g/cm^3$$

$$\Omega_i \equiv \frac{\rho_i}{\rho_{crit}}$$

The geometry is then determined by Ω_{tot}

$$\Omega_{tot} \equiv 1 - \Omega_K = \Omega_m + \Omega_{rad} + \Omega_{vac} + \ldots$$

(X marked over Ω_{rad})

Improper use:

$$\Omega_k = -\frac{k}{a^2 H^2}$$

The Friedmann equations at present time (*today*)

- First Friedmann equation:

$$H_0^2 = \left(\frac{\dot{a}}{a}\right)^2 (today) = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad \rightarrow \quad 1 = \frac{8\pi G}{3H_0^2}\rho - \frac{k}{a^2 H_0^2} + \frac{\Lambda}{3H_0^2}$$

- Normalizing with the critical density, we define:

$$\Omega_m = \frac{8\pi G}{3H_0^2}\rho \qquad \Omega_k = -\frac{k}{a^2 H_0^2} \quad \text{(Improper use)} \qquad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

$$\Omega_{tot} = 1 - \Omega_k = \Omega_m + \Omega_\Lambda$$

- Considering $p \approx 0$ in a matter dominated Universe, from the second Friedmann equation:

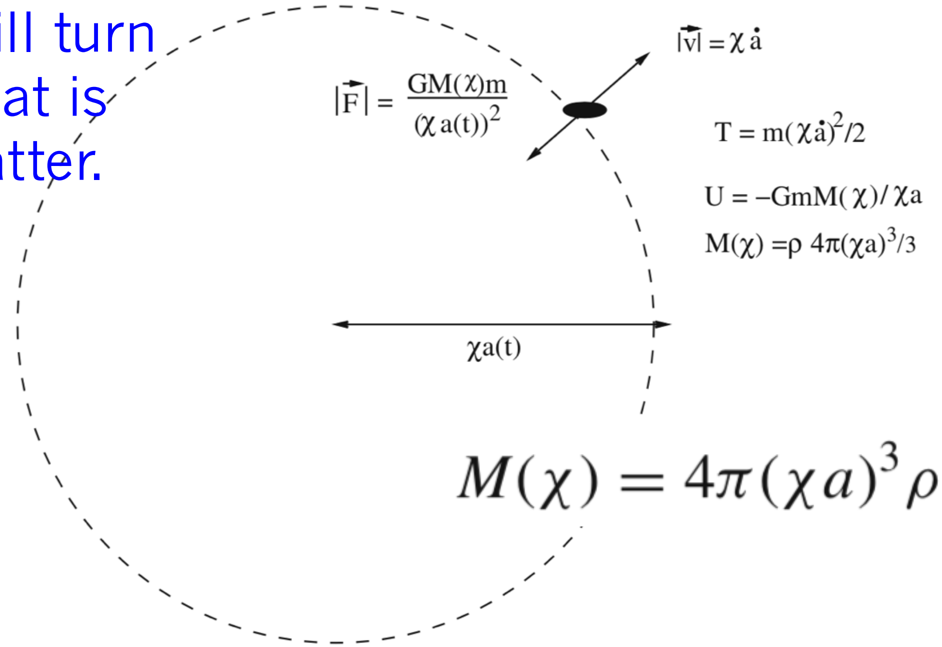
$$\frac{\ddot{a}}{a}(today) = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3} \quad \rightarrow \quad q_0 = -\frac{1}{H_0^2}\frac{\ddot{a}}{a}(today) = \frac{1}{2}\frac{8\pi G}{3H_0^2}\rho - \frac{\Lambda}{3H_0^2}$$

- So that the deceleration parameter, q_0, is:

$$q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$$

The deceleration parameter is a measurable quantity

Newtonian argument (of dubious validity) to find equations that will turn out to be correct in a universe that is dominated by non-relativistic matter.



$$d = a(t)\,c \qquad \dot{d} = \left(\frac{\dot{a}}{a}\right) d + a(t) v_{pec}$$

$$M(\chi) = 4\pi(\chi a)^3 \rho/3.$$

$$|\boldsymbol{F}| = \frac{GM(\chi)m}{\chi^2 a^2},$$

$$|\boldsymbol{F}| = m\ddot{R} = m\chi\ddot{a}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G\rho}{3} \qquad \text{if } \rho \sim \rho_{\mathrm{M}}.$$

Deceleration is now:

$$\left[\frac{\ddot{a}}{a}\right]_{t_0} = -H_0^2 \frac{\Omega_{\mathrm{M}}}{2} \qquad \text{if } \rho \sim \rho_{\mathrm{M}}.$$

$$E = \frac{1}{2} m \chi^2 \dot{a}^2 - G m \frac{4}{3}\pi a^2 \chi^2 \rho$$

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8}{3}\pi G\rho = \frac{2E}{m\chi^2}\,\frac{1}{a^2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{Kc^2}{a^2}$$

The Universe is either flat, open or close?

✓ The mean density ($\Omega=\varrho/\varrho_c$) of the Universe determines its geometry.

Critical density : $\varrho_c \approx 6$ hydrogen atoms per m^3

✓ The structure of the space-time can have three possible geometries

$$\Omega_{tot} = 1 - \Omega_k \qquad \Omega_k = -\frac{k}{a^2 H_0^2}$$

✖ Density > Critical density ($\Omega_{tot}>1$)

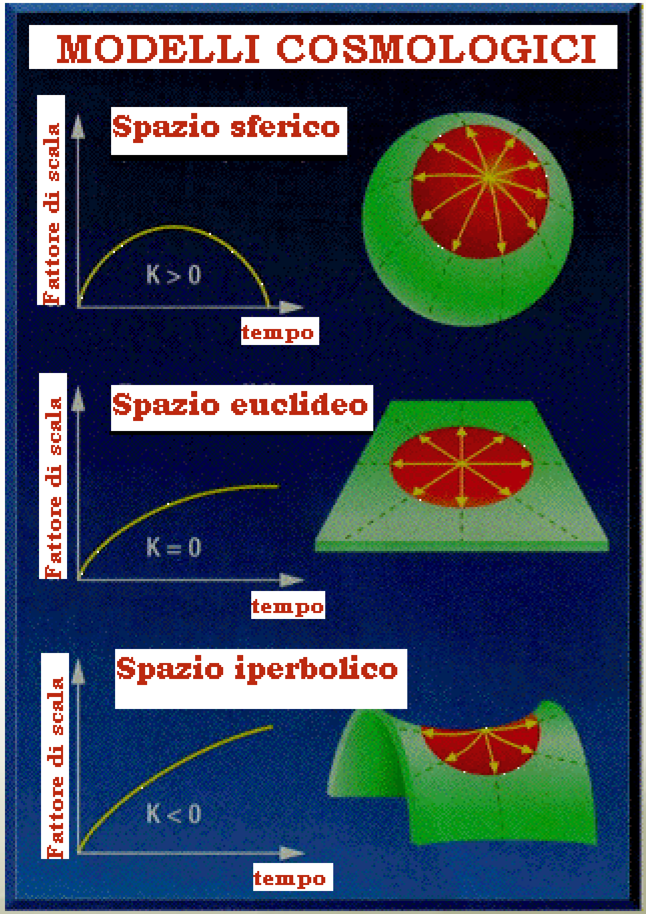
$\Omega_k < 0;$ $k = 1$ → **Close Universe**
Positive Curvature
Finite Volume

✖ Density = Critical density ($\Omega_{tot}=1$)

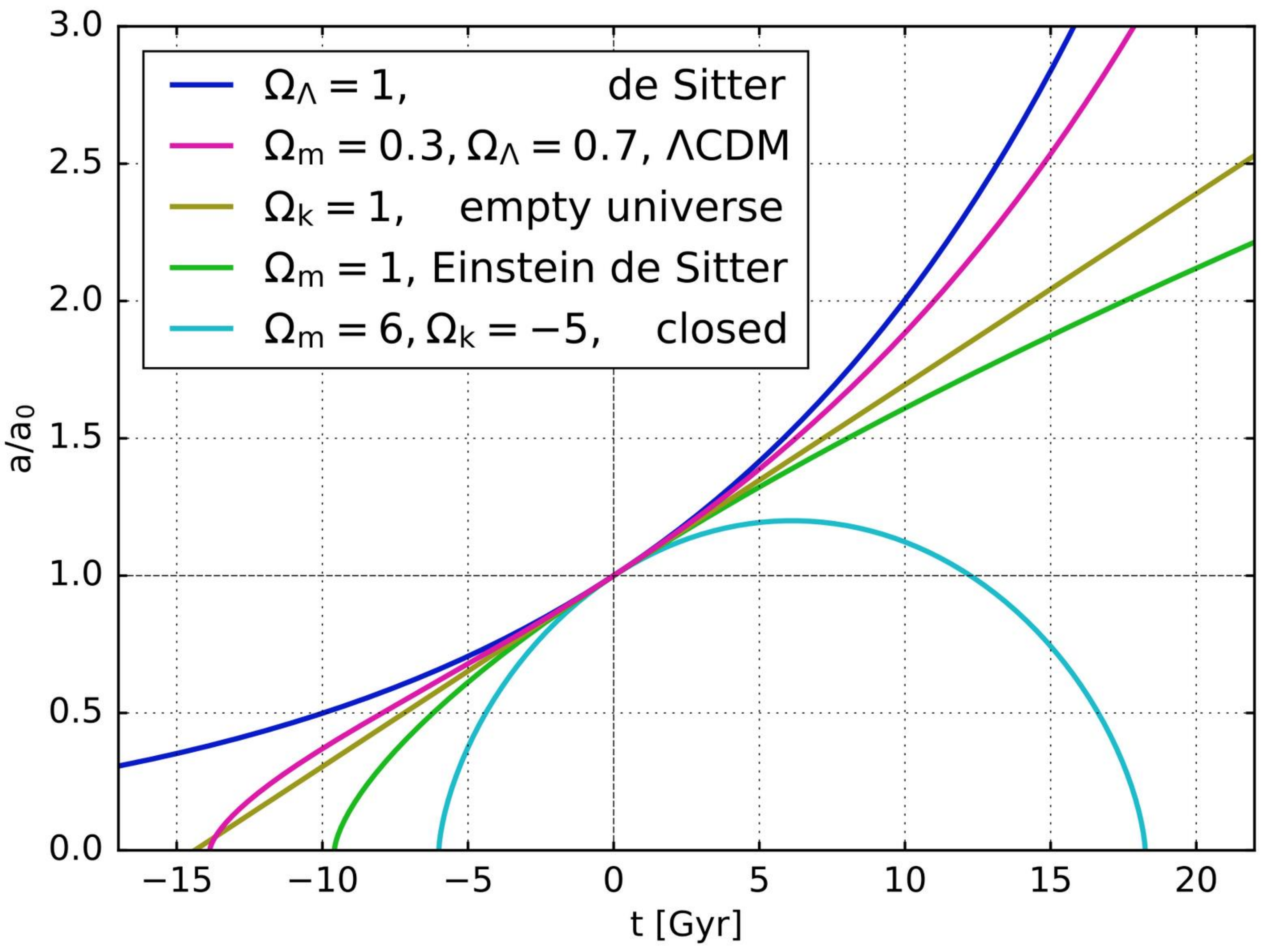
$\Omega_k = 0;$ $k = 0$ → **Flat Universe**
Zero Curvature
Infinite Volume

✖ Density < Critical density ($\Omega_{tot}<1$)

$\Omega_k > 0;$ $k = -1$ → **Open Universe**
Negative Curvature
Infinite Volume



The Universe can change size but NOT the geometry



$\Omega_\Lambda = 1$, de Sitter
$\Omega_m = 0.3, \Omega_\Lambda = 0.7$, ΛCDM
$\Omega_k = 1$, empty universe
$\Omega_m = 1$, Einstein de Sitter
$\Omega_m = 6, \Omega_k = -5$, closed
3.0
2.5
2.0
1.5
1.0
0.5
0.0
a/a_0
−15
−10
−5
0
5
10
15
20
t [Gyr]

The geometry of the Universe

- There is a third equation that can be derived from the Friedmann equation and the Acceleration equation, the Fluid equation (also called continuity equation):

$$\dot{\rho} + 3H(\rho + p) = 0$$

- The energy density is made up of different types of matter characterized by their equation of state:

$$p = w\rho$$

- At least three types of "fluids" are used to describe the energy content of the Universe

  Radiation, w=⅓

  non-relativistic matter, w=0

  vacuum energy, w=-1.

- Inserting the equation of state into the acceleration eqn into we see that:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(1+3w)$$

- Thus, a fluid with w<-1/3 (if dominant) can make the universe accelerate . As we shall see later on, other kinds of energy types with negative w have been suggested, these all fall under the general name "Dark Energy".

- The **third equation**, the **continuity equation**, is the consequence of the **first law of thermodynamics**
- $dU = TdS - pdV$
- Because of homogeneity and isotropy, the Universe is under an adiabatic expansion: $dS = 0$
- $U = \rho V \rightarrow dU = (\dot{\rho}V + \rho\dot{V})dt \rightarrow \dot{\rho}V + \rho\dot{V} = -p\dot{V} \rightarrow \dot{\rho} = -(\rho + p)\dot{V}/V$
- Expansion: $V \sim a^3 \rightarrow \frac{\dot{V}}{V} = \frac{d\ln V}{dt} = 3\frac{\dot{a}}{a} = 3H(t)$

$$\dot{\rho} + 3H(\rho + p) = 0$$

- It can also be obtained, apart from **energy conservation,** from the two Friedmann equations
- First Friedmann equation: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$
- $\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k + \frac{\Lambda a^2}{3}$
- Derivative wrt time: $2\dot{a}\ddot{a} = \frac{8\pi G}{3}\dot{\rho}a^2 + \frac{8\pi G}{3}\rho 2a\dot{a} + \frac{\Lambda}{3}2a\dot{a} \rightarrow \ddot{a} = \frac{4\pi G}{3\dot{a}}\dot{\rho}a^2 + \frac{8\pi G}{3}\rho a + \frac{\Lambda}{3}a$
- Second Friedmann equation: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \rightarrow \ddot{a} = -\frac{4\pi Ga}{3}(\rho + 3p) + \frac{\Lambda}{3}a$
- Subtracting: $0 = \frac{4\pi G}{3\dot{a}}\dot{\rho}a^2 + \frac{4\pi Ga}{3}(2\rho + \rho + 3p) \rightarrow \dot{\rho}\frac{a^2}{\dot{a}} + 3a(\rho + p) = 0$

$$\dot{\rho} + 3H(\rho + p) = 0$$

EoS of the fluid: $\boldsymbol{p = w\rho}$ (ρ is the energy density)

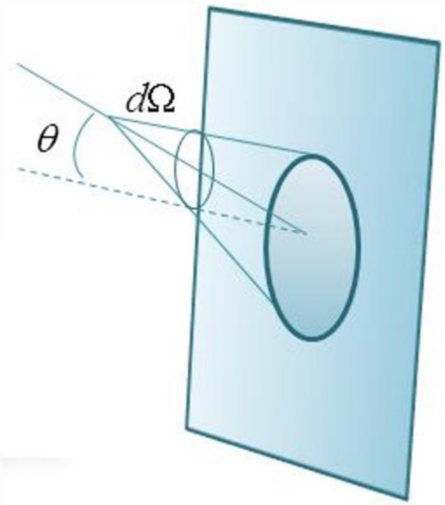
- Case of low-density gas of non-relativistic massive particles: $p = \frac{\rho}{\mu c^2} kT$ ($\rho \approx$ the mass density $\times$ c^2: kinetic energy is negligible; μ is the mass of the particle)

For a non-relativistic gas: $\frac{1}{2}\mu\langle v^2\rangle = \frac{3}{2}kT$ → $\boldsymbol{w} = \frac{kT}{\mu c^2} = \boldsymbol{\frac{1}{3} \cdot \frac{\langle v^2\rangle}{c^2} \ll 1}$

A gas of highly relativistic massive particles (with $\langle v^2\rangle \sim c^2$): → $\boldsymbol{w = \frac{1}{3}}$

A gas of mildly relativistic particles (with $0 < \langle v^2\rangle < c^2$): → $\boldsymbol{0 < w < \frac{1}{3}}$

- Case of radiation pressure



Hyp: monoenergetic photons, incident at angle ϑ: $E_\gamma = h\nu$; $p_\gamma = h\nu/c$

Number of photons incident on surface S for time unit: $\frac{u}{h\nu} \cdot c \cdot \frac{d\Omega}{4\pi} \cdot S\cos\vartheta$

where u is the energy density

The momentum transferred by a hit: $2 \cdot \frac{h\nu}{c} \cdot \cos\vartheta$

The total momentum (for time unit) transferred by all the hits: $F = \frac{dQ}{dt} = \left(\frac{u}{h\nu} \cdot c \cdot \frac{d\Omega}{4\pi} \cdot S\cos\vartheta\right)\left(2 \cdot \frac{h\nu}{c} \cdot \cos\vartheta\right) = 2uS\cos^2\vartheta \cdot \frac{d\Omega}{4\pi}$

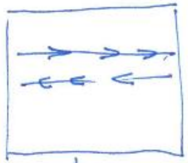
The radiation pressure is then: $p = \frac{F}{S} = \int 2u\cos^2\vartheta \cdot \frac{d\Omega}{4\pi} = \int_0^{\pi/2} 2u\cos^2\vartheta \cdot \frac{d\cos\vartheta}{2} = \frac{1}{3}u$ → $p = \frac{1}{3}u$ $\boldsymbol{w = \frac{1}{3}}$

- Case of dark energy

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} = \frac{8\pi G}{3}\left(\rho - \frac{3k}{8\pi G a^2} + \frac{\Lambda}{8\pi G}\right)$$ → $\rho_\Lambda = \frac{\Lambda}{8\pi G}$

If Λ constant: ρ_Λ constant

$\dot\rho + 3H(\rho + p) = 0$ → $0 + 3H(\rho + p) = 0$ → $p = -\rho_\Lambda$ $\boldsymbol{w = -1}$



A particle in a box

Non-relativistic case

$V_x = \frac{p_x}{m}$ $\quad \Delta p_x = 2p_x$

$T_{two\,hits} = \frac{2 L m}{p_x}$

$F = \frac{p_x^2}{m L_x}$

$P = \frac{\langle p^2 \rangle}{3 m V} = \frac{2}{3V}\left(\frac{1}{2} m \langle v^2 \rangle\right)$

$PV = \frac{2}{3}\,\frac{3}{2} kT = kT$ $\quad$ EOS

Relativistic case

$V_x = \frac{p_x}{E}$ $\quad \Delta p_x = 2p_x$

$T_{two\,hits} = \frac{2L_x}{V_x} = \frac{2EL_x}{p_x}$

$F = \frac{\Delta p_x}{\Delta t} = \frac{2p_x}{2EL_x} p_x = \frac{p_x^2}{E L_x}$

$P = \frac{F}{L_y L_y} = \frac{p_x^2}{EV}$ $\quad$ pressure

$\langle p^2 \rangle = \langle p_x^2 \rangle + \langle p_y^2 \rangle + \langle p_z^2 \rangle = 3 \langle p_x^2 \rangle$

$P_{tot} = g \int \frac{d\vec{p}}{(2\pi\hbar)^3} f(\vec{p}) \frac{p^2}{3E(p)}$

$E = \sqrt{p^2 + m^2}$

$$\rho = g \int \frac{d^3p}{(2\pi\hbar)^3} f \cdot E(p)$$

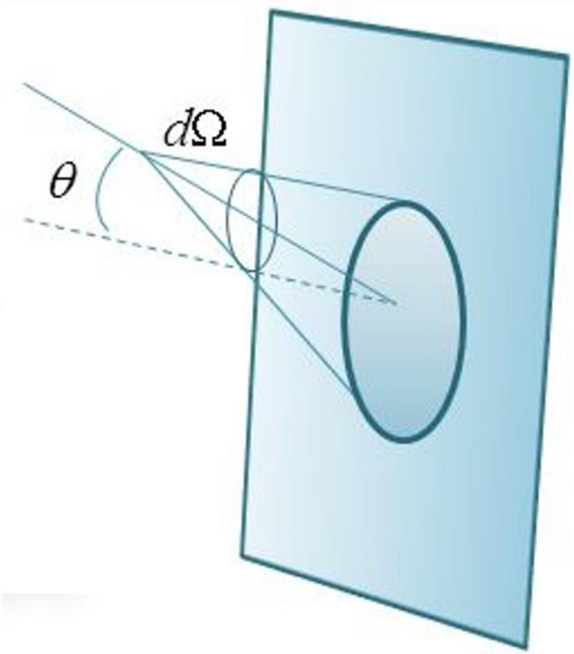
$$P = g \int \frac{d^3p}{(2\pi\hbar)^3} f \frac{p^2}{3E(p)}$$

f(p) is the Bose-Einstein or Fermi-Dirac distribution

Ultra-relativistic case $\quad E \sim P \quad \rightarrow \quad P = \frac{E}{3V} \quad \rightarrow w = \frac{1}{3}$

Radianza del corpo nero e pressione di radiazione

Se I_γ è la radianza (W/m²), l'energia (per unità di tempo) che incide sulla superficie S è:



$$W = \frac{d\Omega}{4\pi} \cdot I_\gamma \cdot S \cdot \cos\vartheta$$

Connessione tra radianza (W/m²) e densità di energia u: $u = \frac{I_\gamma}{c}$

La quantità di moto (per unità di tempo) è:

$$\frac{W}{c} = \frac{d\Omega}{4\pi} \cdot \frac{I_\gamma}{c} \cdot S \cdot \cos\vartheta = \frac{d\Omega}{4\pi} \cdot u \cdot S \cdot \cos\vartheta$$

Quantità di moto totale (per unità di tempo) trasferita nell'urto speculare:

$$2\frac{W}{c}\cos\vartheta = 2\frac{d\Omega}{4\pi} \cdot u \cdot S \cdot \cos\vartheta\cos\vartheta$$

La forza esercitata è:

$$F = \frac{dQ}{dt} = \int 2\frac{d\Omega}{4\pi} \cdot \cos^2\vartheta \cdot u \cdot S = uS\int_0^{\pi 72} \cos^2\vartheta \, d\cos\vartheta = uS\frac{1}{3}$$

The radiation pressure is then: $p = \frac{F}{S} = \frac{1}{3}u$ → $p = \frac{1}{3}u$ $\quad w = \frac{1}{3}$

The geometry of the Universe

From the fluid equation (or continuity equation) + the EoS, in the simple and easy case of only one fluid with *w* constant:

$$\dot{\rho} = -3(1+w)\rho\frac{\dot{a}}{a}$$

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

$$\frac{d}{dt}\ln\rho = -3(1+w)\frac{d}{dt}\ln a$$

$$\rho = \rho_0 a^{-3(1+w)}$$ → Implications….

Evolution: solutions

- Equation of State (EoS). Simple and useful example: $\boldsymbol{p = w\rho}$ with $\boldsymbol{w = const.}$
- $\boldsymbol{\rho = \rho_0 a^{-3(1+w)}}$
- First Friedmann equation, with $k = 0$ and $\Lambda = 0$: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
- $\dot{a}^2 = \frac{8\pi G}{3} a^2 \rho_0 a^{-3(1+w)} = \alpha^2 a^{-(1+3w)} \quad \rightarrow \quad \dot{a} = \alpha a^{-\frac{1+3w}{2}} \qquad \delta = -\frac{1+3w}{2}$
- $\frac{da}{a^\delta} = \alpha dt$
- If $\delta = 1 \quad (\boldsymbol{w = -1}) \rightarrow \quad d\ln a = \alpha dt \quad \rightarrow \quad \boldsymbol{a = a_0 e^{\alpha t}}$
- If $\delta \neq 1 \quad (\boldsymbol{w \neq -1}) \rightarrow \quad d\left(a^{1-\delta}\right) = \alpha(1-\delta)dt \quad \rightarrow \quad a^{1-\delta} = const \cdot t$

  $\rightarrow \quad \boldsymbol{a} = a_0 t^{1/(1-\delta)} = \boldsymbol{a_0 t^{2/3(1+w)}}$
- $H = \frac{\dot{a}}{a} = \frac{a_0 \gamma t^{\gamma-1}}{a_0 t^\gamma} = \frac{\gamma}{t} \quad (\boldsymbol{w \neq -1})$
- For $\boldsymbol{w = -1}$ $\quad \rho = \rho_0 a^{-3(1+w)} = const. = \rho_0 \quad \rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 + \frac{\Lambda}{3} \quad \rightarrow \quad H = const.$

Evolution: Solutions

Equation-of-state (simple & useful example): $p = w\rho \quad w = const.$

$$\rho = \rho_0 a^{-3(1+w)}$$

Non-relativistic matter ("dust"): $p = 0$

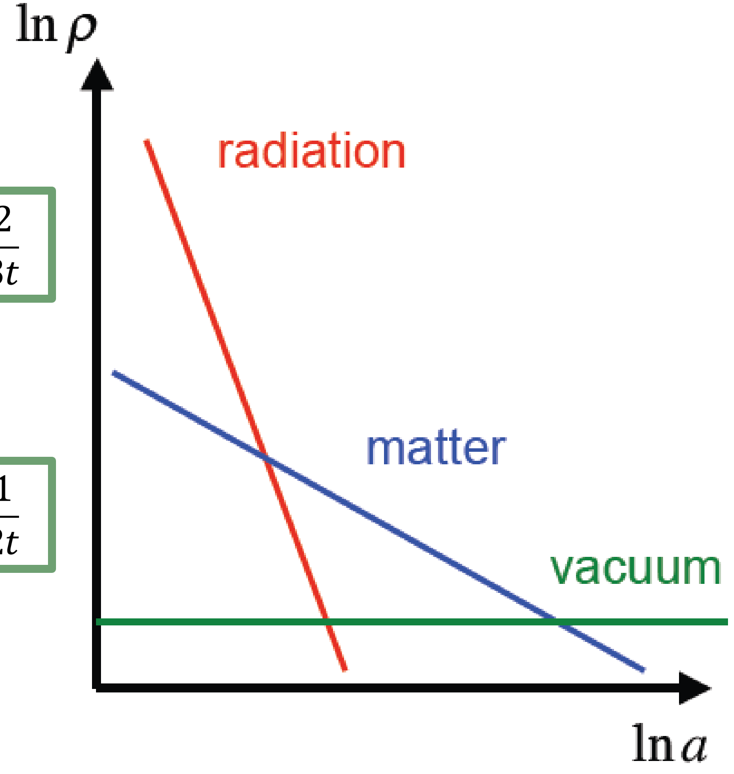
$$\rho = \rho_0 a^{-3} = \rho_0 (1+z)^3 \qquad a(t) \propto t^{2/3} \qquad H = \frac{2}{3t}$$

Radiation: $p = \frac{1}{3}\rho$

$$\rho = \rho_0 a^{-4} = \rho_0 (1+z)^4 \qquad a(t) \propto t^{1/2} \qquad H = \frac{1}{2t}$$

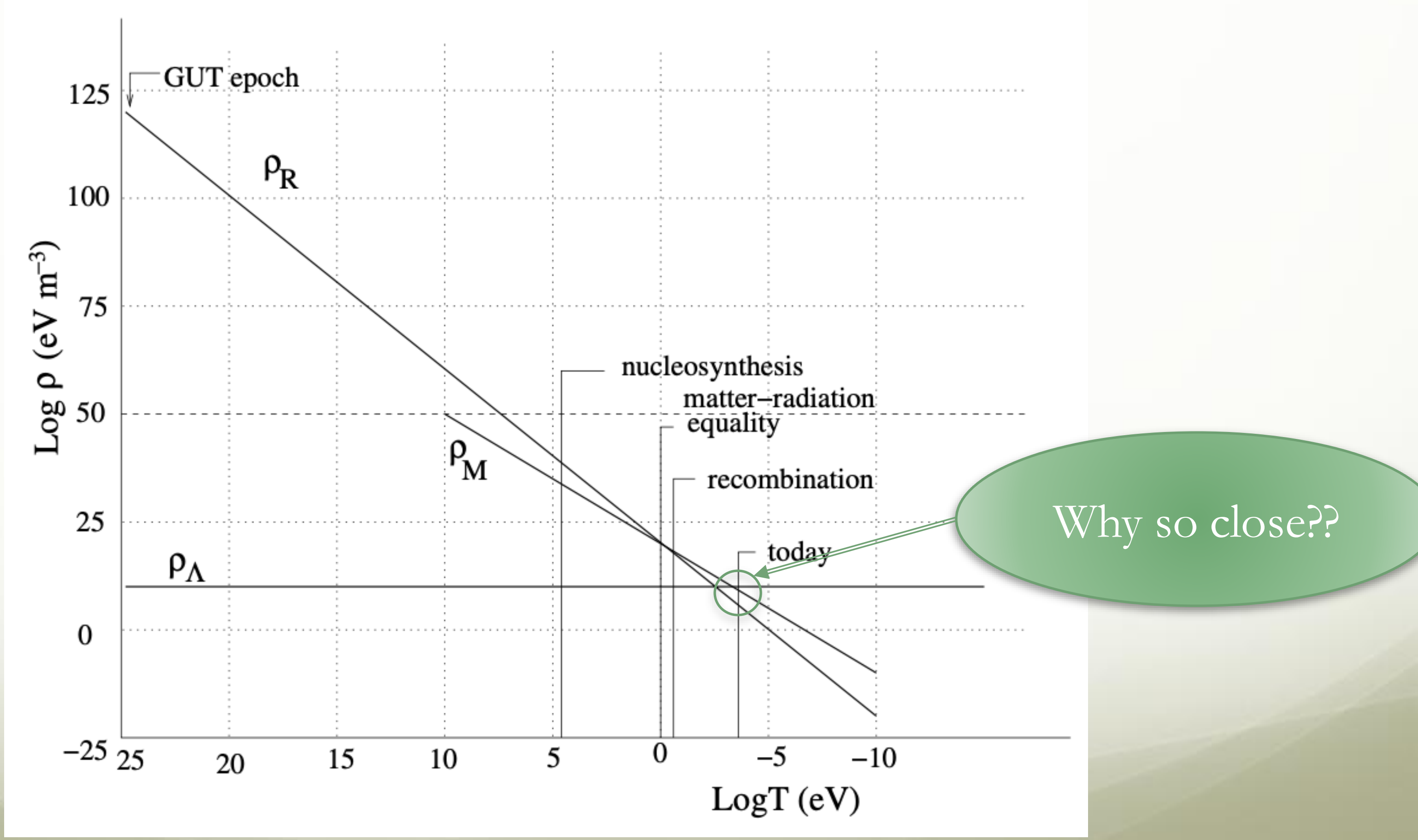
Vacuum energy: $p = -\rho$

$$\rho, H = const \qquad a(t) \propto e^{Ht} \qquad H = const.$$



The energy density of matter, radiation and vacuum (assumed constant) as a function of temperature. The temperature scale starts at the expected "grand unification" scale of $\sim 10^{16}$ GeV.

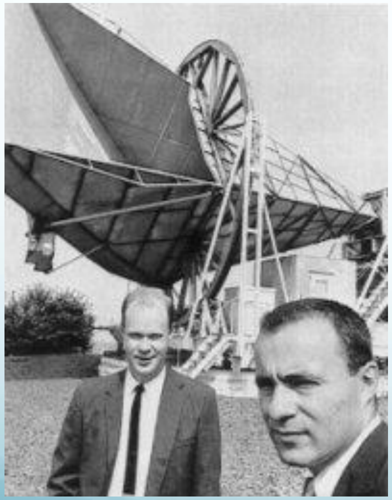
We suppose that the CDM particles have masses greater than ~ 10 GeV so the line of ϱ_M starts at 10 GeV.



... going to cosmological scale: the basis of the Big Bang model

- ✓ The expansion of the Universe – E. Hubble (1929)

- ✓ The abundance of light elements: H, He, Li – G. Gamow (1948)



- ✓ The Cosmic Microwave Background (CMB) – Penzias e Wilson (1965)

The detailed studies on these items through high-precision experimental tools allowed us to define the Concordance ΛCDM model.

The Cosmic Microwave Background (CMB)

Discovered by Penzias and Wilson (1965)

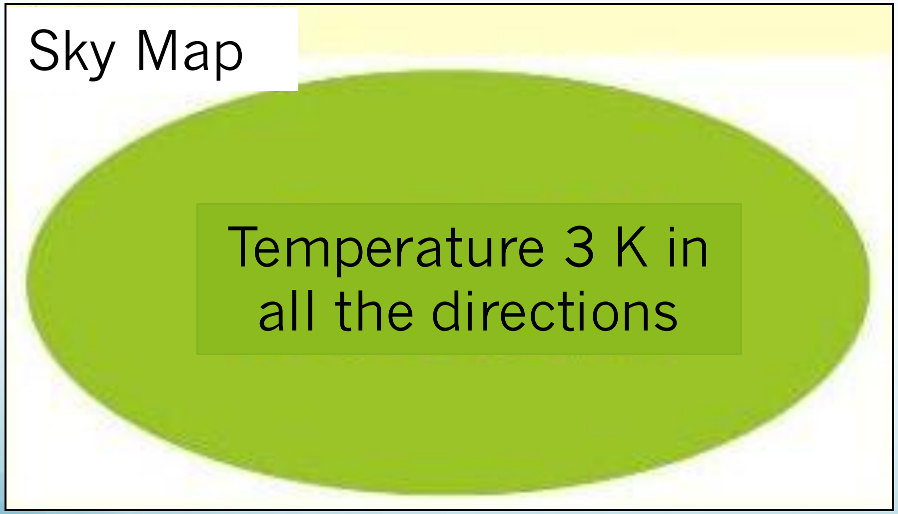
The measured temperature of the radiation was

3 K → -270 °C

it appears on the microwave band

Radio-Antenna used by Penzias and Wilson

UNIFORM distribution in the sky



No astrophysical source able to produce such a uniform radiation!!

Relic radiation of Big Bang

CMB was predicted by Gamow in 1946; detected and explained in 1965

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic ... polarized, and ... observed excess noise temperature is the one given by Dicke, R... d Wilkinson (1965) in a companion letter in this issue.

The total antenna temperature measured at ... 2.3° K is due to atmospheric absorption. The calc... ses in the antenna and back-lobe response is ...

The radiometer used in t... zias and Wilson 1965). It em... db) comparison switch, and a li... 1965). Measurements were m... put and the reference termina-ti... ometer were well matched so that a ro... existed throughout the measurement; thus err... ive temperature due to impedance mismatch can be n... in the measured value of the total antenna temperature is 0.3... y from uncertainty in the absolute calibration of the reference termi...

Cosmic "noise" discovered!

The ... ibution to the antenna temperature due to atmospheric absorption was obtained by recording the variation in antenna temperature with elevation angle and employing the secant law. The result, 2.3° ± 0.3° K, is in good agreement with published values (Hogg 1959; DeGrasse, Hogg, Ohm, and Scovil 1959; Ohm 1961).

The contribution to the antenna temperature from ohmic losses is computed to be 0.8° ± 0.4° K. In this calculation we have divided the antenna into three parts: (1) two ...

Note added in proof.—The highest frequency at which the background temperature of the sky had been measured previously was 404 Mc/s (Pauliny-Toth and Shakeshaft 1962), where a minimum temperature of 16° K was observed. Combining this value with our result, we find that the average spectrum of the background radiation over this frequency range can be no steeper than $\lambda^{0.7}$. This clearly eliminates the possibility that the radiation we observe is due to radio sources of types known to exist, since in this event, the spectrum would have to be very much steeper.

A. A. Penzias
R. W. Wilson

May 13, 1965
Bell Telephone Laboratories, Inc
Crawford Hill, Holmdel, New Jersey

COSMIC BLACK-BODY RADIATION*

One of the basic problems of cosmology is the singularity characteristic of the familiar cosmological solutions of Einstein's field equations. Also puzzling is the presence of matter in excess over antimatter in the universe, for baryons and leptons are thought to be conserved. Thus, in the framework of conventional theory we cannot understand the origin of matter or of the universe. We can distinguish three main attempts to deal with these problems.

1. The assumption of continuous creation (Bondi and Gold 1948; Hoyle 1948), which avoids the singularity by postulating a universe expanding for all time and a continuous but slow creation of new matter in the universe.

2. The assumption (Wheeler 1964) that the creation of new matter is intimately related to the existence of the singularity, and that the resolution of both paradoxes may be found in a proper quantum mechanical treatment of Einstein's field equations.

3. The assumption that the singularity results from a mathematical over-idealization,

* This research was supported in part by the National Science Foundation and the Office of Naval Research of the U.S. Navy.

high pressure, such as the zero-mass scalar, capable of speeding the universe through the period of helium formation. To have a closed space, an energy density of 2×10^{-29} gm/cm^3 is needed. Without a zero-mass scalar, or some other "hard" interaction, the energy could not be in the form of ordinary matter and may be presumed to be gravitational radiation (Wheeler 1958).

One other possibility for closing the universe, with matter providing the energy content of the universe, is the assumption that the universe contains a net electron-type neutrino abundance (in excess of antineutrinos) greatly larger than the nucleon abundance. In this case, if the neutrino abundance were so great that these neutrinos are degenerate, the degeneracy would have forced a negligible equilibrium neutron abundance in the early, highly contracted universe, thus removing the possibility of nuclear reactions leading to helium formation. However, the required ratio of lepton to baryon number must be $> 10^9$.

We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
Princeton, New Jersey

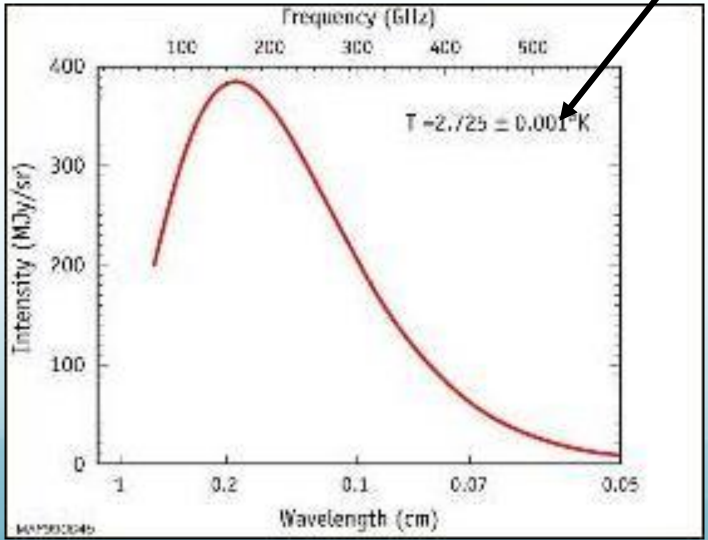
The oldest image of Universe

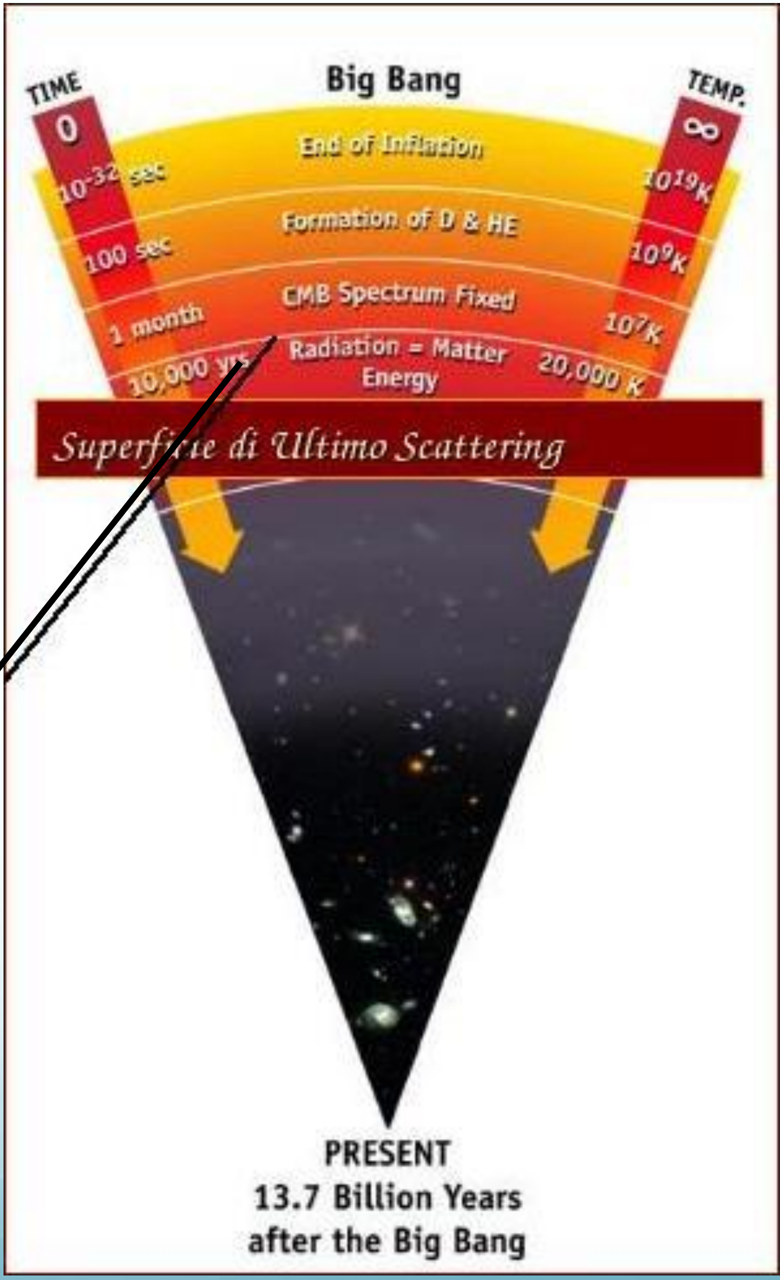
380.000 years after Big Bang the Universe becomes transparent for the light and the electromagnetic radiation starts to propagating "freely"

The relic radiation holds "memory" of the Universe when it was 0.003% of present age

COBE satellite (1992)

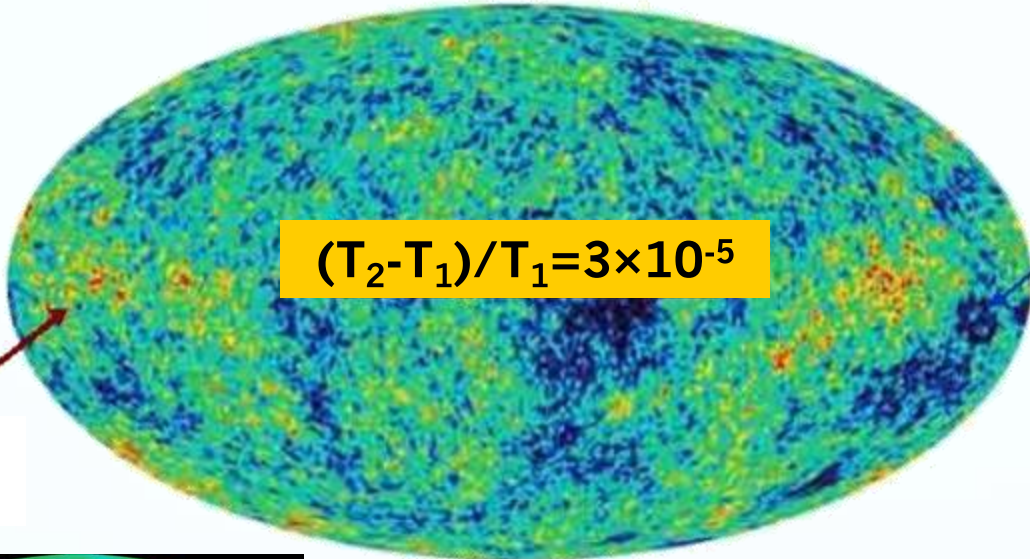
CMB spectrum





Is the CMB really uniform?
Fluctuation of CMB: the galaxies' seeds

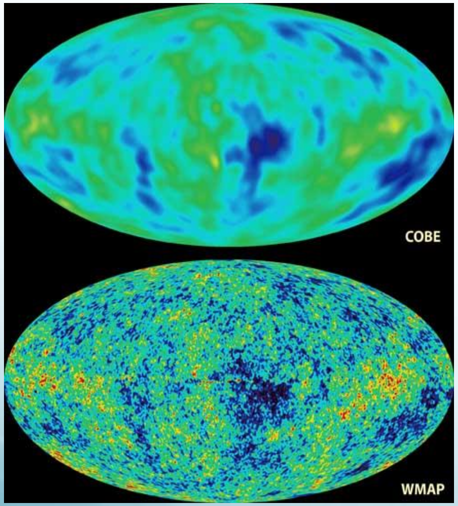
"cold" regions



"hot" regions

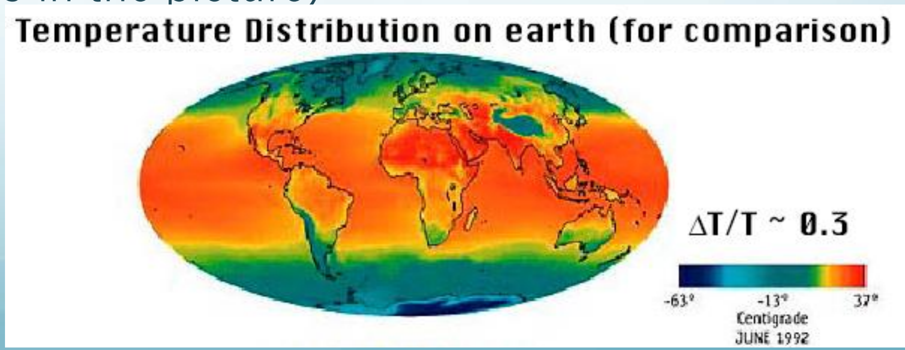
Sky map of the Universe 380000 years after the Big Bang.

WMAP



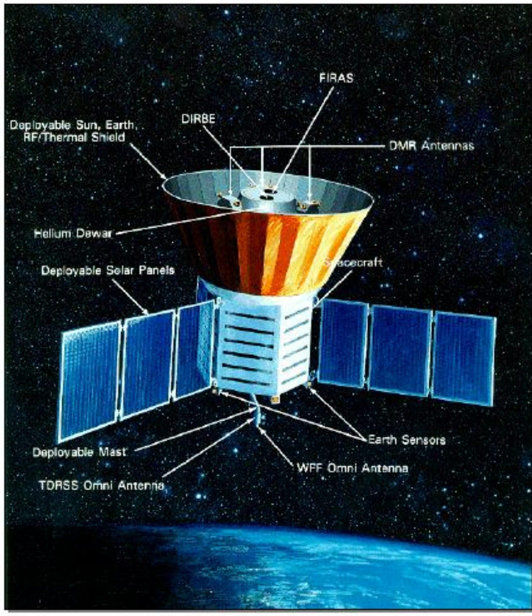
The temperature fluctuations are the trace of the density fluctuations of matter: the galaxies' seeds that will grow up through the gravitational amplification

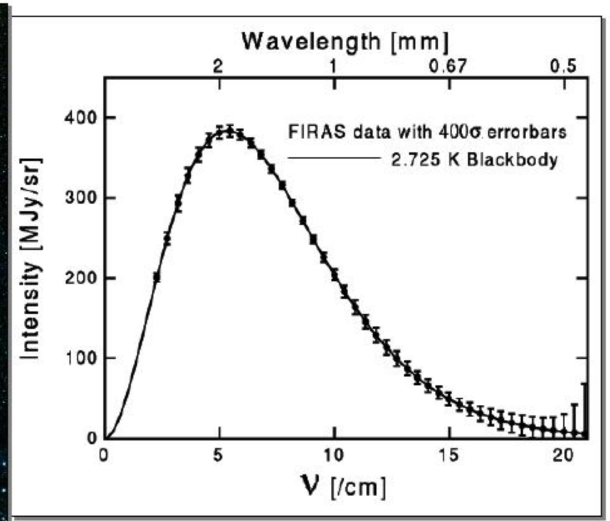
On 1992, COBE detected by first small temperature variations (different colors in the picture)



On 2003 WMAP and on 2013 PLANCK focused the imagine measured by COBE

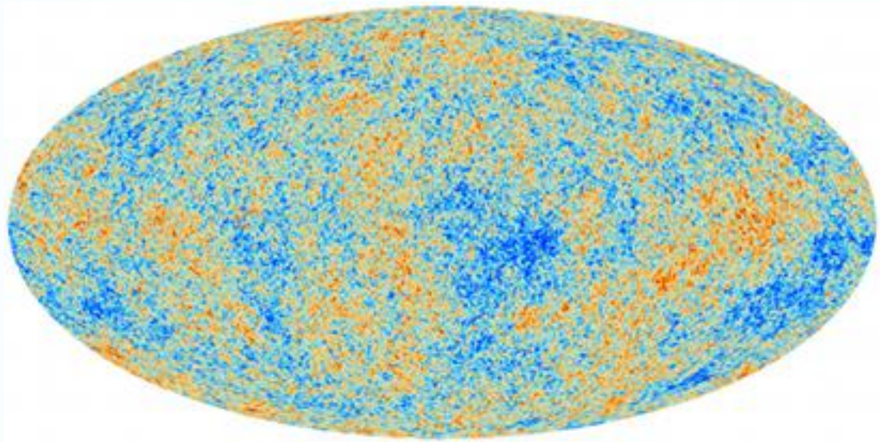
The CMB –
A perfect black body spectrum

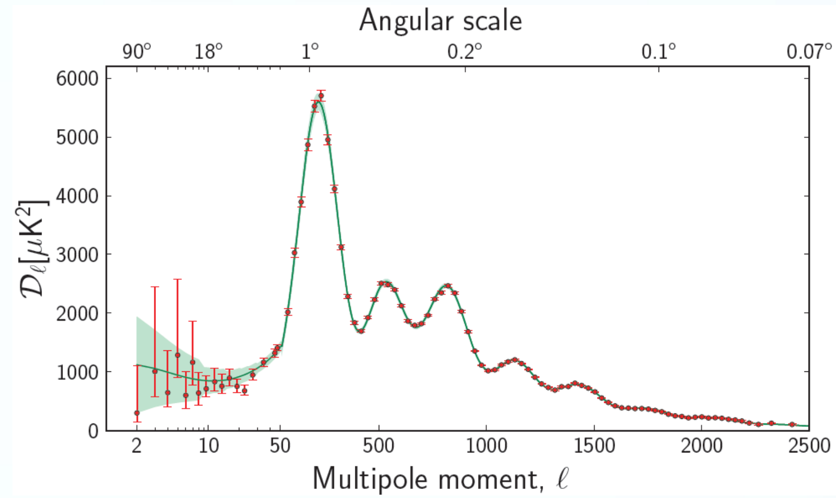


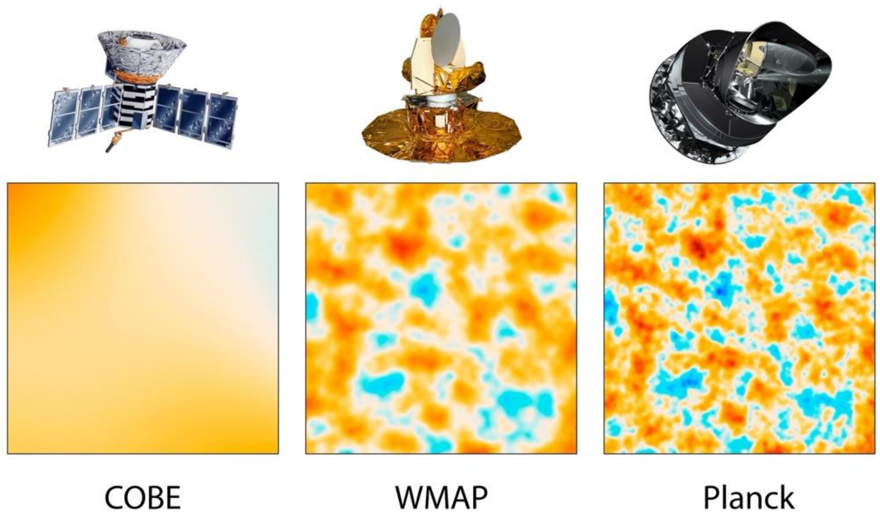


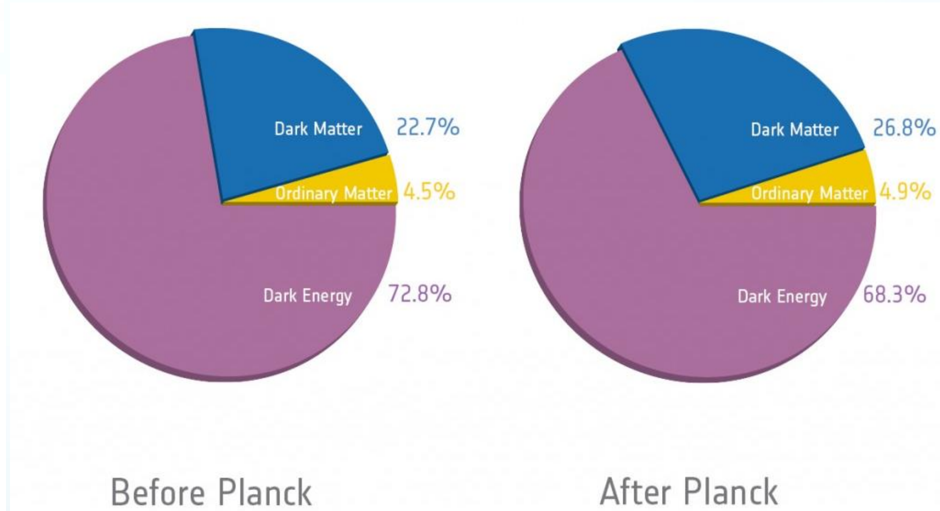
J. Mather and G. Smoot, Nobel laureates on 2006

Planck 2013 result

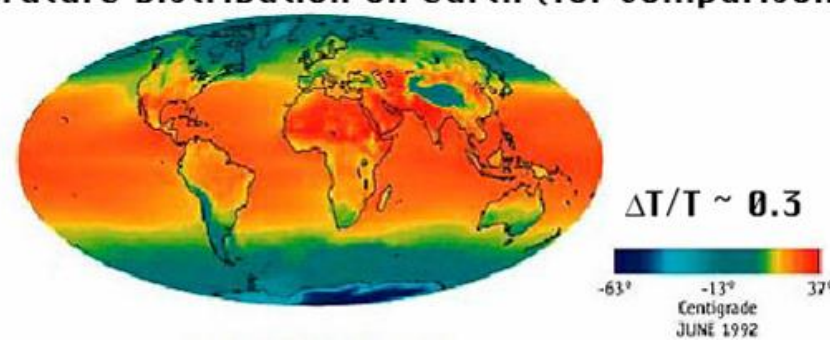








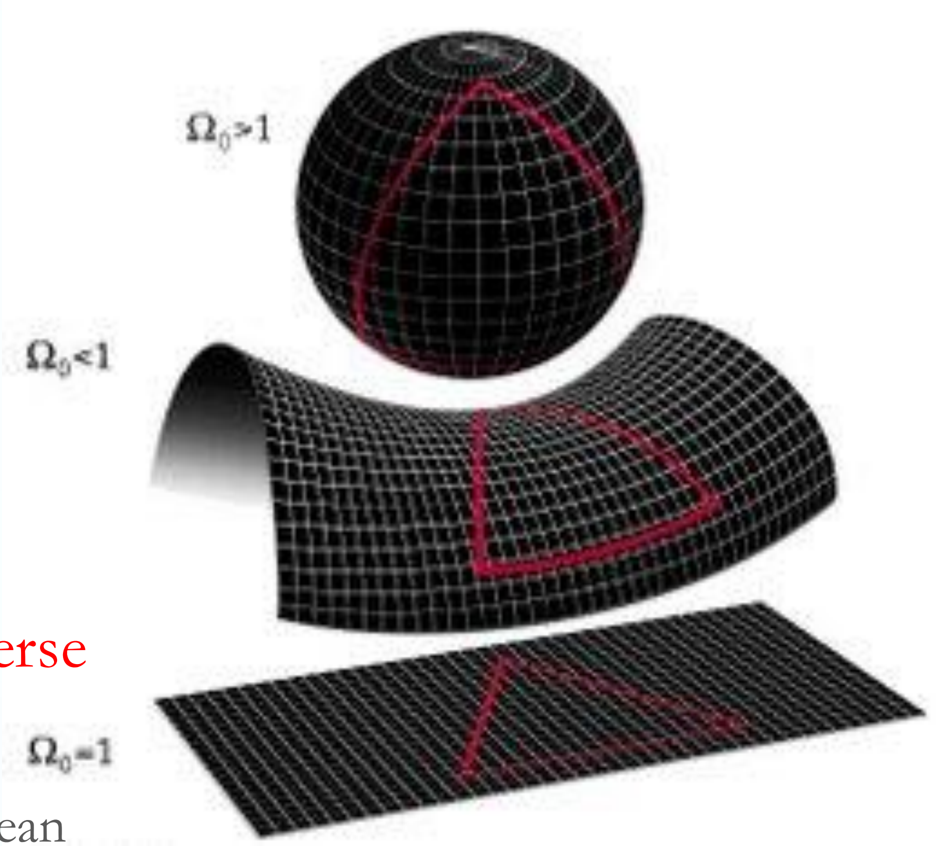
Temperature Distribution on earth (for comparison)



J. Mather and G. Smoot,
Nobel laureates on 2006

The anisotropy of CMB

- can be used to measure the geometry of the Universe
- can be used to estimate the baryon density



Measuring the geometry of the Universe

Distances are no longer trivial like in Euclidean geometry

$\Longrightarrow$ they can be used to measure the geometry!

- Angular diameter distance: it preserves the Euclidean relation between sizes and angles
- Luminosity distance: it preserves the Euclidean relation between luminosities and fluxes

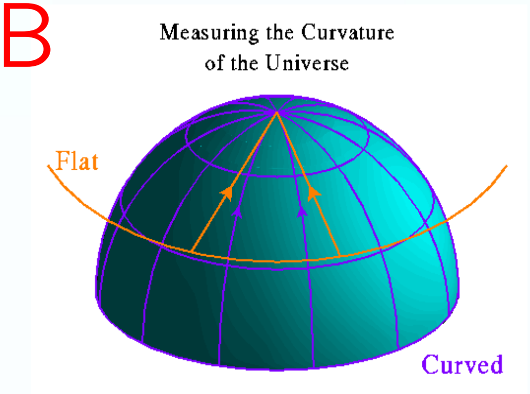
Ω = density/critical density

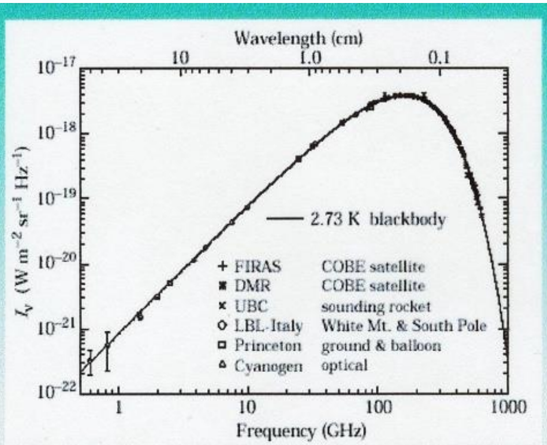
6 atoms of H/m^3

Necessity of 'standard' candles or rulers to measure the geometry!

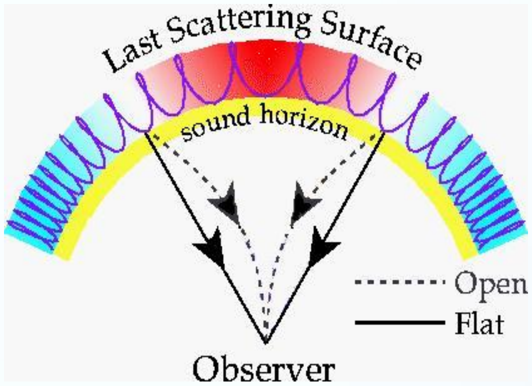
Anisotropy of CMB

Measuring the geometry of the Universe: the first acoustic peak in the CMB power spectrum is related to the sound horizon, the corresponding angular scale can be used to measure the geometry (angular diameter distance)





Ω value and other cosmological parameters depend on the anisotropy of **CMB vs angular scale**

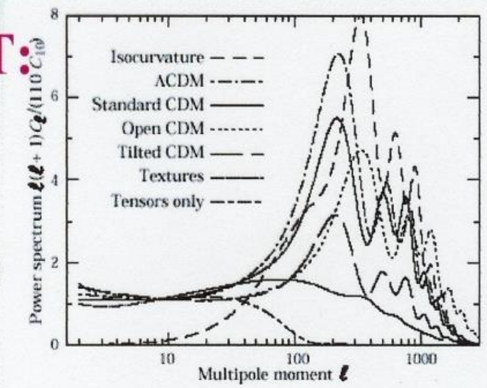


Temperature fluctuations ΔT/T:

angular correlation function

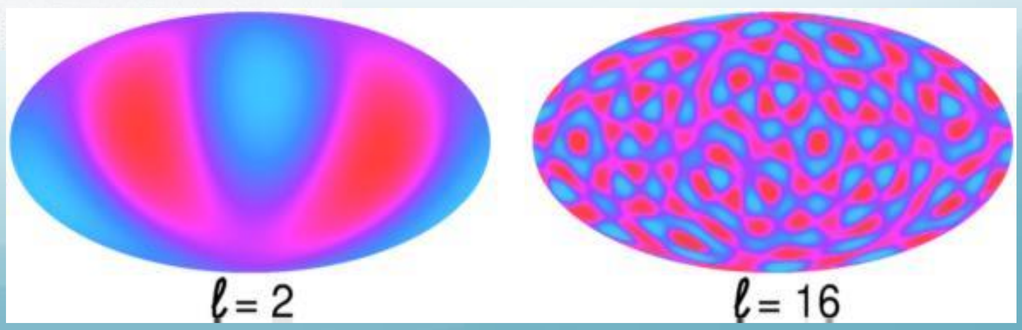
$$\left\langle \frac{\Delta T}{T}(\hat{n}) \frac{\Delta T}{T}(\hat{n}') \right\rangle = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) C_l P_l(\cos\theta)$$

$$\cos\theta = \hat{n} \cdot \hat{n}'$$

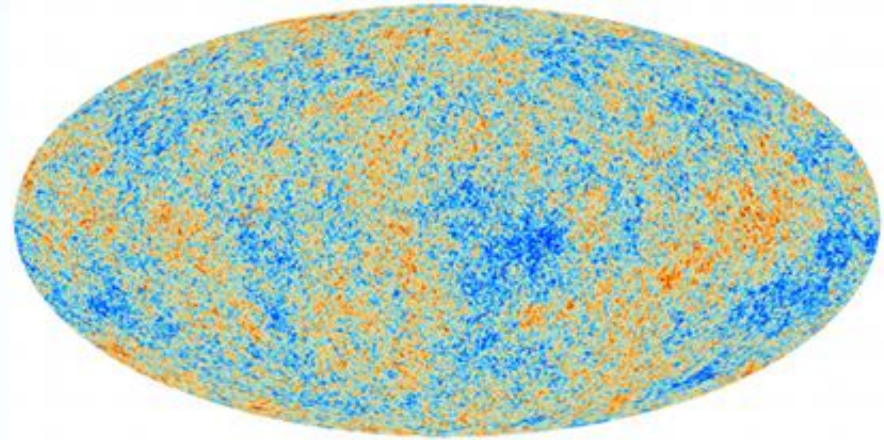


$$l \approx \frac{180^o}{\vartheta}; \qquad \vartheta_h \approx 1^o \sqrt{\Omega_o}$$

If Ω_o=1 a peak at l ~ 200 is expected

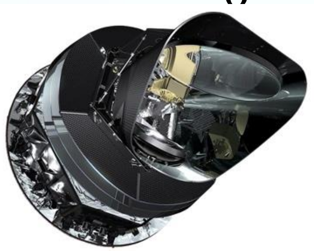


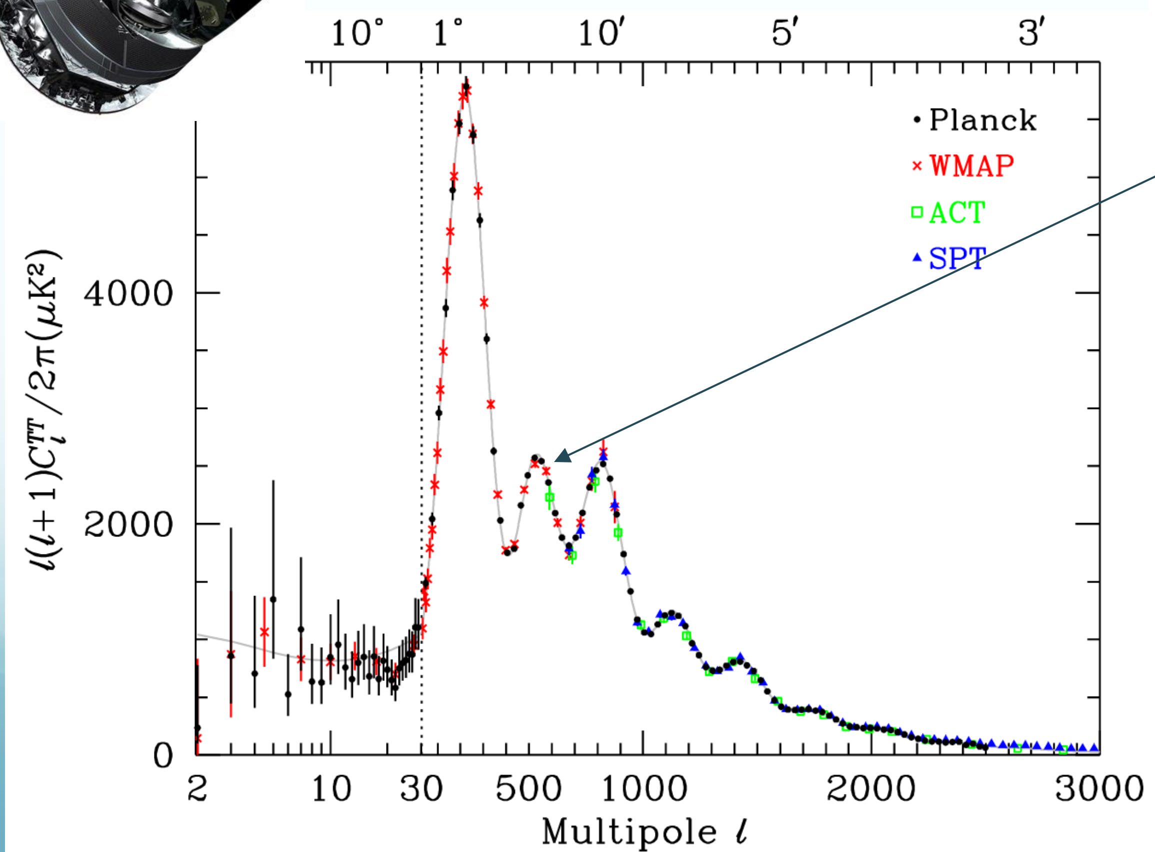
“Precision” Cosmology

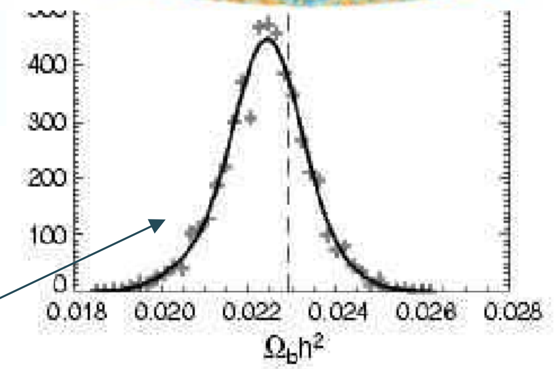


The Universe is flat

$$W_0 = 1.0002 \pm 0.0026$$







… and the baryons are “too few” (…and we already know it...)

$\Omega_b \sim 4\%$

$\Omega_\nu < 1\ \%$

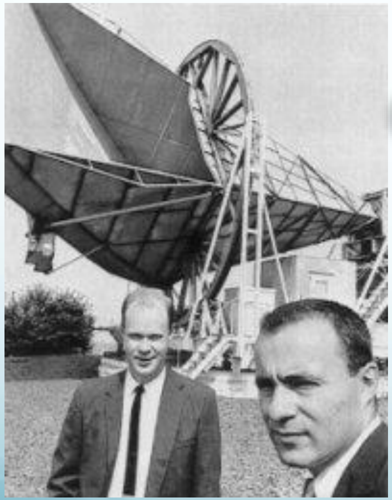
Ω = density/critical density

6 atoms of H/m^3

... going to cosmological scale: the basis of the Big Bang model

- ✓ The expansion of the Universe – E. Hubble (1929)

- ✓ The abundance of light elements: H, He, Li – G. Gamow (1948)



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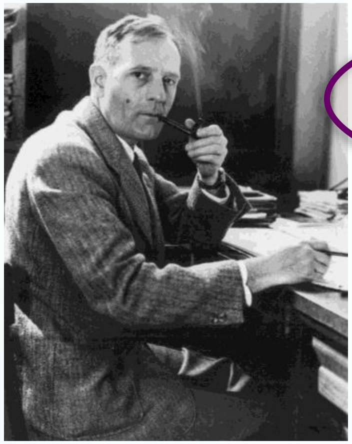

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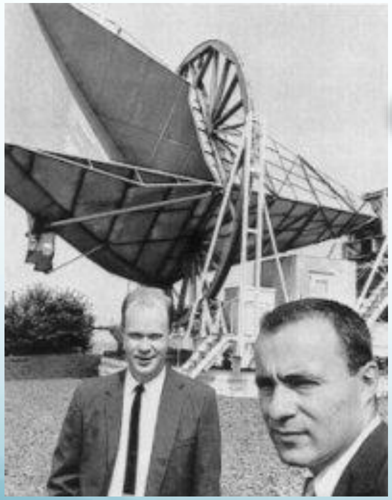

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The expansion of the Universe

- ✓ In 1917 the Universe was considered as static and composed only by our Galaxy and by the empty spaces around it.
- ✓ In twenties, E. Hubble through the new Mount Wilson telescope of 2.5 m measured:

M31- Andromeda

The Milky Way

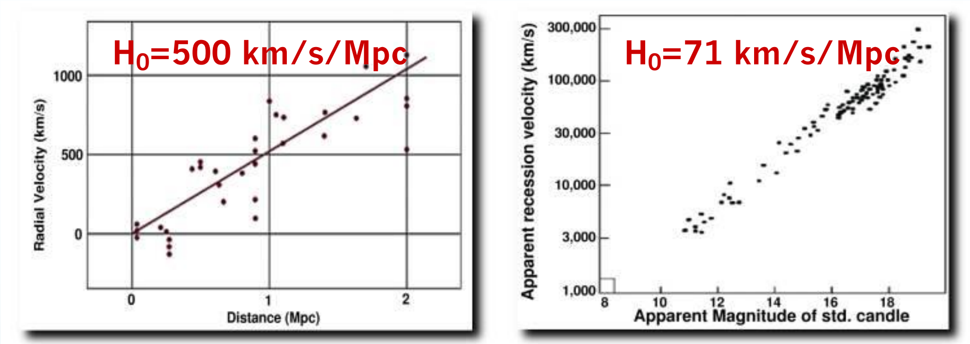
- ☑ the distances of some nebulas, proving that they were external galaxies
- ☑ their recession velocities

The galaxies move with recession velocities proportional to the distance $v = H_0 d$

General expansion of time-space:

The galaxies do not go away in the space, but it is the space moving, dragging the galaxies

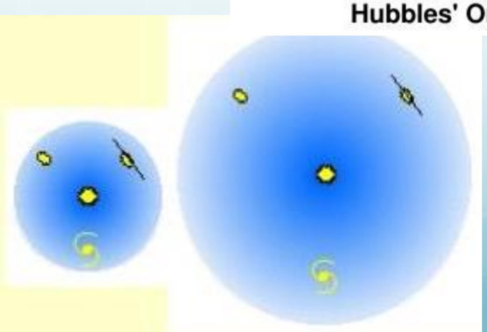
The physical sizes of galaxies do not change → the gravity "segregates" them through the expansion



Hubbles' Original Data

Modern Data

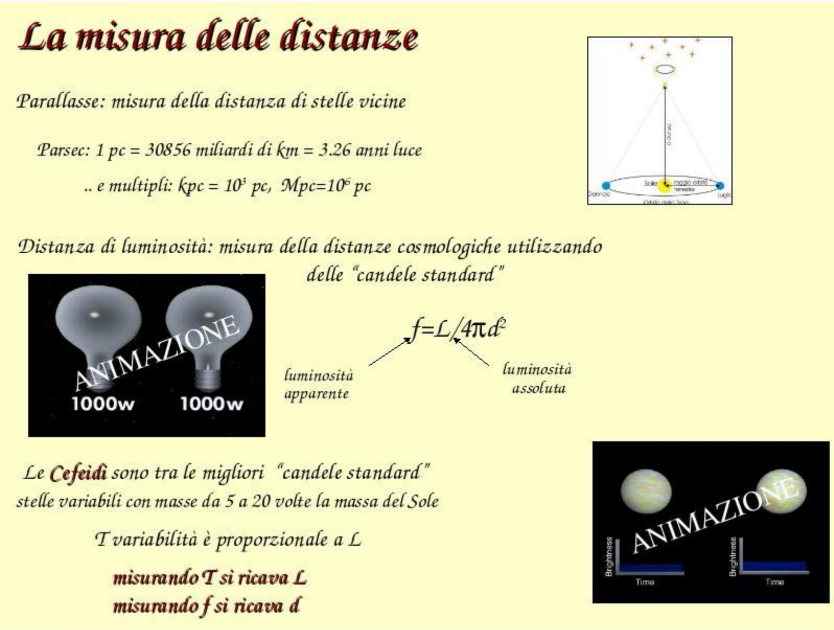
The effect is observable over intergalactic scale: the closest star to the Sun (Proxima Centauri), 4.22 light-years, moves away due to the only expansion with a velocity of 10 cm/s !!



Expansion without a centre:

An observer everywhere in the space sees the same expansion → the only expansion law compatible with the Cosmological Principle

The measurement of the distance



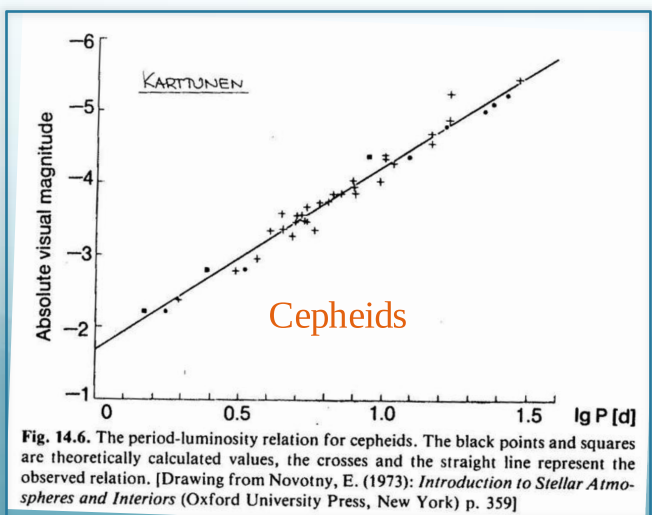
Apparent (m) vs absolute (M) magnitude

$$m = -2.5\log_{10} F + C$$

$$M - m = -2.5\log_{10} \frac{Fd^2}{F(10\,pc)^2}$$

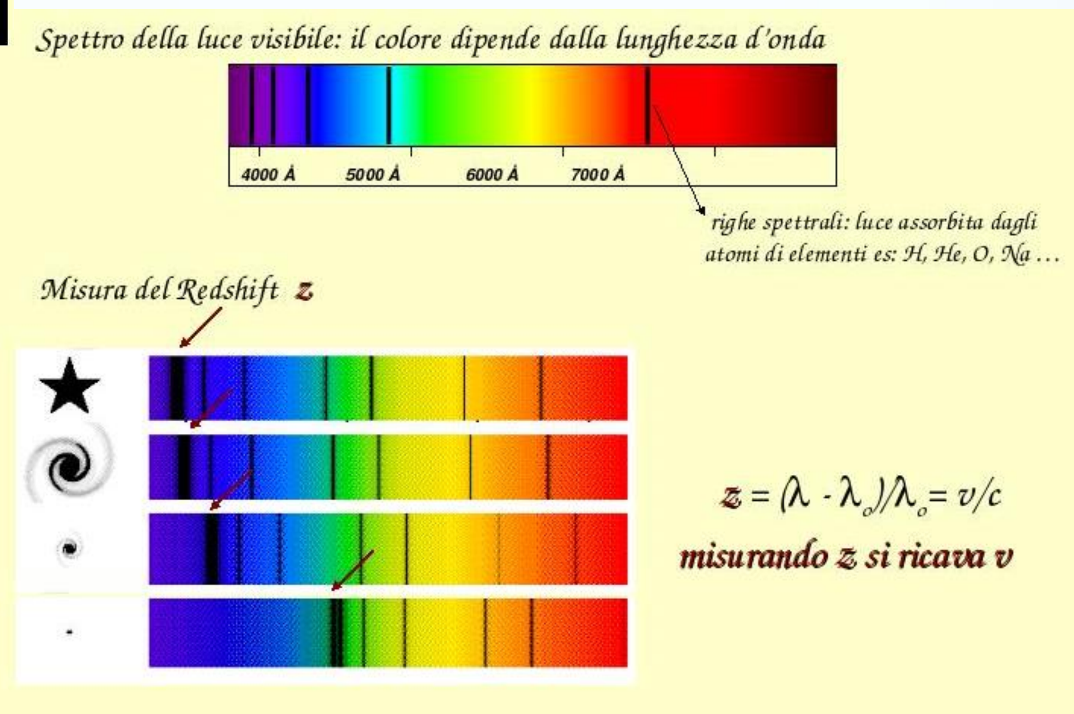
$$M - m = -5\log_{10} \frac{d}{10\,pc}$$

$$M = A + B\log_{10}(T)$$



The measurement of the velocity

Redshift

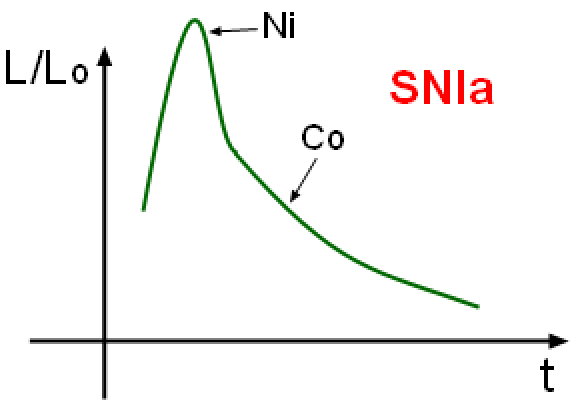


How to measure longer distance?
We need standard candles

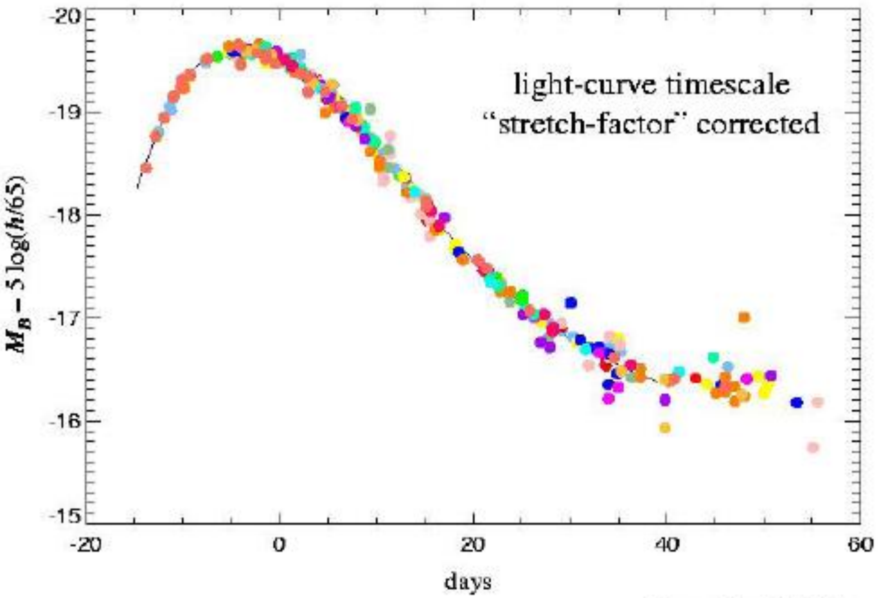
Supernovae Type 1A

- When a slowly-rotating**, carbon-oxygen white dwarf** accretes matter from a **companion**, it cannot exceed the Chandrasekhar limit of about 1.38 solar masses, beyond which it would no longer be able to support its weight through electron degeneracy pressure and begin to collapse.

- The typical visual absolute magnitude of Type Ia supernovae is $\mathbf{M_v = -19.3}$ (≈ 5 billion times brighter than the Sun), with little variation.



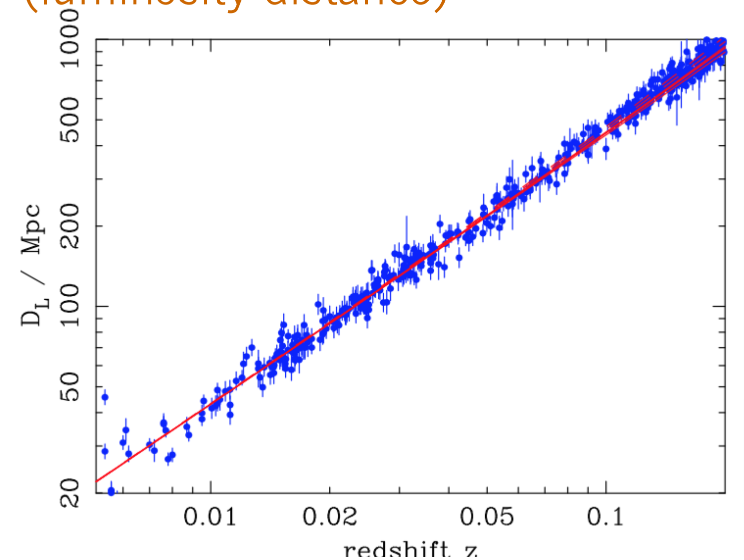
Light curve

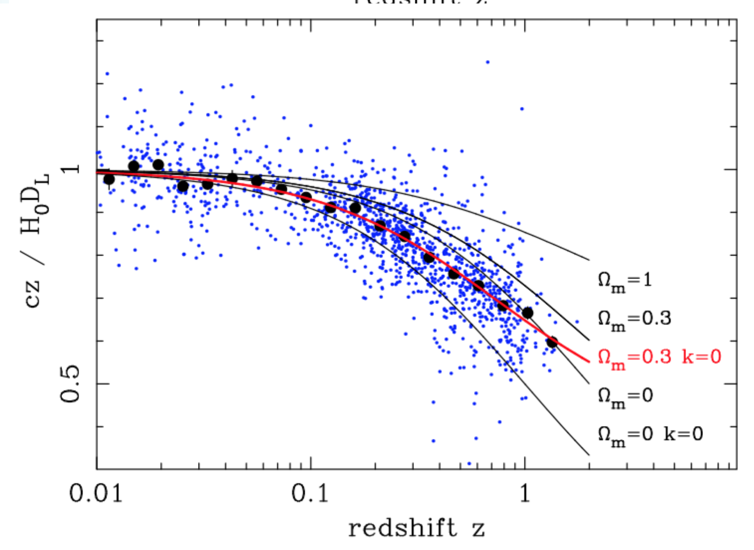


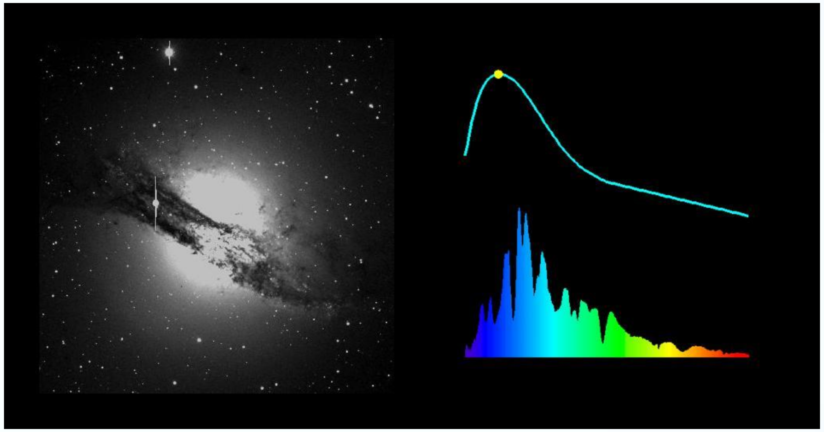
The similarity in the absolute luminosity profiles of nearly all known Type Ia supernovae has led to their use as a **standard candle** in extragalactic astronomy. The cause of this uniformity in the luminosity curve is still an **open question**.

"Measuring" the Universe

The observed luminosities of a special subclass of **supernovae** (type 1A) can be used as standard candles up to large distances (luminosity distance)



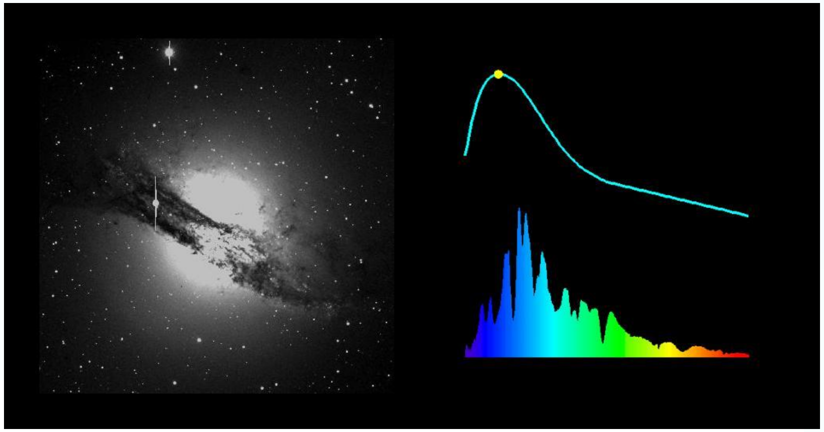




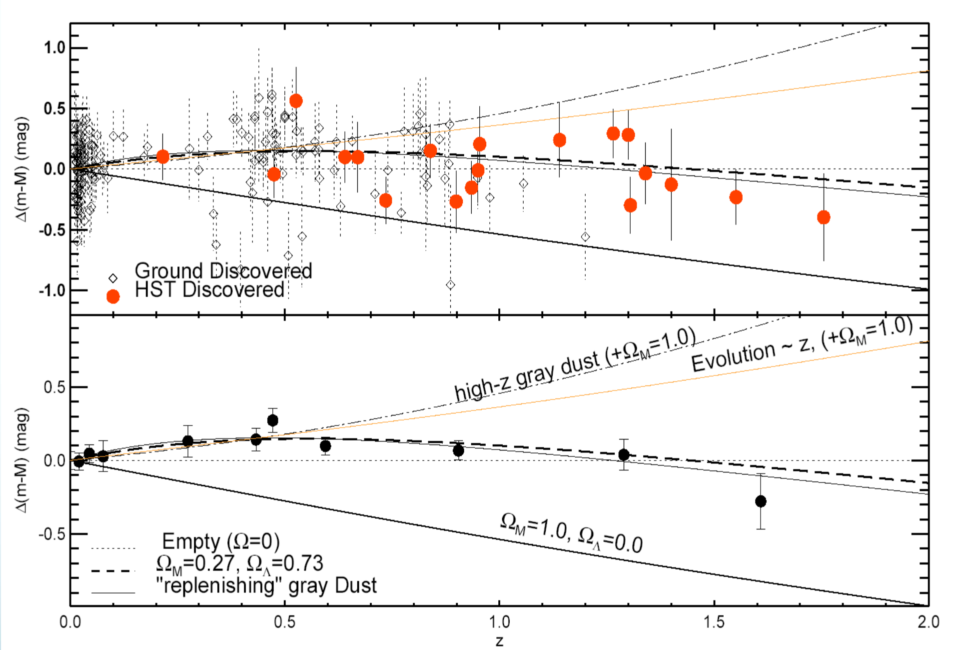
- The type 1a supernova Hubble diagram, based on over 1200 publicly available supernova distance estimates.
- The first panel shows that for $z \ll 1$ the large-scale Hubble flow is indeed linear and uniform
- The second panel shows an expanded scale, with the linear trend divided out, and with the redshift range extended to show how the Hubble law becomes nonlinear. ($\Omega_r = 0$ is assumed)
- Larger points with errors show median values in redshift bins. Comparison with the prediction of Friedmann models appears to favor a vacuum-dominated Universe.

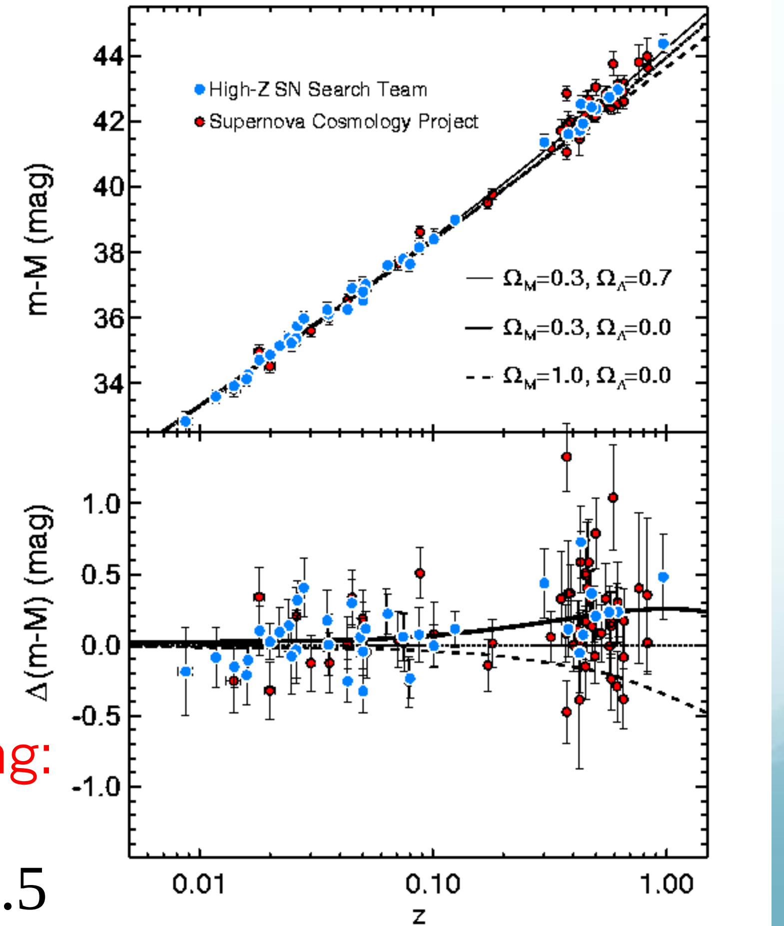
The expansion of the universe is accelerating

Measuring the geometry



The observed luminosities of a special subclass of **supernovae** (type 1A) can be used as **standard candles** up to large distances (luminosity distance)





The expansion of the universe is accelerating:

S. Perlmutter, B.P. Schmidt and A.G. Riess, Nobel laureates on 2011

$$\frac{1}{2}\Omega_m - \Omega_{vac} \approx -0.5$$

A cosmological constant?

Quantum gravity would predict its value to be 10^{120} times the observed value, perhaps it could be zero only in the presence of an unknown symmetry.
A vacuum energy? Does it evolve with time? A quintessence field?

The dark energy? A mystery

Cosmology

About it :
1) It should emit/absorb no light
2) It should have negative pressure, with magnitude comparable to its energy density in order to produce accelerated expansion
3) It should be "approximately" homogeneous
4) It should not interfere with production of structure but it could decide the future of Universe

Particle physics

A direct remnant of string theory ??

Are dark matter and dark energy connected through axion physics?

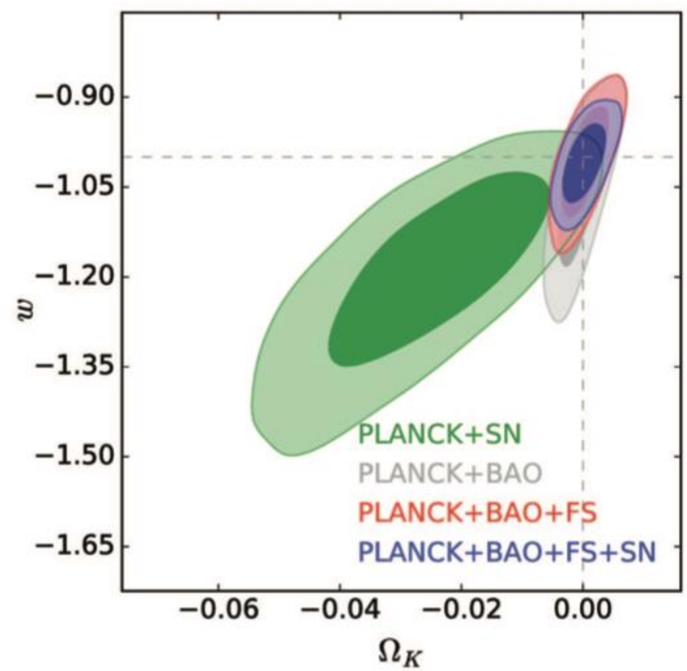
Is there a case of "vacuum energy" or "quintessence" in particle physics?

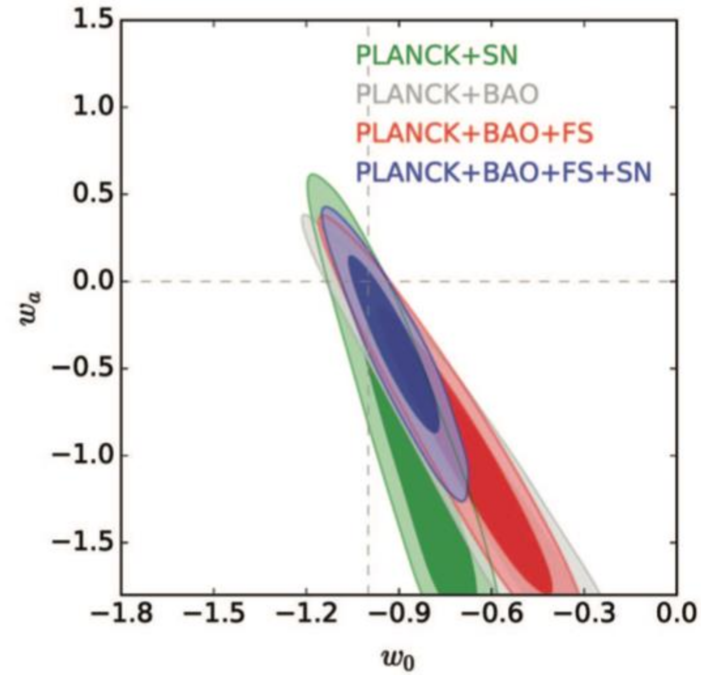
If elementary particles could couple to quintessence field , there could be exotic signatures detectable at accelerator and by astrophysical experiments

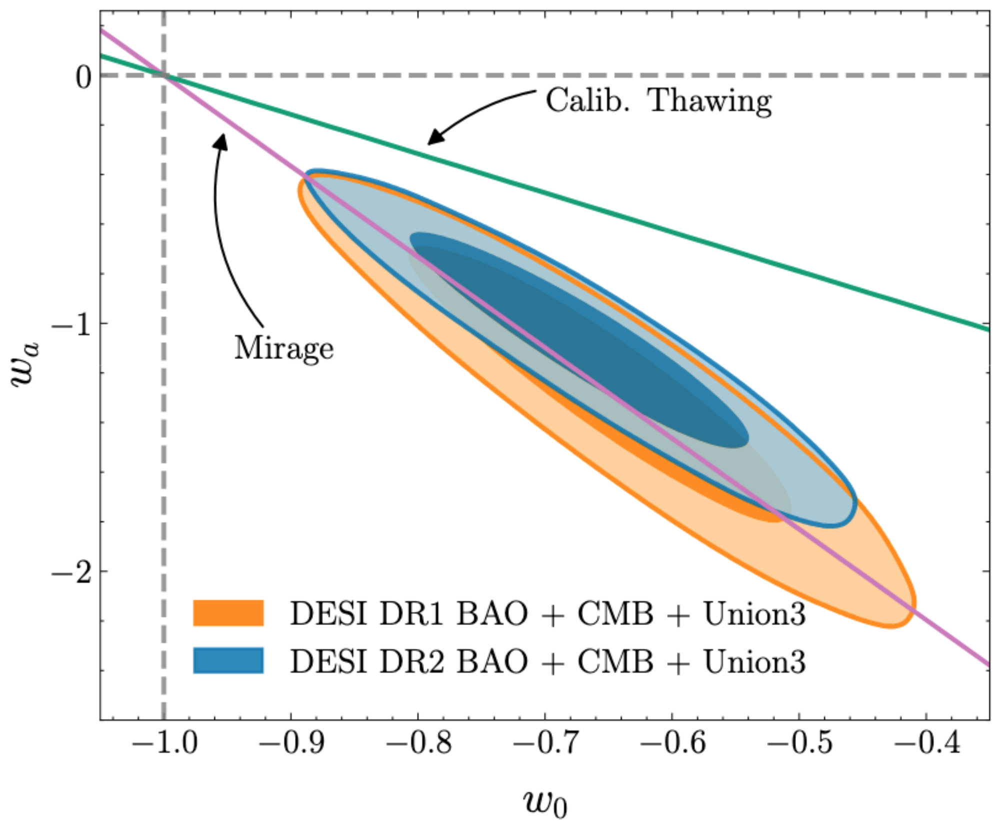
- Models for Dark Energy
- EoS: $p = w\rho$
- Top: constant w and non-zero space curvature $\Omega_K = 1 - \Omega_{tot}$
- Bottom: flat Universe and evolving EoS with $w(a) = w_0 + w_a(1 - a(t))$

- Present values: $w = -1.01 \pm 0.04$

Legenda:
- PLANCK: data on CMB
- SN: Supernova
- BAO: Baryonic Acoustic oscillations
- FS: full shape of redshift-space galaxy clustering



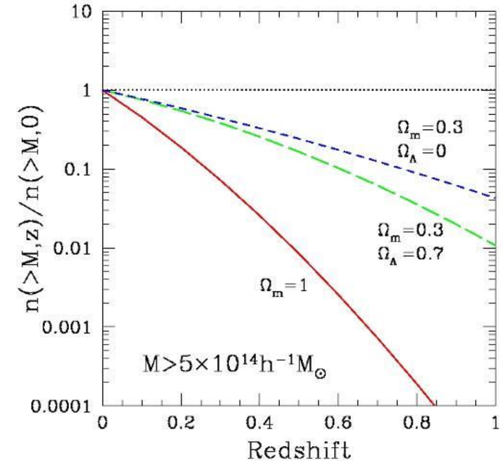




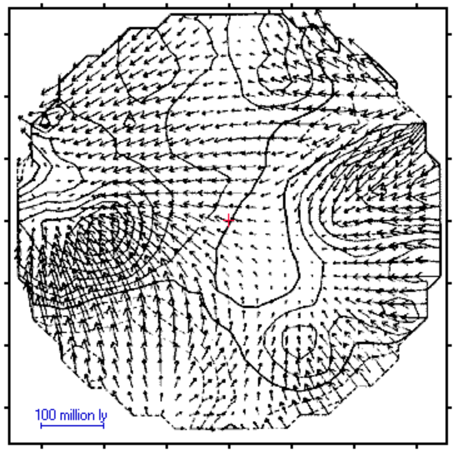
Calib. Thawing
Mirage
DESI DR1 BAO + CMB + Union3
DESI DR2 BAO + CMB + Union3
w_a
w_0
0
−1
−2
−1.0
−0.9
−0.8
−0.7
−0.6
−0.5
−0.4

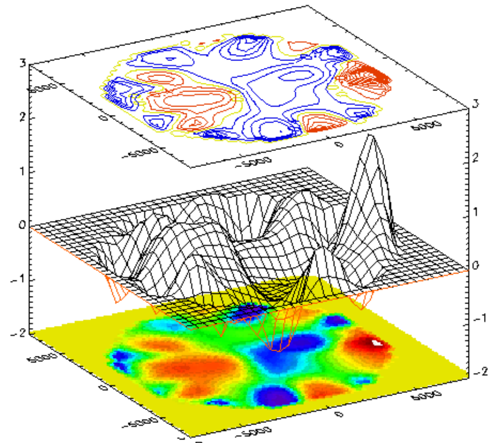
Gravitational probes: a summary

- **Galaxy clusters:** the evolution of their abundance is produced by **gravitational instability**

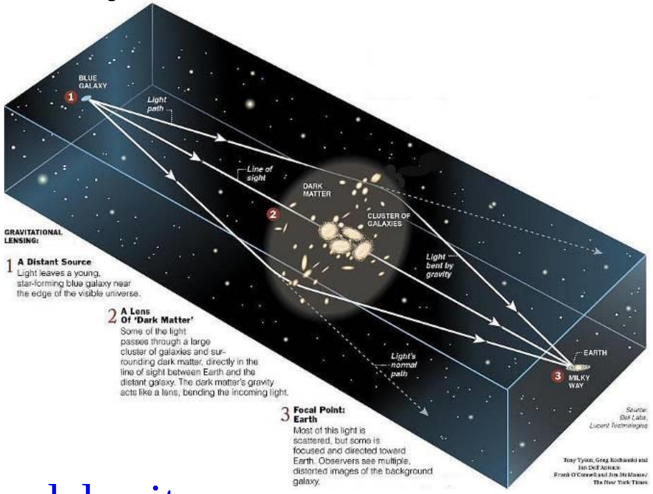


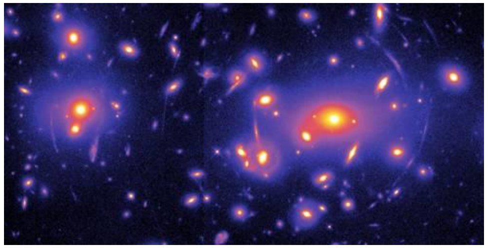
- **Cosmic flows:** peculiar motions are originated by the matter distribution





- **Gravitational lensing:** the light distortion is produced by the matter distribution



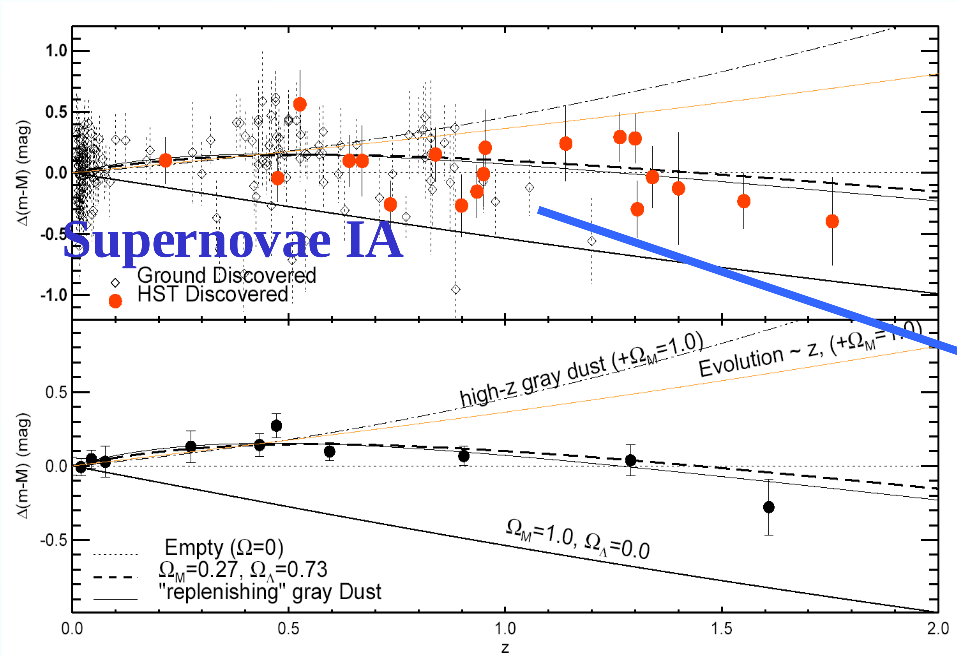


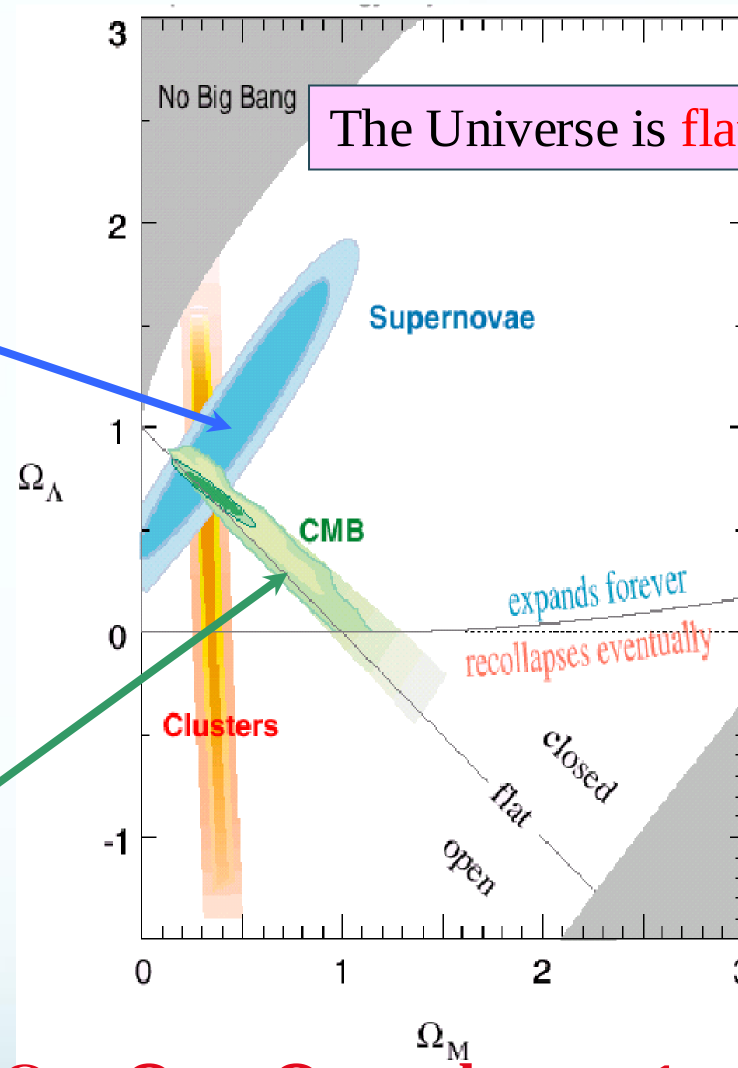
All gravity-based methods now converge on the same value:

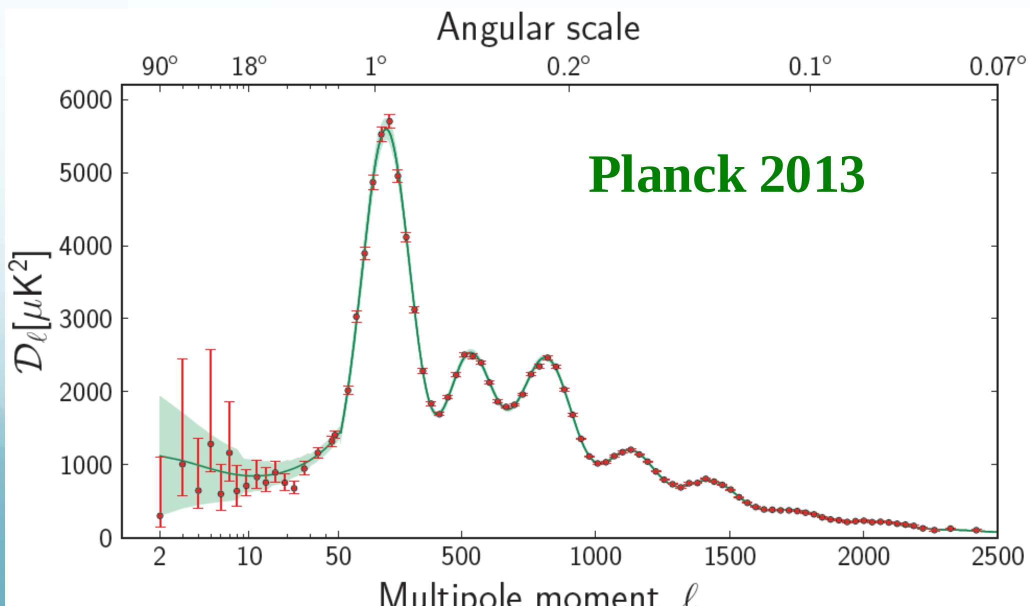
$$\Omega_m \approx 0.3 \pm 0.1$$ but…

- **no constraints on vacuum energy:** in fact, in the simplest models, it represents a uniform background without fluctuations
- **self-contained** inside the GR theory: if GR is uncorrect at large scales, these constraints are meaningless

"Concordance ΛCDM model"







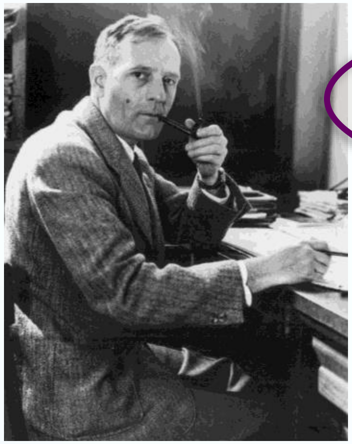
$$\Omega = \Omega_\Lambda + \Omega_M = \text{close to } 1$$

Ω = density/critical density

6 atoms of H/m³

$$\Omega_\Lambda \approx 0.69$$
$$\Omega_M \approx 0.31$$

... going to cosmological scale: the basis of the Big Bang model



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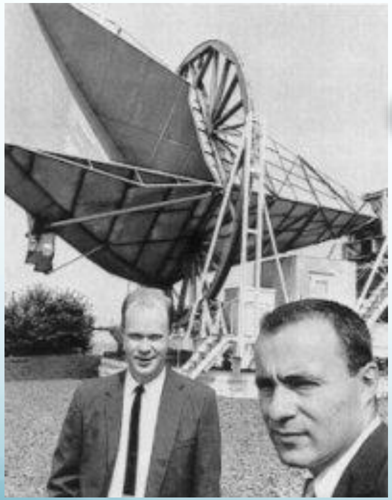
- ✓ The abundance of light elements: H, He, Li – G. Gamow (1948)



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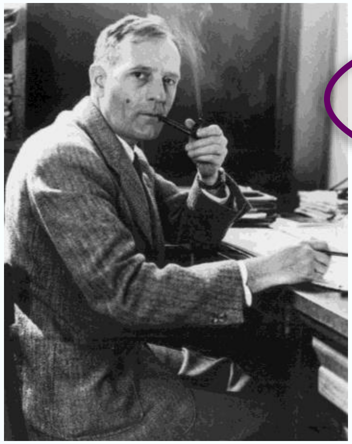

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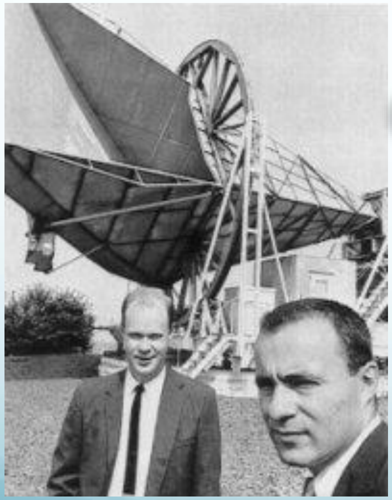


The detailed studies on these items through high-precision experimental tools allowed us to define the Concordance ΛCDM model.

The abundance of the elements

In the years 1950-60 the common thinking was that all the chemical elements were produced in the inner of the stars through nuclear processes

Problem

In all the regions of the cosmo, where is possible to measure the abundance of the chemical elements, one observed:

- 75% of the mass is in H
- 23-25% of the mass is in He
- less than 2% in other elements

?

The ratio 3:1 between H and He is ***UNIVERSAL***

Stellar Nucleosynthesis:

H → He → C → N → O → Mg → … → Fe

The mass of the star determines:

- where the chain is interrupted
- the needed time (tens of millions years!)

The primordial nucleosynthesis

- "Light" elements H, He, Li

  They are produced in the primordial Universe a lot of time before the birth of the stars
- "Heavy" elements C, N, O, …, Fe "metals"

  They are produced through the stellar nucleosynthesis

Gamow, Alpher and Herman 1948-50

Nucleosynthesis in the early Universe and in the stars

<u>1940:</u> Gamow introduced the theory (with the hope that the relative abundances of all elements could be explained) which credits that the light elements would largely be formed during the first time of rapid expansion of the Universe

- He introduced the fundamental idea of the Big-Bang nucleosynthesis
- When the Universe expanded and cooled down to $T=7.5\ 10^9$ K (650 keV):
- → the ratio n/p freezed to the value ~ 1/7
- It remained at this value until $T=10^9$ K (90 keV) → deuterium and heavier nuclei were formed
- When $T << 10^9$ K: all the n were already decayed in p or incorporated in ^{4}He
- The nucleosynthesis through the charged particles stopped because the thermal energies were not enough to allow the interacting nuclei to overcome the coulombian barrier
- We now know that nuclear reactions froze at $T \sim 30$ keV leaving most nuclei in the form of hydrogen and helium. Nucleosynthesis started up again once stars were formed providing "gravitational confinement" for astronomical "fusion reactors."

The first 3 minutes

QUARKS

Baryogenesis

Baryons
protons, neutrons

Leptons
electrons, neutrinos

1 second

Less frequent interactions
1 neutrons every 7 protons
photons

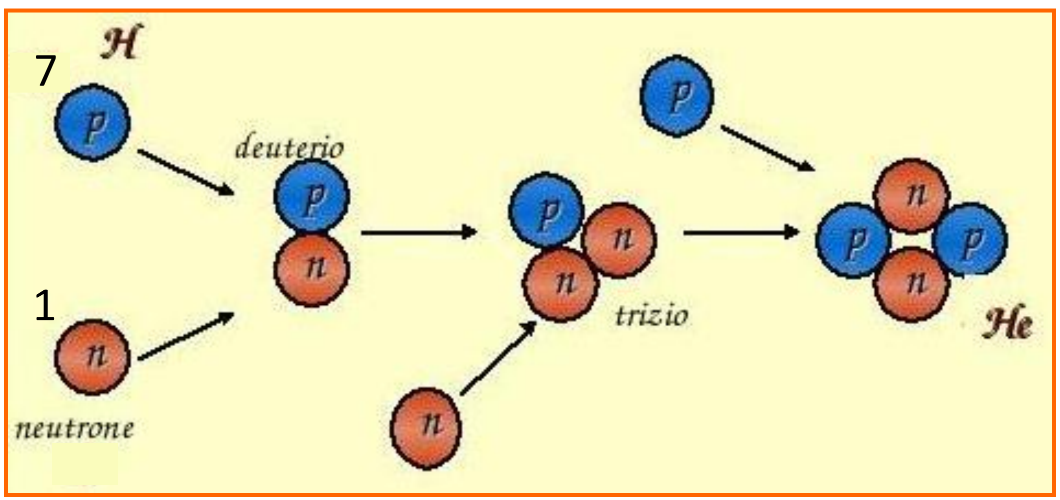
$$\frac{n}{p} \approx \frac{1}{7}$$

100 seconds

Formation of the atomic nuclei

H, He

It will **never** happen in the history of the Universe



$$Y = \frac{2n}{p+n} = \frac{2}{\frac{p}{n}+1} \approx \frac{2}{7+1} = 0.25$$

Two neutrons produce 1 nucleus of He

25% He   75% H

Gamow:

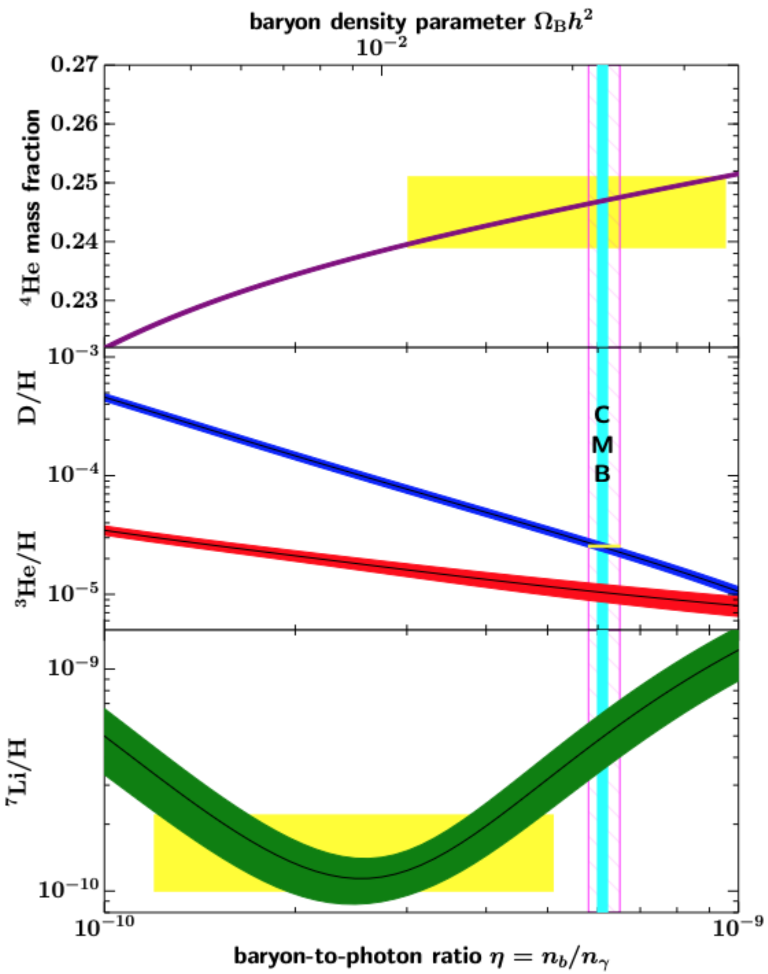
- The Big Bang radiation survives but, because of expansion, cools down
- Gamow prevision of: "relic" radiation = 5 K → -268 °C

Composition of the Universe (baryons)

Density parameters: Ω = density/critical density ← 6 atoms of H/m^3

The cosmological theory of Big Bang foresees, in the most credited scenarios, Ω=1. Indications from anisotropy measurements of CMB

$\Omega = 1$



Baryons (that is protons, nuclei and atoms):

To explain the observations, from nucleosynthesis we know that:

$\Omega_B = 4\%$

Gamow thought that subsequently heavier elements have been formed through a succession of n (insensitive to the Coulomb barrier) captures and β decays

The Gamow Universe was likened to a "giant fusion reactor".

Then:

discovery that in nature there are no stable nuclei with A = 5 and 8

-> implied that the nucleosynthesis of elements heavier than ^4He was impossible, this cannot provide stable nuclei either by reacting with the protons (A = 5), or by reacting with other ^4He nuclei (A = 8)

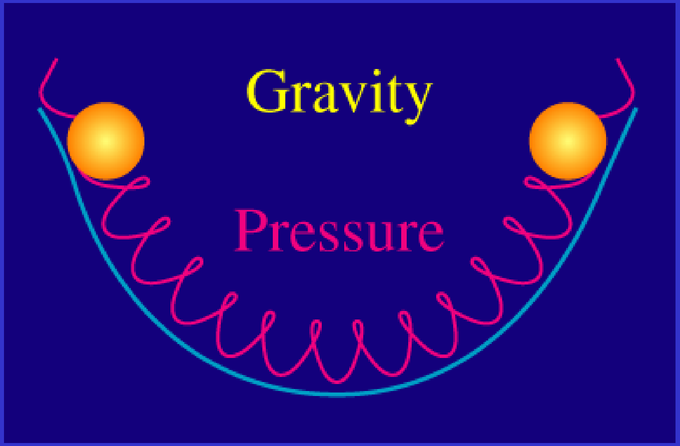
1957: Fowler & Hoyle, and Cameron, hypothesized that the nucleosynthesis of elements happens in the stars

Calculations on the abundance of intergalactic ^4He confirmed, however, that there was an initial phase of nucleosynthesis in the early Universe, which was stopped with the production of ^4He (actually stopped up to **^7Li**).

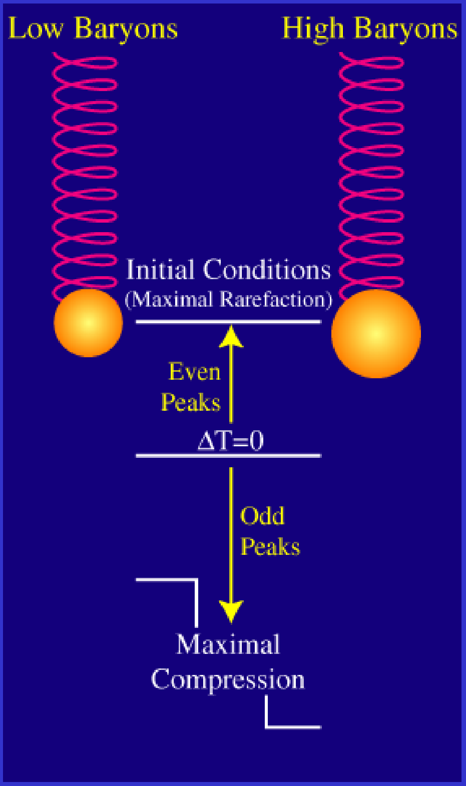
CMB peaks as a baryometer

Another precise estimate of Ω_b: the CMB acoustic peaks

What is doing the pushing that compresses the fluid? GRAVITY



The photon-baryon fluid is sitting in the **gravitational potential** wells that are the seeds of the structure in the universe. As gravity tries to compress the fluid, radiation pressure resists resulting in **acoustic oscillations.**

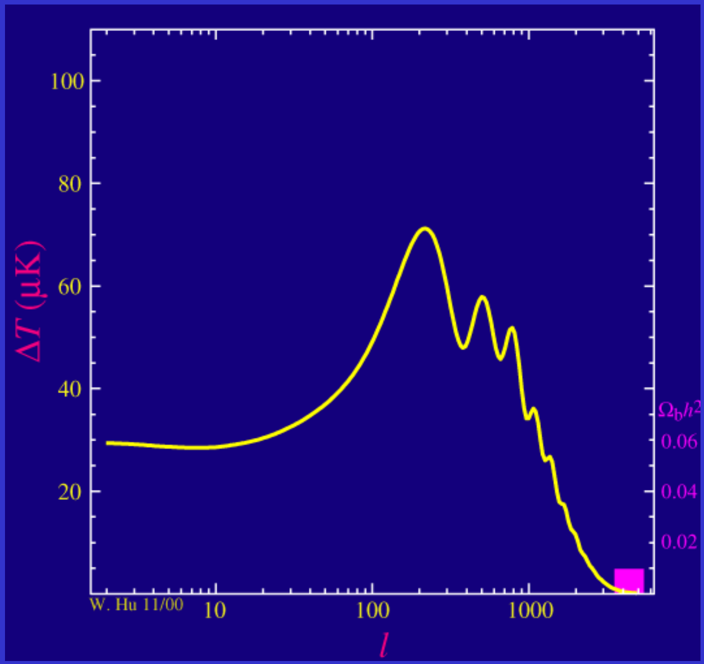


Baryons load down the photon-baryon plasma and add inertial (and gravitational) mass to the oscillating system.

Since the **odd** acoustic peaks (I, III, V, …) are associated with the plasma compression, they are enhanced by the amount of baryons. Viceversa the even peaks (II, IV, VI, …) are associated with plasma rarefaction and consequently they are relatively suppressed.

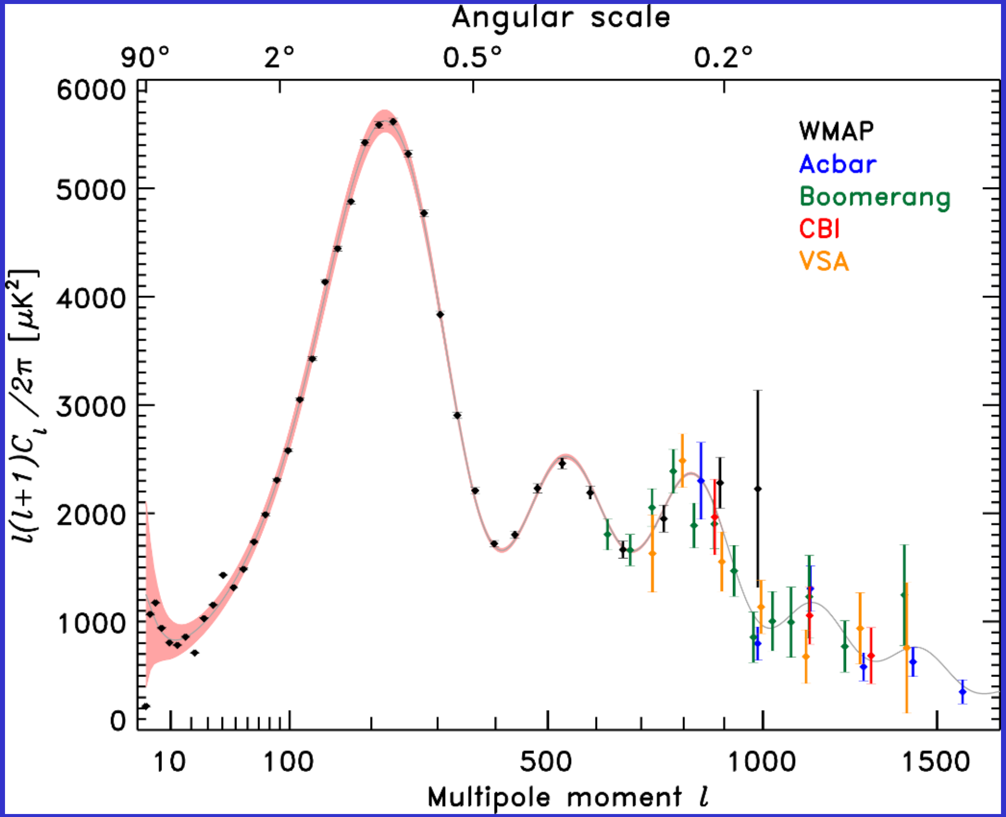
L. Moscardini

CMB peaks as a baryometer



The difference between the amplitudes of **odd and even** peaks allows to measure the **baryon density**

Present CMB data

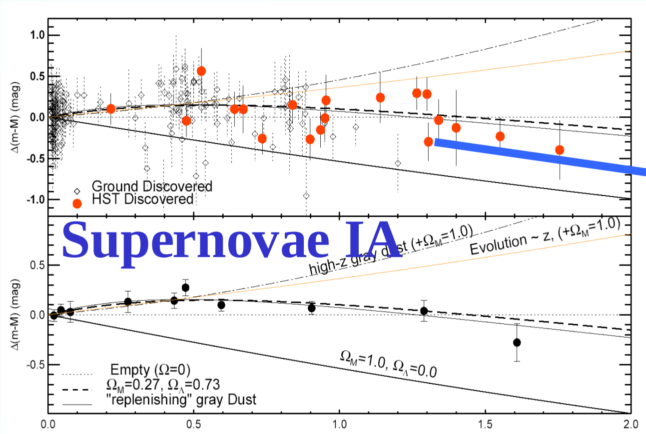


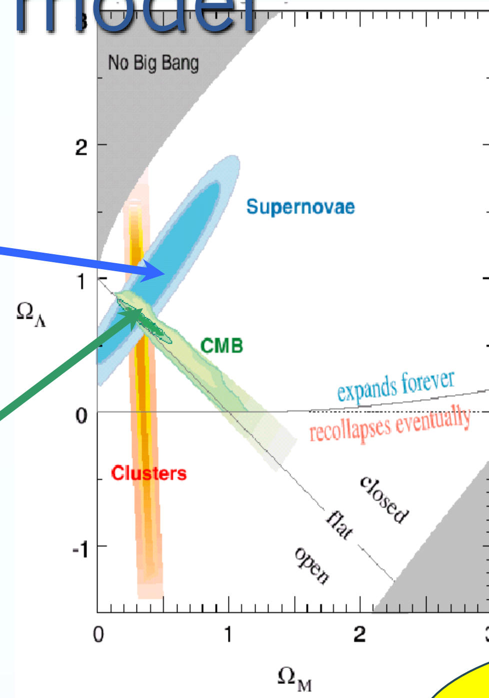
$$\Omega_b = 0.044^{+0.001}_{-0.001}$$

from a cosmological point of view we need DARK MATTER

L. Moscardini

"Concordance ΛCDM model"





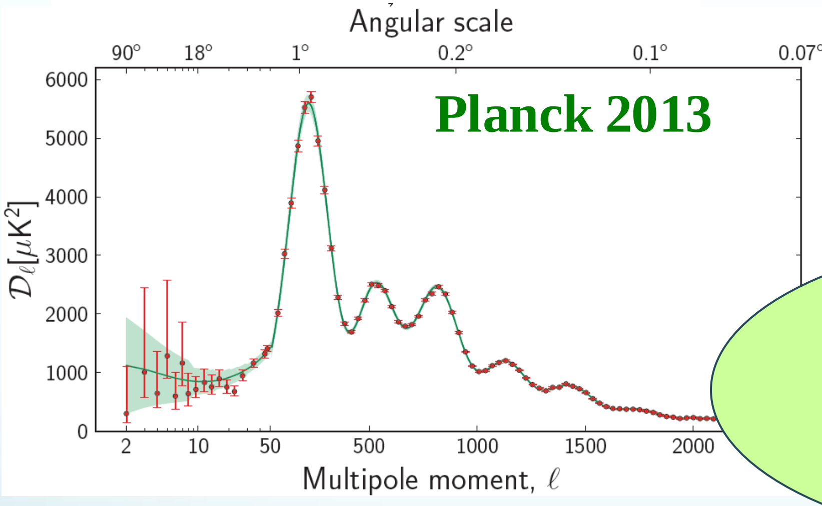
$\Omega = \Omega_\Lambda + \Omega_M =$ close to 1

Ω = density/critical density

6 atoms of H/m³

$$\Omega_\Lambda \approx 0.69$$
$$\Omega_M \approx 0.31$$

The Universe is flat



Primordial Nucleosynthesis

Observations on:
- light nuclei abundance
- microlensings
- visible light.

Structure formation in the Universe

The baryons give "too small" contribution

$\Omega_b \sim 4\%$

Non baryonic Cold Dark Matter is dominant

$\Omega_{CDM} \sim 27\%$, $\Omega_{HDM,\nu} < 1\%$

~ 90% of the matter in the Universe is non baryonic

A large part of the Universe is in form of non baryonic Cold Dark Matter particles

Constituents of our universe

- Let us now discuss the various constituents of our universe.
- $\boldsymbol{\Omega_{tot} = 1}$ (flat geometry, Euclidean space)
- $\boldsymbol{\Omega_\Lambda \approx 0.69}$; $\boldsymbol{\Omega_m \approx 0.31}$; $\boldsymbol{\Omega_r \ll 1}$.
- $\boldsymbol{\Omega_m = \Omega_b + \Omega_{CDM} + \Omega_{HDM}}$
- $\boldsymbol{\Omega_b \approx 0.04}$
- $\boldsymbol{\Omega_{CDM} \approx 0.27}$; $\boldsymbol{\Omega_{HDM,\nu} < 0.01}$

Constituents of our universe: radiation

- CMB, blackbody radiation at a temperature of 2.73 K, hence the corresponding energy density is

$$\rho_r = a_B T^4 = 7.56 \times 10^{-15} \text{erg cm}^{-3}\ \text{K}^{-4}\ 2.73^4\ \text{K}^4 \approx 4.2 \times 10^{-13}\ \text{erg cm}^{-3}$$

- Converting this to equivalent mass density, we get: $\rho_r \approx \frac{4.2\times10^{-13}}{c^2} = 4.7 \times 10^{-34}\ \text{g cm}^{-3}$
- Thus, $\Omega_r \approx \frac{4.7\times10^{-34}\ \text{g cm}^{-3}}{1.88\times10^{-29}\ h^2\ \text{g cm}^{-3}} = 2.45 \times 10^{-5}\ h^{-2}$
- Clearly $\boldsymbol{\Omega_r \ll 1}$. However, since $\rho_r \propto a^{-4}$, their contribution will be significant at early times.

Constituents of our universe: baryons

- Galaxy survey and CMB fluctuation data indicate $\boldsymbol{\Omega_m \approx 0.31}$
- However, the baryon density can be estimated from abundances of light elements produced in Big Bang nucleosynthesis (BBN). These imply $\boldsymbol{\Omega_b \approx 0.04}$
- The matter in stars can be computed from the luminosity function of galaxies (i.e., the number of galaxies per unit volume in a given luminosity range). These observations imply $L_{B,*} \approx 2 \times 10^8 h L_\odot \mathrm{Mpc}^{-3}$
- If all stars had mass-to-light ratio like the Sun, this would imply a mass density of stars

$$\rho_* \approx L_{B,*} \frac{M_\odot}{L_\odot} \quad \rightarrow \quad \boldsymbol{\Omega_* \approx 10^{-3}}$$

- In general, observations of galaxies imply a mass-to-light ratio of 2− 10, so even if we take a typical vale of 5, the stellar density will be $\boldsymbol{\Omega_* \approx 5 \times 10^{-3}}$.
- A substantial fraction of baryons are as photoionized gas in the intergalactic space: $\boldsymbol{\Omega_{IGM} \approx 0.01 h^{-3/2}}$
- There is (cold) gas locked up in atoms, molecules etc, detected via absorption/emission lines from galaxies. These make up to $\boldsymbol{\Omega_{cold\ gas} \approx 5 \times 10^{-4} h^{-1}}$.
- There is also matter in hot gas in clusters, detected in X-ray, which give $\boldsymbol{\Omega_{cluster} \approx 0.0016 h^{-3/2}}$.
- There are also contributions from the circumgalactic medium and other sources. However, these do not add up to the value implied by the BBN. This is sometimes called the **missing baryon problem**.
- It is believed that the rest is in the **warm-hot intergalactic medium (WHIM)** which is difficult to detect directly.
- At high redshifts z ~ 2− 3, the relative contributions of the components are different.

Hot versus Cold dark matter

Hot dark matter (HDM).

Candidates: neutrino

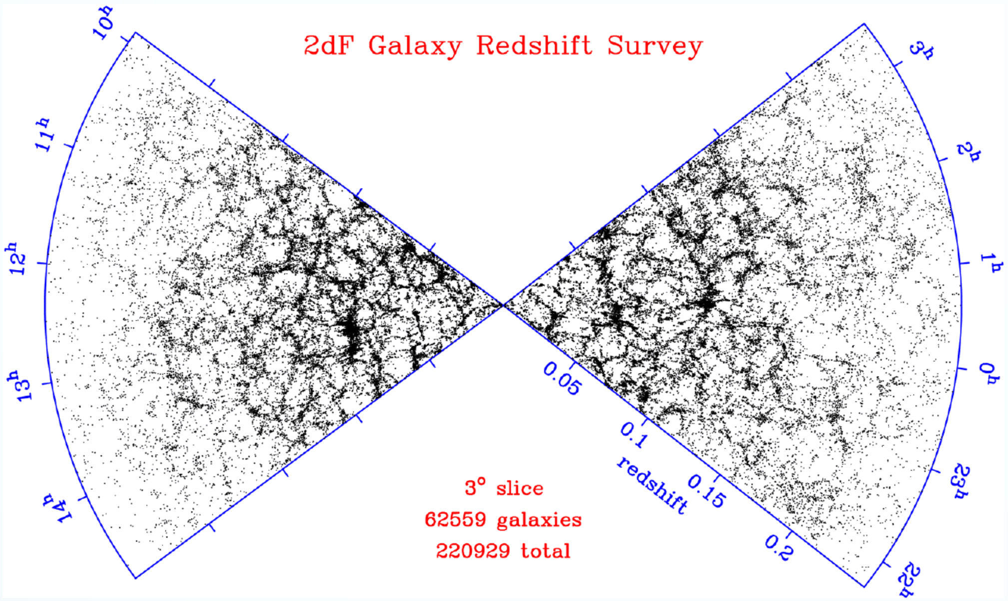
It is still relativistic when it decouples from thermal background, so free-streaming affects also large scales

Cold dark matter (CDM).

Candidates: neutralinos, axions, axinos, mirror DM, etc…

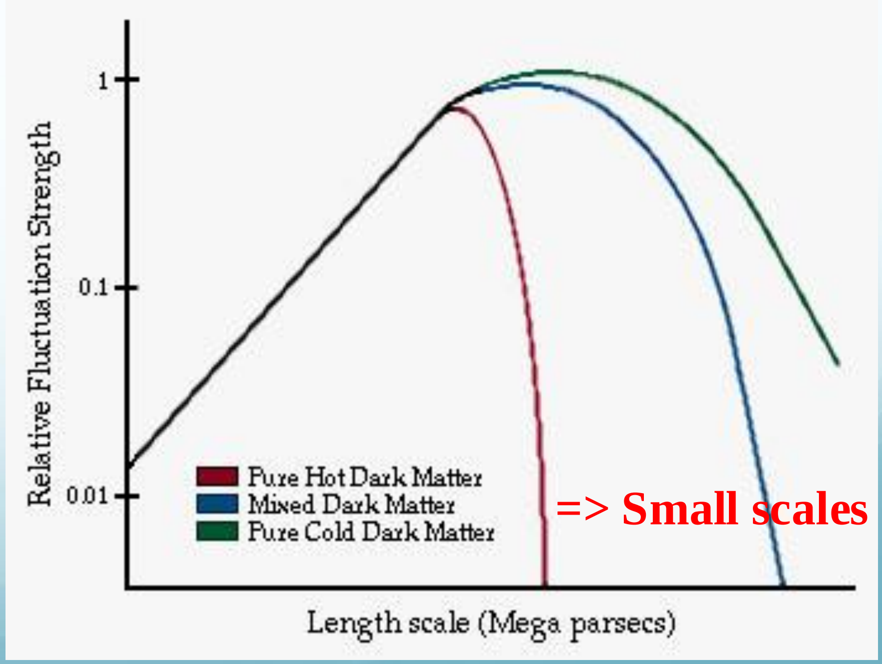
It is no more relativistic when it decouples from thermal background, so free-streaming affects only small scales

HDM vs CDM models



HDM: first objects are very large (superclusters); smaller objects like galaxies form later by fragmentation.
Top-down scenario

CDM: first objects are small (globular clusters?); larger objects (galaxies, clusters) form later by hierarchical merging.
Bottom-up scenario



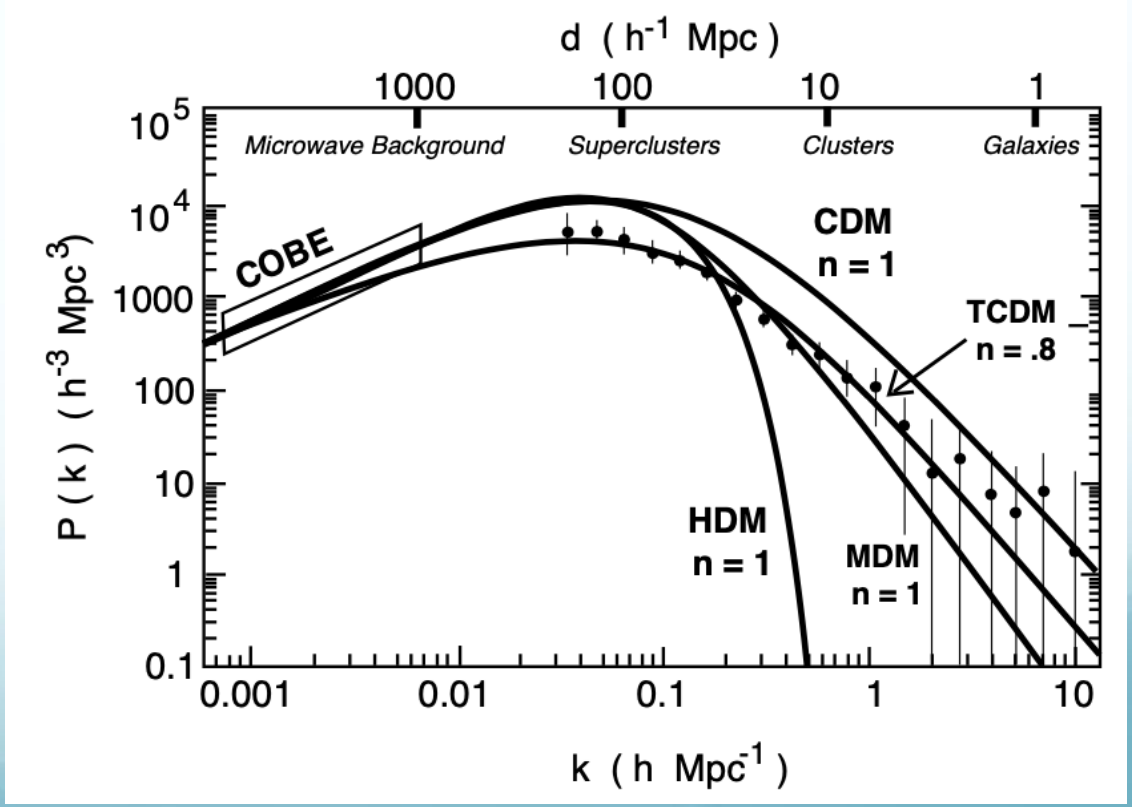
=> Small scales

But HDM does not produce small scale structure !

But, starting from late '80s, we have evidences in favour of a **bottom-up** structure formation (hierarchical) model, where objects formed first at small scales.

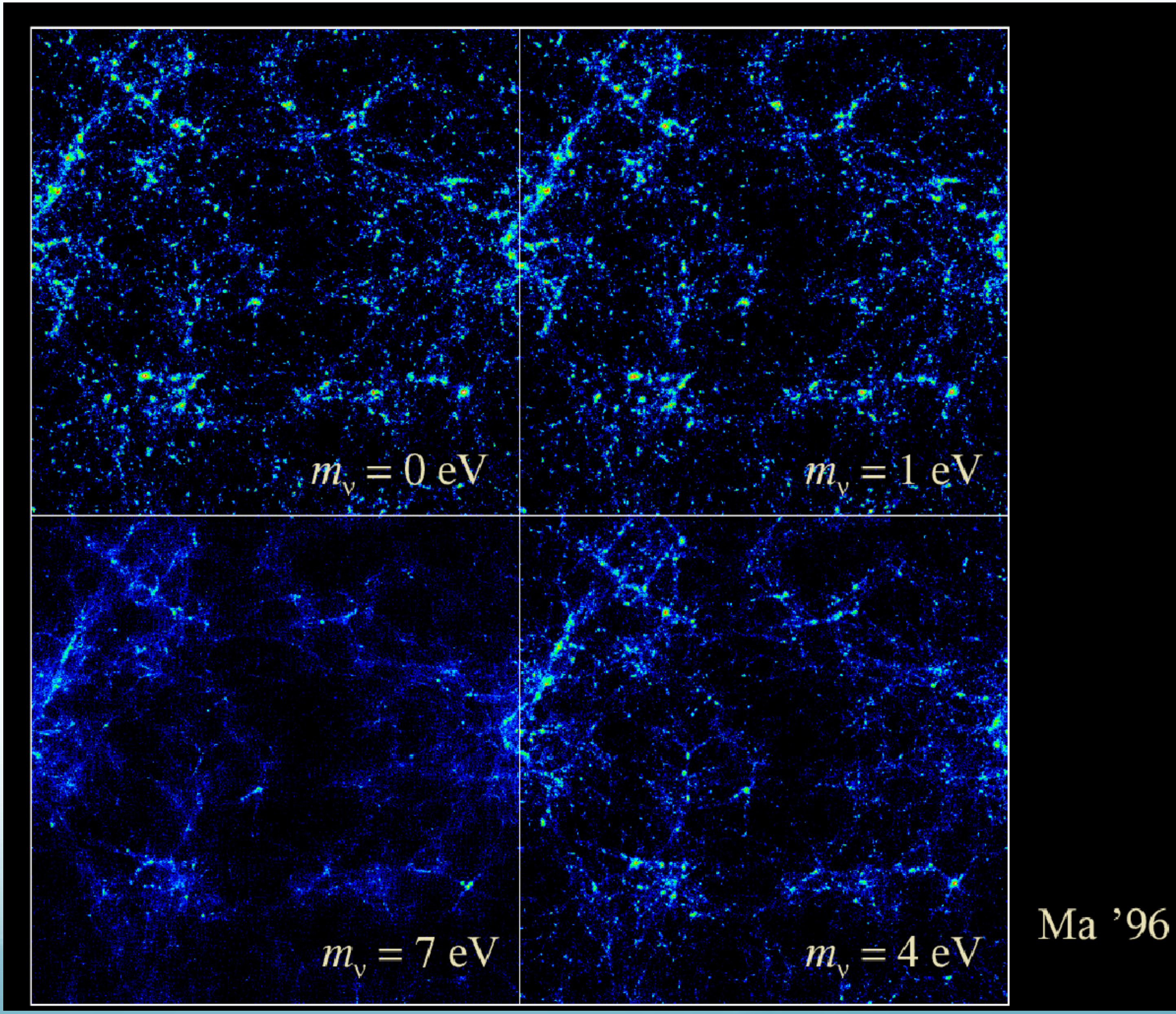
Now this is confirmed by observational data. A **cold** (i.e. non-relativistic when it decoupled from the thermal background) **dark matter** (CDM) component is strongly favoured

dark matter cannot be dominated by neutrinos!



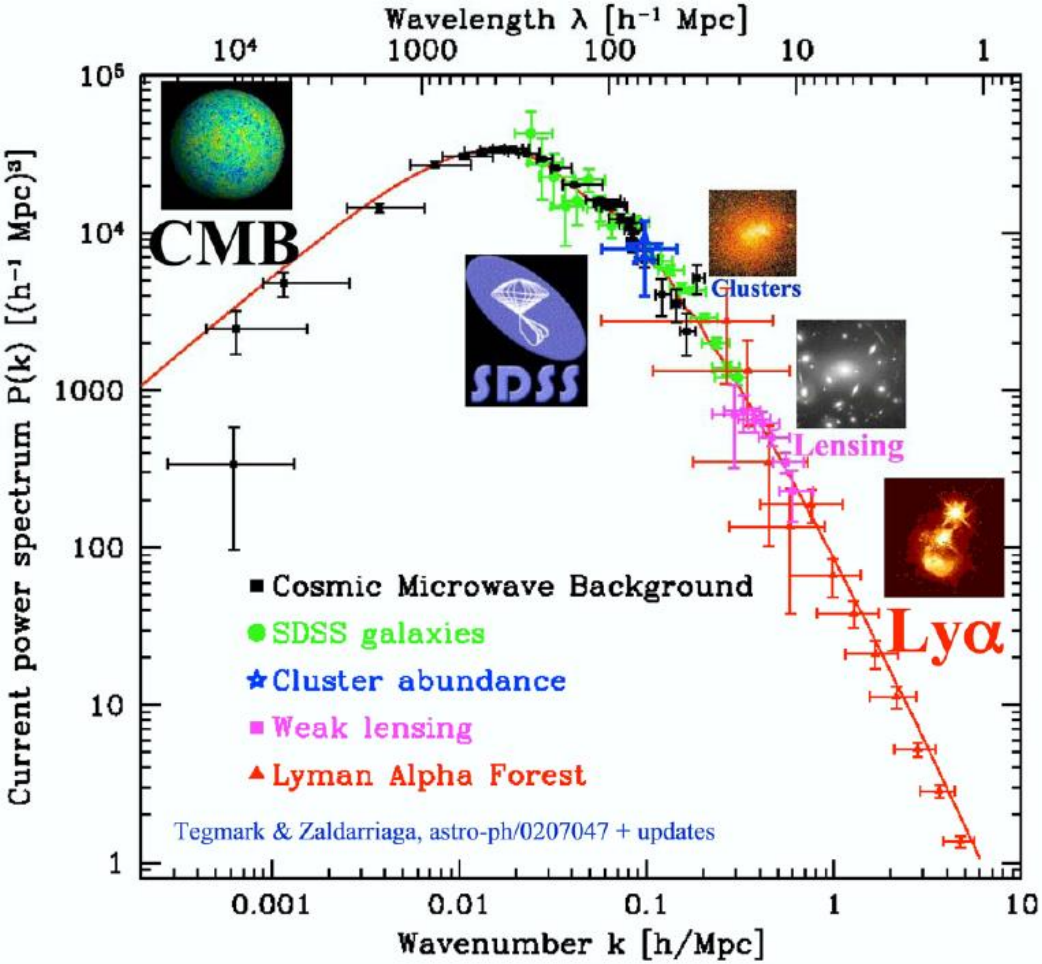
N-body results

<u>There is less clustering in models with massive neutrinos</u>



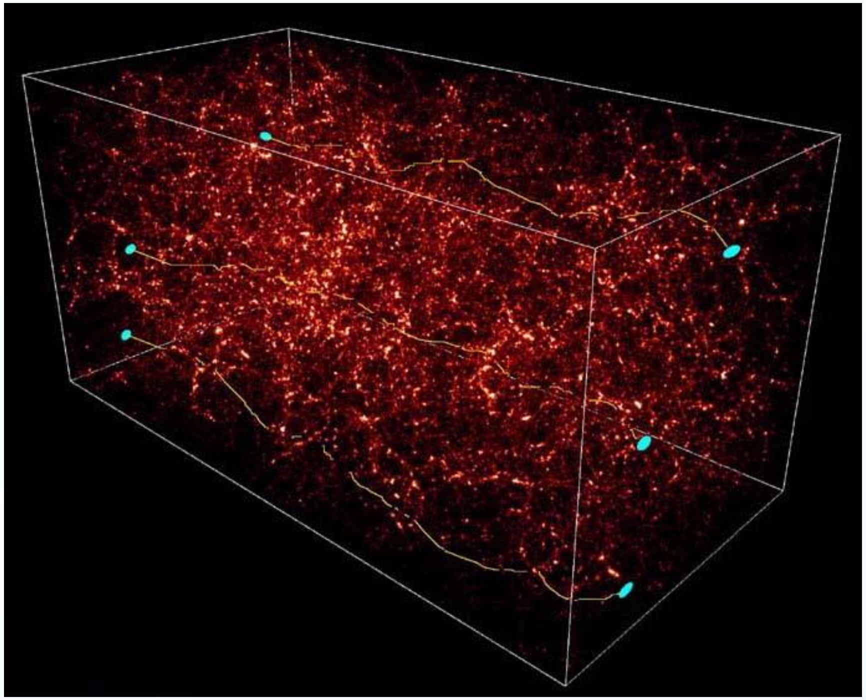
Cosmological observables

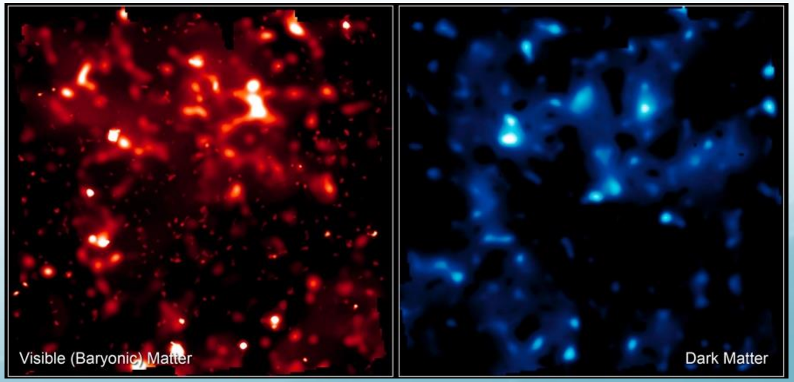
- Cosmic microwave background (CMB)
- Galaxy surveys & large scale structure (LSS)
- Lyman alpha forest
- Galaxy clusters
- Gravitational lensing
- …



The Dark Matter in the Universe

- A large part of the Universe is made of Dark Matter and Dark Energy
- The so-called “baryonic” matter is only ≈4% of the total budget
- (Concordance) ΛCDM model and precision cosmology
- Non-baryonic Cold Dark Matter (≈27%) is the dominant component (≈87%) among the matter.
- The Dark Matter is fundamental for the formation of the structures and galaxies in the Universe
- CDM particles, possibly relics from Big Bang, with no em and color charges → beyond the SM





Dark Matter Candidates

Requirements for a good DM candidate:

- must have lifetime $\tau_{DM} >> \tau_U$
- must be electrically neutral (otherwise not dark)
- must have correct relic density: $\Omega_{DM} \simeq 0.27$
- must have mass for gravitational interaction
- must be trapped in the gravitational well of the galaxies
- non-relativistic velocities
- can be either or not thermally produced by "elementary" particles in the early Universe

Boltzmann equation for DM density calculation

- Assumptions: binary interactions of particle X in the thermal bath
- $X + X \leftrightarrow$ thermal bath particles $\quad l = (e^{\pm}, \mu^{\pm}, \tau^{\pm}, u, d, s, c, b, t, W^{\pm}, Z^{0})$
- Early hot universe, X effectively massless $(T \gg m_X)$, in thermal equilibrium with SM particles
- Annihilation rate: $\Gamma_{XX\to l\bar{l}} = \langle \sigma_{XX\to l\bar{l}}\, v \rangle n_X$; expansion rate of the Universe H
- If $\Gamma \ll H$, the number of particles conserved (equation of continuity, Friedmann eq.):

$$\frac{dn}{dt} + 3Hn = 0 \quad \rightarrow \quad n \propto a^{-3}$$

- If $\Gamma \gg H$, the number of particles follows the equilibrium (non-relativistic particles):

$$n_{eq} = g \left(\frac{mT}{2\pi}\right)^{3/2} exp\left(-\frac{m}{T}\right)$$

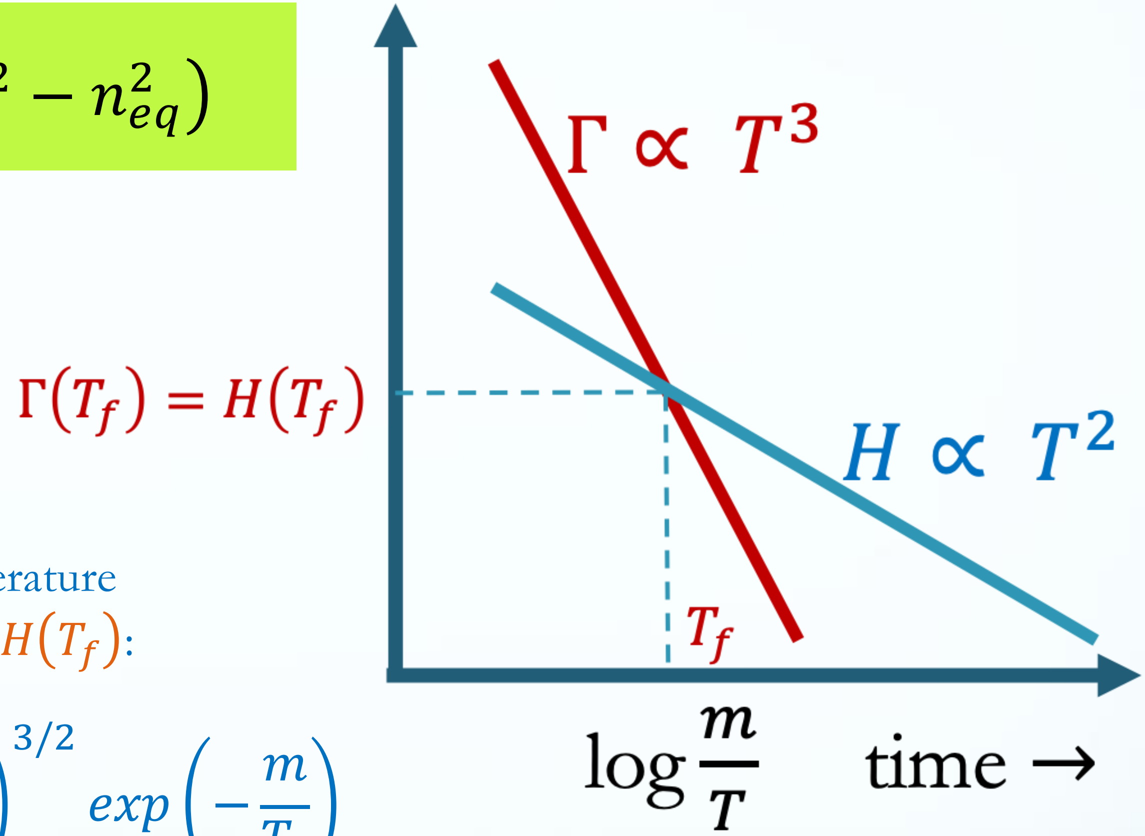
- Putting all together:

quadratic for binary processes

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{eq}^2\right)$$

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle\left(n^2 - n_{eq}^2\right)$$

- $\Gamma = \langle\sigma v\rangle n \propto T^3$
- $H = \sqrt{\frac{4\pi^3}{45} g_*}\, T^2/m_{pl}$
- In hot early Universe: $\Gamma \geq H$
- As the universe expands, temperature drops, and eventually $\Gamma(T_f) = H(T_f)$:



$$\sqrt{\frac{4\pi^3}{45} g_*}\,\frac{T_f^2}{m_{pl}} = \langle\sigma v\rangle\, g\left(\frac{mT_f}{2\pi}\right)^{3/2} exp\left(-\frac{m}{T_f}\right)$$

- **Freeze-out!** Thermal bath of particles becomes nearly transparent to X.
- Ratio of number density to entropy density remains constant.
- Roughly: $x_f = \frac{m}{T_f} \approx 30$

$$\boldsymbol{T_f \approx m/30}$$

The condition $\Gamma(T_f) = H(T_f)$:

$$\sqrt{\frac{4\pi^3}{45} g_*} \frac{T_f^2}{m_{pl}} = \langle\sigma v\rangle\, g \left(\frac{mT_f}{2\pi}\right)^{3/2} exp\left(-\frac{m}{T_f}\right)$$

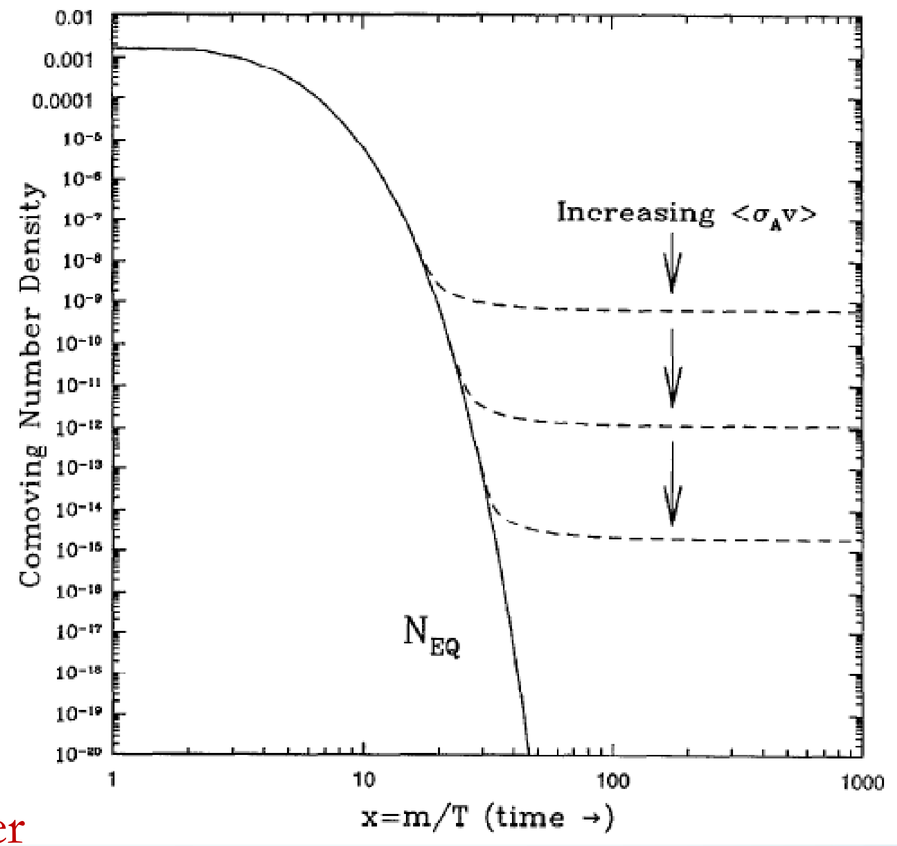
- Defining $x_f = \frac{m}{T_f}$

$$x_f^{1/2} e^{-x_f} = \sqrt{\frac{4\pi^3}{45} g_*} \frac{(2\pi)^{3/2}}{m_{pl} m g \langle\sigma v\rangle}$$

- Roughly: $x_f \sim \ln(m\langle\sigma v\rangle)$
- Solving the eq. we found $x_f = \frac{m}{T_f} \approx 30$ $\Longrightarrow$ $\boldsymbol{T_f \approx m/30}$
- … and $\Omega_X h^2 = \frac{m_X n_X}{(\rho_c/h^2)} \approx \left(\frac{2.5 \times 10^{-10} GeV^{-2}}{\langle\sigma v\rangle}\right)$
- approximate solution for the Boltzmann equation, independent from m_X
- that is: $\Omega_X h^2 \approx \frac{0.1\, pb}{\langle\sigma v\rangle}$

- $\Omega_X h^2 \approx \frac{0.1\,pb}{\langle \sigma v \rangle}$
- We remind this is for non-relativistic X −particles
- Increasing annihilation cross section, the X −particles remain in equilibrium for more time and decouple later. Their abundance decreases
- Hence very interesting results:
  - ✓ X −particles are well within non-relativistic regime
  - ✓ A value of the relic density corresponding to that estimated for CDM ($\Omega_X h^2 \approx 0.1$) implies an annihilation cross section on the order of the weak interaction

Numerical solution for the Boltzmann equation



- ✓ dimensionally, for electroweak scale masses and couplings, one gets
- ✓ $\rightarrow \langle \sigma v \rangle \sim \frac{\alpha^2}{m^2} \simeq 1\,pb \left(\frac{200\,GeV}{m}\right)^2$
- ✓ New physics at electroweak scale with stable neutral particle $\Longrightarrow$ CDM candidate
- ✓ Sometimes dubbed "WIMP-miracle"

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle\left(n^2 - n_{eq}^2\right)$$

Approximate solution:

- $n \sim n_{eq}(T)$ $T \gg T_f$
- $n\, a^3 \sim$ constant $T \ll T_f$

- ❑ $\Gamma = \langle\sigma v\rangle n \propto T^3$
- ❑ $H = \sqrt{\frac{4\pi^3}{45} g_*}\, T^2/m_{pl}$
- ❑ As the universe expands, temperature drops, and eventually $\Gamma(T_f) = H(T_f)$

$$\langle\sigma v\rangle\, n_{eq}(T_f) = \sqrt{\frac{4\pi^3}{45} g_*}\frac{T_f^2}{m_{pl}}$$

$$\boldsymbol{T_f \approx m/30}$$

At the freeze-out epoch: $\frac{n(a_f)}{n_\gamma(a_f)} \sim \frac{n(a_f)}{T_f^3} \sim \frac{n_{eq}(T_f)}{T_f^3} \sim \sqrt{\frac{4\pi^3}{45} g_*(T_f)}\frac{T_f^2}{m_{pl}}\frac{1}{T_f^3\langle\sigma v\rangle} \sim \sqrt{g_*(T_f)}\frac{30}{m_{pl} m\langle\sigma v\rangle}$

After freeze-out, the expansion is adiabatic separately for the CDM particles and for the particles in thermal equilibrium:

Today:

- $n(a_0) = n(a_f)\left(\frac{a_f}{a_0}\right)^3$
- $n_\gamma(a_0) = n_\gamma(a_f)\left(\frac{g_f a_f^3}{g_0 a_0^3}\right)$

$$\left[\frac{n}{n_\gamma}\right]_0 = \left[\frac{n}{n_\gamma}\right]_f \frac{g_0}{g_f}$$

$$n_{\gamma,0} \approx 413\ \mathrm{cm}^{-3}$$

$$n_0 \sim \left[\frac{n}{n_\gamma}\right]_f \frac{g_0}{g_f} 413\ \mathrm{cm}^{-3} \sim \frac{413\ \mathrm{cm}^{-3}\ 30}{\sqrt{g_*(T_f)} m_{pl} m\langle\sigma v\rangle}$$

$$\Omega_X h^2 = \frac{m\, n_0}{(\rho_c/h^2)} \approx \left(\frac{2.5\times 10^{-10} GeV^{-2}}{\langle\sigma v\rangle}\right)$$

Relativistic Freeze-out - $\Gamma(T_f) = H(T_f)$

- Relativistic particles at the decoupling time
- HDM particles
- The mass dependence does NOT drop down
- For neutrinos: $\Omega_\nu h^2 \simeq \frac{\sum m_\nu}{94\ eV}$

Non-relativistic Freeze-out - $\Gamma(T_f) = H(T_f)$

- Non-relativistic particles at the decoupling time
- CDM particles
- The mass dependence does drop down
- For CDM particles: $\Omega_X h^2 \approx \frac{0.1\ pb}{\langle \sigma v \rangle}$

Relic DM particles from primordial Universe

SUSY
(as neutralino or sneutrino
in various scenarios)

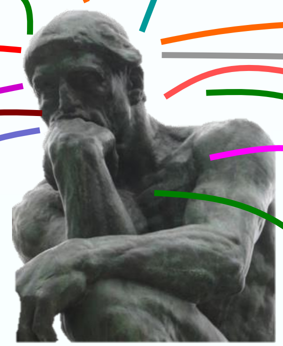
the sneutrino in the Smith
and Weiner scenario

sterile ν

electron interacting dark matter

a heavy ν of the 4-th family

even a suitable particle not
yet foreseen by theories



axion-like (light pseudoscalar
and scalar candidate)

self-interacting dark matter

mirror dark matter

Kaluza-Klein particles (LKK)

heavy exotic canditates, as
"4th family atoms", ...

Elementary Black holes,
Planckian objects, Daemons

invisible axions, ν's

etc…

Moreover, several questions arise about:

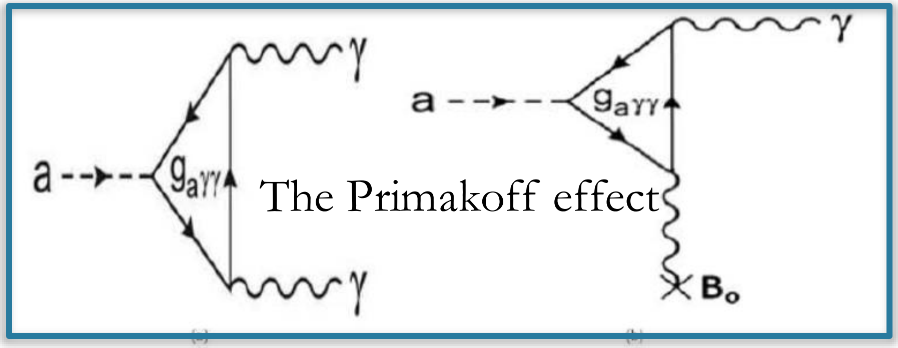
- interaction type with ordinary matter and its description
- related nuclear and particle physics
- halo model and parameters
- halo composition. DM multicomponent also in the particle sector?
- non thermalized components?
- caustics?
- clumpiness?
- etc.

Axions

- **Light pseudoscalar particles** in many theories Beyond Standard model
- Peccei-Quinn Axion (QCD) solves **strong CP problem**
- The piece in the Lagrangian responsible for CP violation in QCD: $\mathcal{L}_{\theta QCD} = \frac{\theta_{QCD}}{32\pi^2} \mathrm{Tr} G_{\mu\nu}\tilde{G}^{\mu\nu}$
- G is the gluon field strength tensor and $\tilde{G}$ its dual: $\tilde{G}^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu} G_{\alpha\beta}$
- This term is called topological since it is a total derivative and does not affect the classical equations of motion.
- However, it has important effects on the quantum theory. This term is odd under CP, and produces a neutron electric dipole moment (EDM): $d_n \approx 3.6 \times 10^{-16}\, \theta_{QCD}\, e\, cm$
- Experimentally: $|d_n| < 3.0 \times 10^{-26} e\, cm$ → $\boldsymbol{\theta_{QCD} < 10^{-10}}$
- If there were only QCD, then θ_{QCD} can be set to zero by symmetry.
- In the real world, the weak interactions violate CP. The physically measurable parameter is $\theta_{QCD} = \tilde{\theta}_{QCD} + others$, where $\tilde{\theta}$ is the "bare" quantity. Thus, the smallness of θ_{QCD} is a fine-tuning problem **since it involves a precise cancellation** between two dimensionless terms generated by different physics.
- Peccei Quinn: Global symmetry $U(1)_{PQ}$ spontaneously broken by a scalar field. The Goldstone boson is the **axion**

Axions

- Light pseudoscalar particles in many theories Beyond Standard model
- Peccei-Quinn Axion (QCD) solves strong CP problem $\theta_{QCD} < 10^{-10}$ from limits on the neutron's EDM.
- Dark matter candidate
- Production in the stars (i.e. Sun)



$$m_a \sim \frac{\Lambda^2}{f_a}$$

$$\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$$ ← Coupling to electromagnetic field
ADMX, DM Radio, LC Circuit

$$\frac{a}{f_a} G_{\mu\nu}\tilde{G}^{\mu\nu}$$ ← Coupling to gluon field
CASPEr Electric

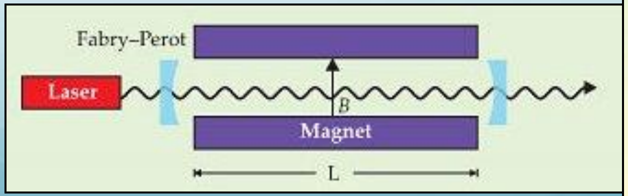
$$\frac{\partial_\mu a}{f_a} \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f$$ ← Coupling to fermions
CASPEr Wind, QUAX

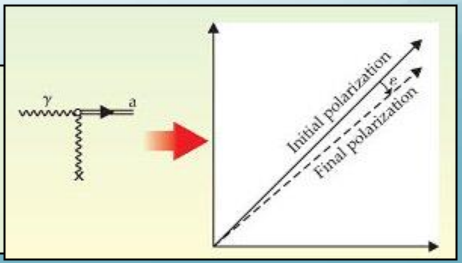
Axions

Theoretical motivation: to explain the absence of CP violation in strong interactions (strong CP problem)

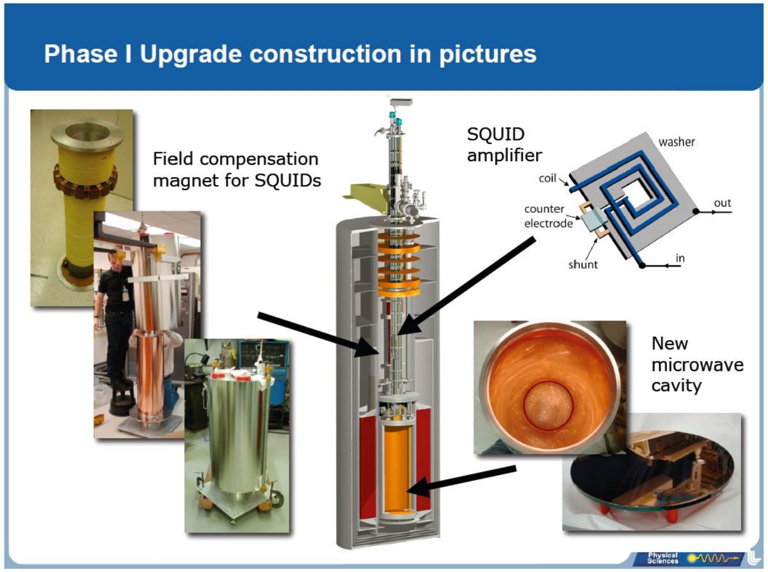
Variety of experiments

- **Axions as Dark Matter**
  - Haloscopes: Microwave cavities in solenoid magnet
  - Look for dark matter axions (low mass) converting to photons in B-Field PRL 51:1415 (1983)
  - **New techniques being explored: NMR, LC-circuit, Axion Wind**
- **Axions from the Sun**
  - Helioscopes: Axions generated from the sun
    - PRL 51:1415 (1983), PRD 39:2089 (1989)
    - **CAST, IAXO**
  - Bragg scattering on crystals, noble liquids (g_{ae})
- **Axions in the Lab**
  - Photon regeneration and polarization changes
    - **PVLAS, ALPS**
  - Modifications to short range forces
    - **ARIADNE, Torsion-balance**





CDM AXION SEARCHES



ADMX (Axion Dark Matter eXperiment)

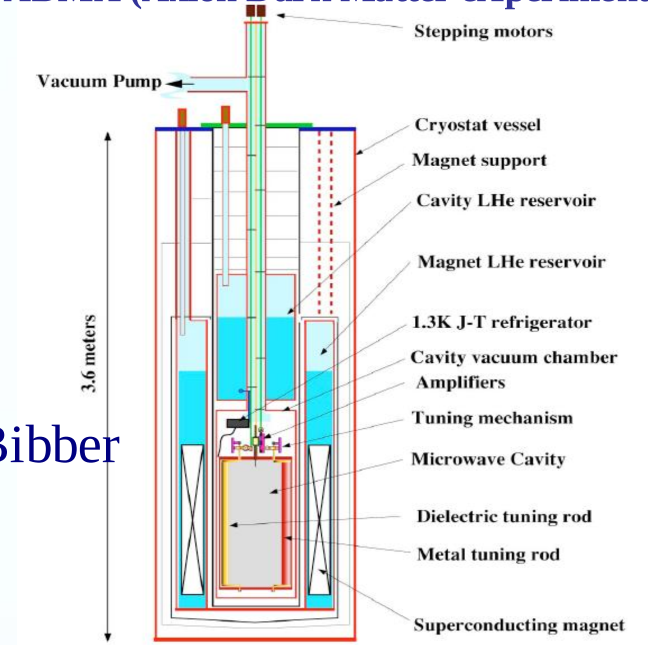
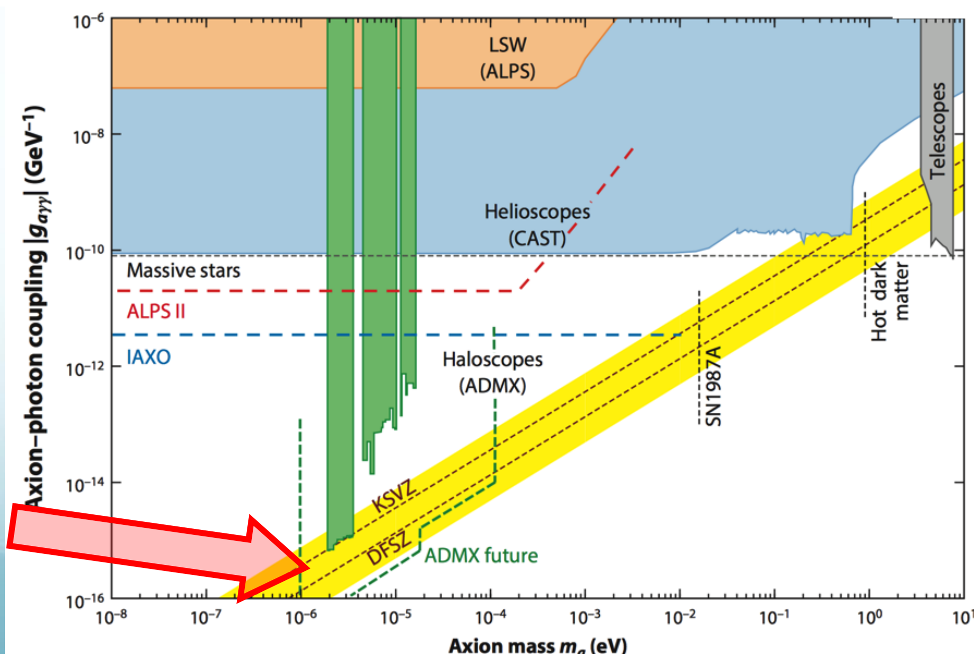


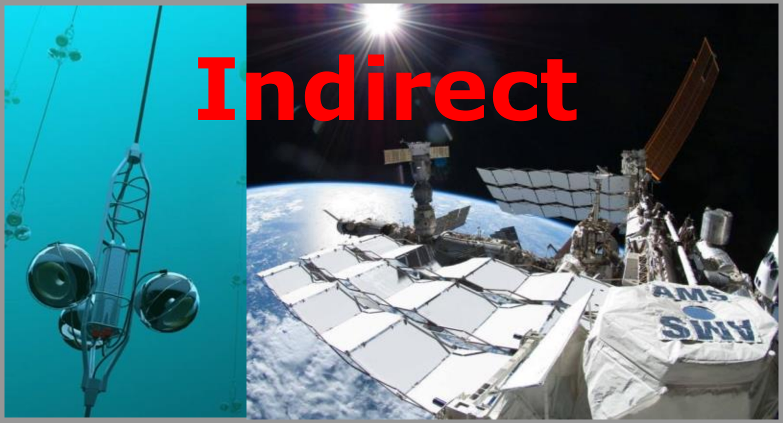
FIG. 1. Sketch of the rf cavity axion detector.

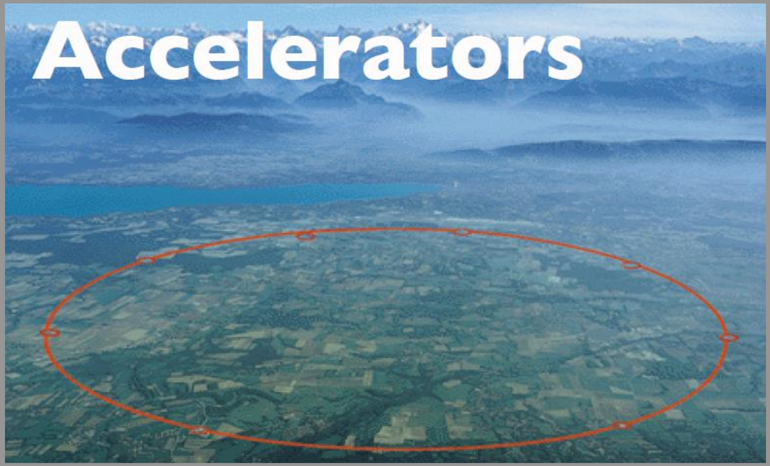
K. Van Bibber



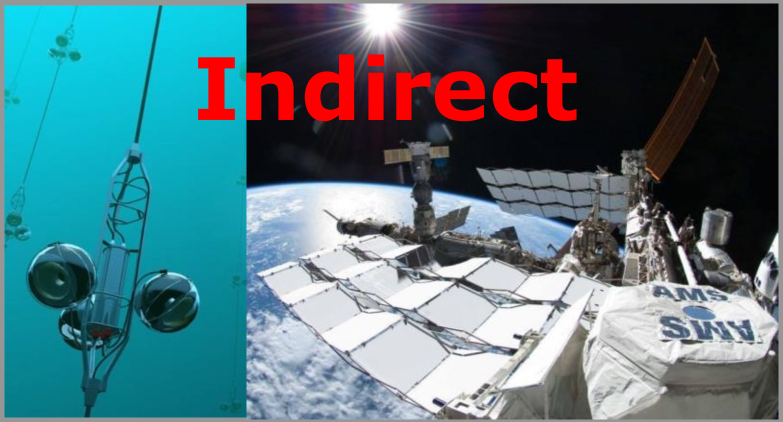
Axions as CDM

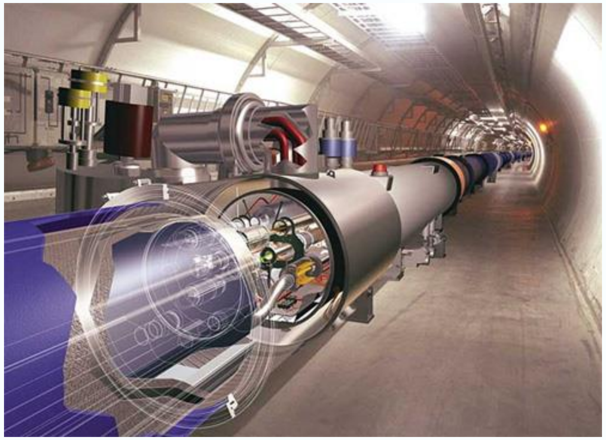
Hunting the Dark Matter particles





Hunting the Dark Matter particles





What accelerators can do:

to demostrate the existence of some of the possible DM candidates

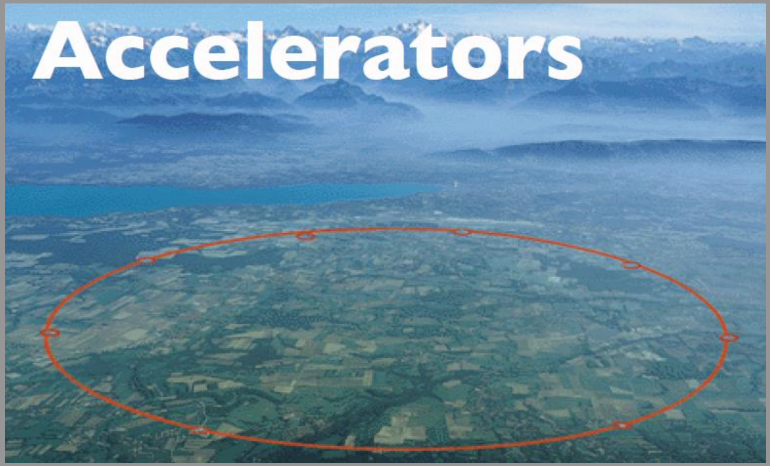
What accelerators cannot do:

to credit that a certain particle is the Dark Matter solution or the “single” Dark Matter particle solution…

+ DM candidates and scenarios exist (even for neutralino candidate) on which accelerators cannot give any information

DM direct detection method using a model independent approach and a low-background widely-sensitive target material

Hunting the Dark Matter particles



Hunting the Dark Matter particles

- **High-energy neutrinos**
- **Gamma-rays**
- **Antimatter in the space (anti-protons)**
- **Antimatter in the space (positrons)**
- **Effects of DM on astrophysical objects**

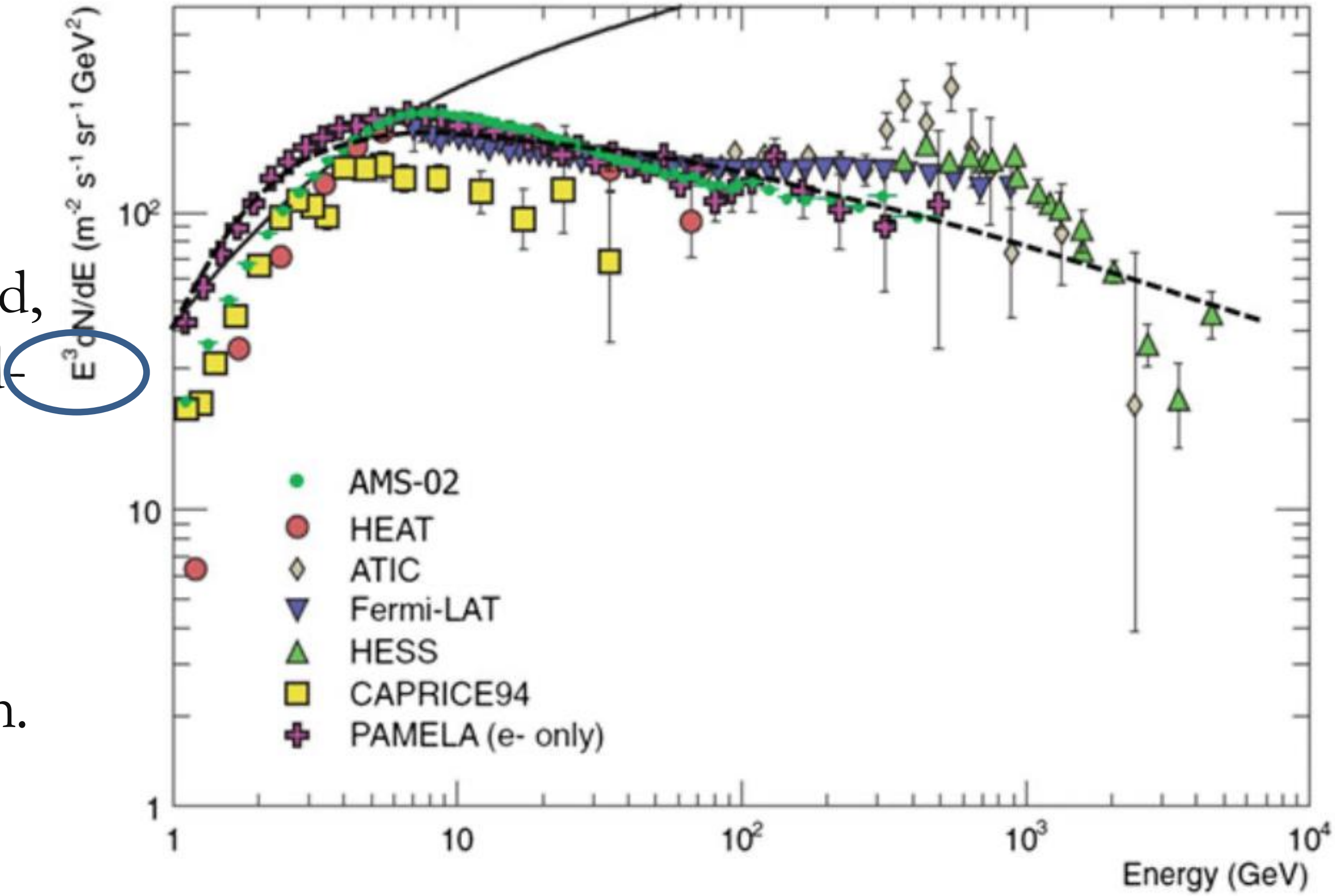
But:

- **model dependent results**
- **strong modeling of the background is needed**
- **other sources of positrons/gamma-rays/anti-matter/… are present**

Electrons and positrons

Electrons and positrons constitute about 1% of the CRs

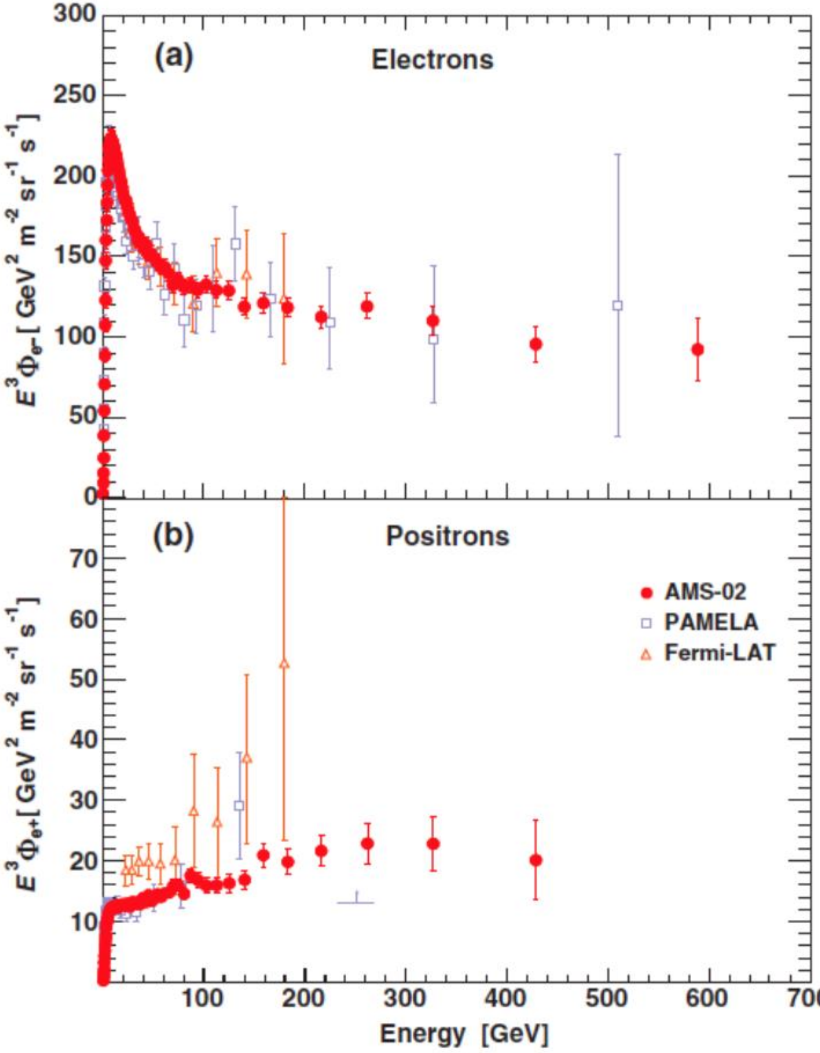
- The electron plus positron energy spectrum from different space-based, balloon, and ground-based experiments.
- The *black full line* shows for reference the proton spectrum.

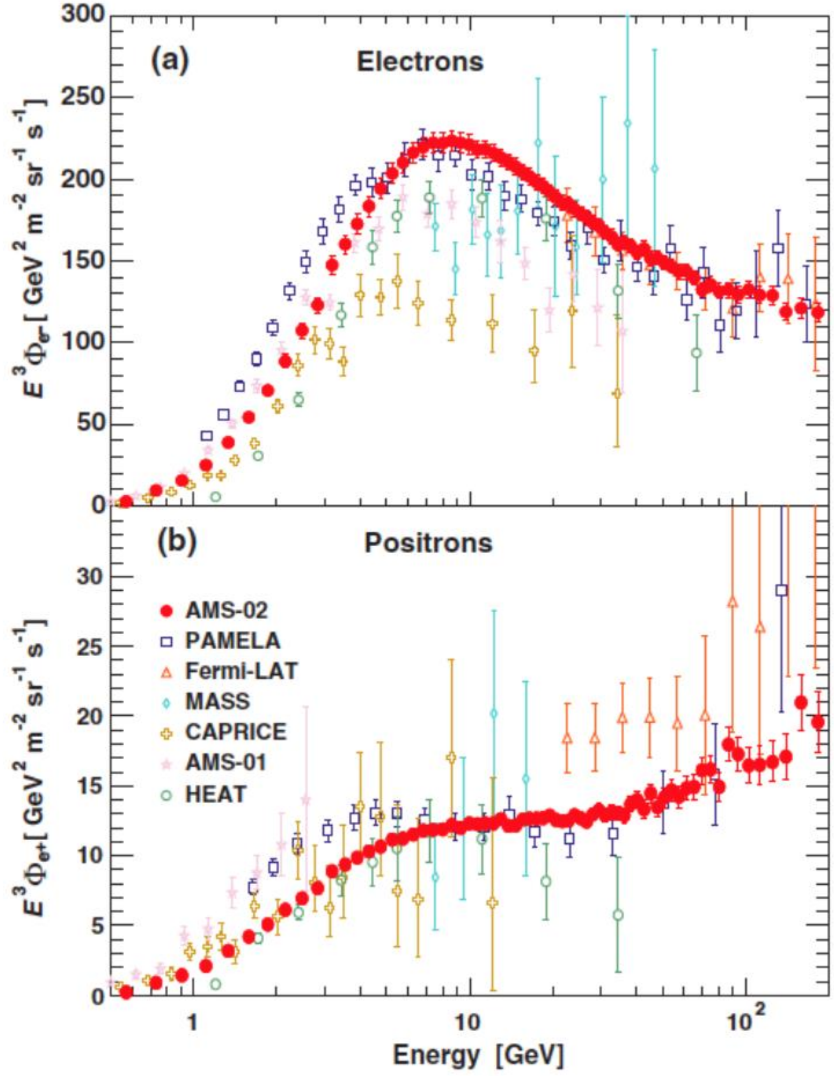


- The theoretical calculation (*dashed line*) based on the prediction of secondary electrons produced by the interaction of CRs with the ISM.

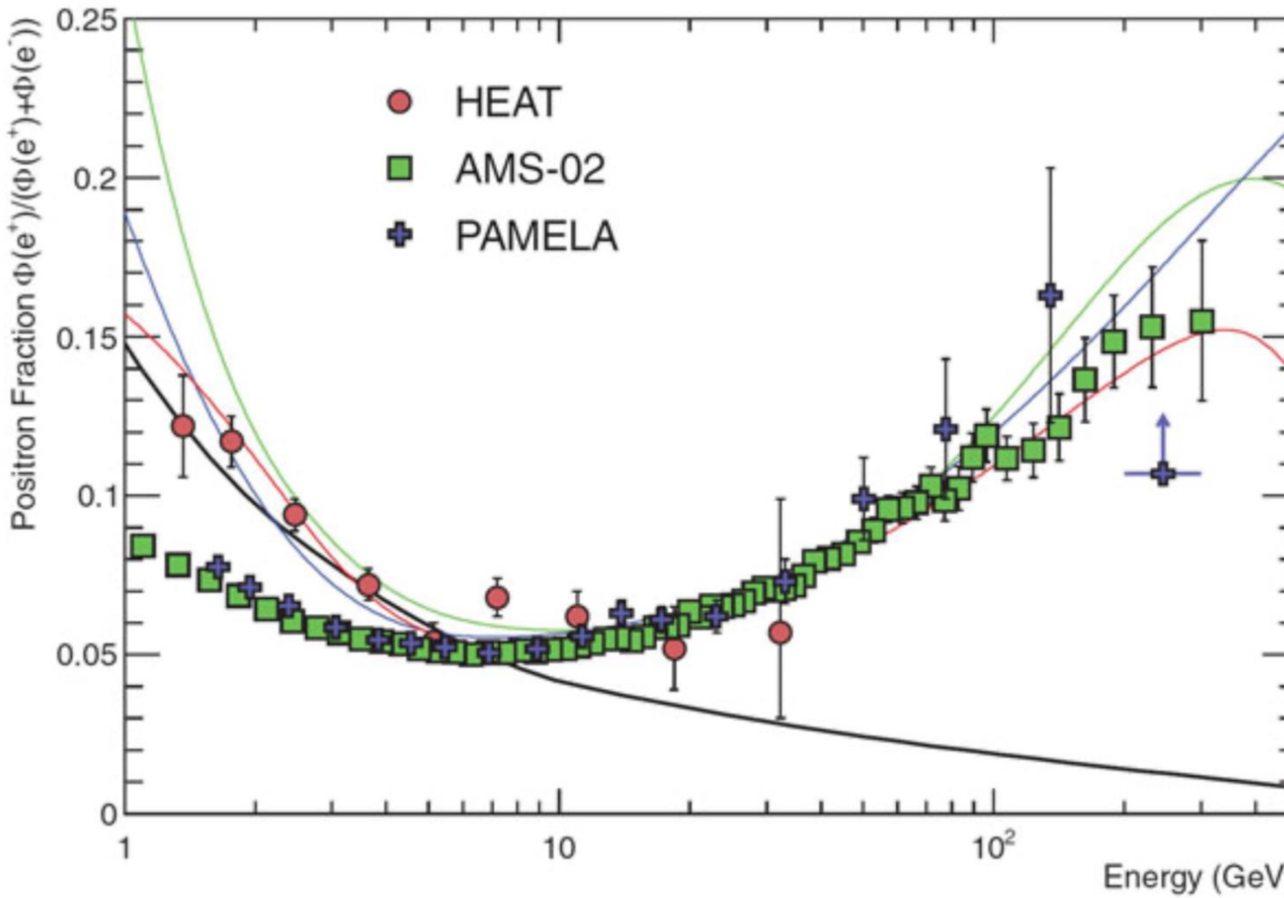
Electrons/positrons fluxes

No sharp structures





- The positron fraction (ratio of the flux of e^+ to the total flux of $e^+ + e^-$) as a function of the energy measured HEAT, PAMELA, and AMS-02.
- The heavy *black line* is a model of pure secondary production using a detailed propagation model of CRs
- The *three thin lines* show three representative attempts to model the positron excess with different phenomena: dark matter decay (*green*); propagation physics (*blue*); production in pulsars (*red*).
- The ratio below 10 GeV is dependent on the polarity of the solar magnetic field.



PAMELA DETECTOR

<u>TRD</u>

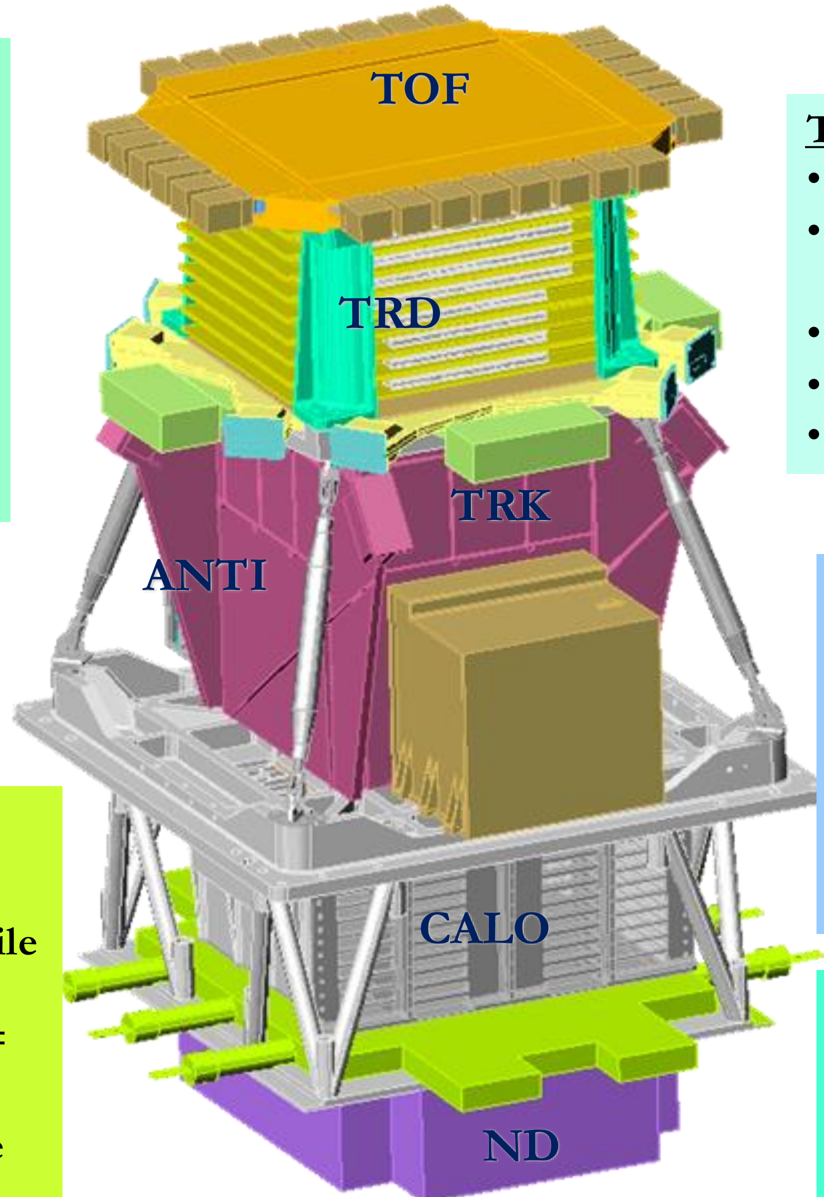
- Threshold detector: signal from $e^{\pm}$, no from p.
- 9 radiator planes (carbon fiber) and straws tubes (4mm diameter) filled with Xe/CO_2 mixture.
- 10^2 e/p separation (E > 1 GeV/c).

<u>Anticoincidence system</u>

- Defines tracker acceptance
- Plastic scint. + PMT

<u>Si-W Calorimeter</u>

- Imaging Calorimeter: reconstructs shower profile discriminating e/p
- Energy Resolution for $e^{\pm}$ $\Delta E/E = 15\% / E^{1/2}$.
- Si-X / W / Si-Y structure
- 22 W planes
- $16.3\ X_0 / 0.6\ \lambda_0$



<u>Time-of-flight</u>

- Level 1 trigger
- particle identification (up to 1GeV/c)
- dE/dx
- Plastic scintillator + PMT
- Time Resolution ~ 70 ps

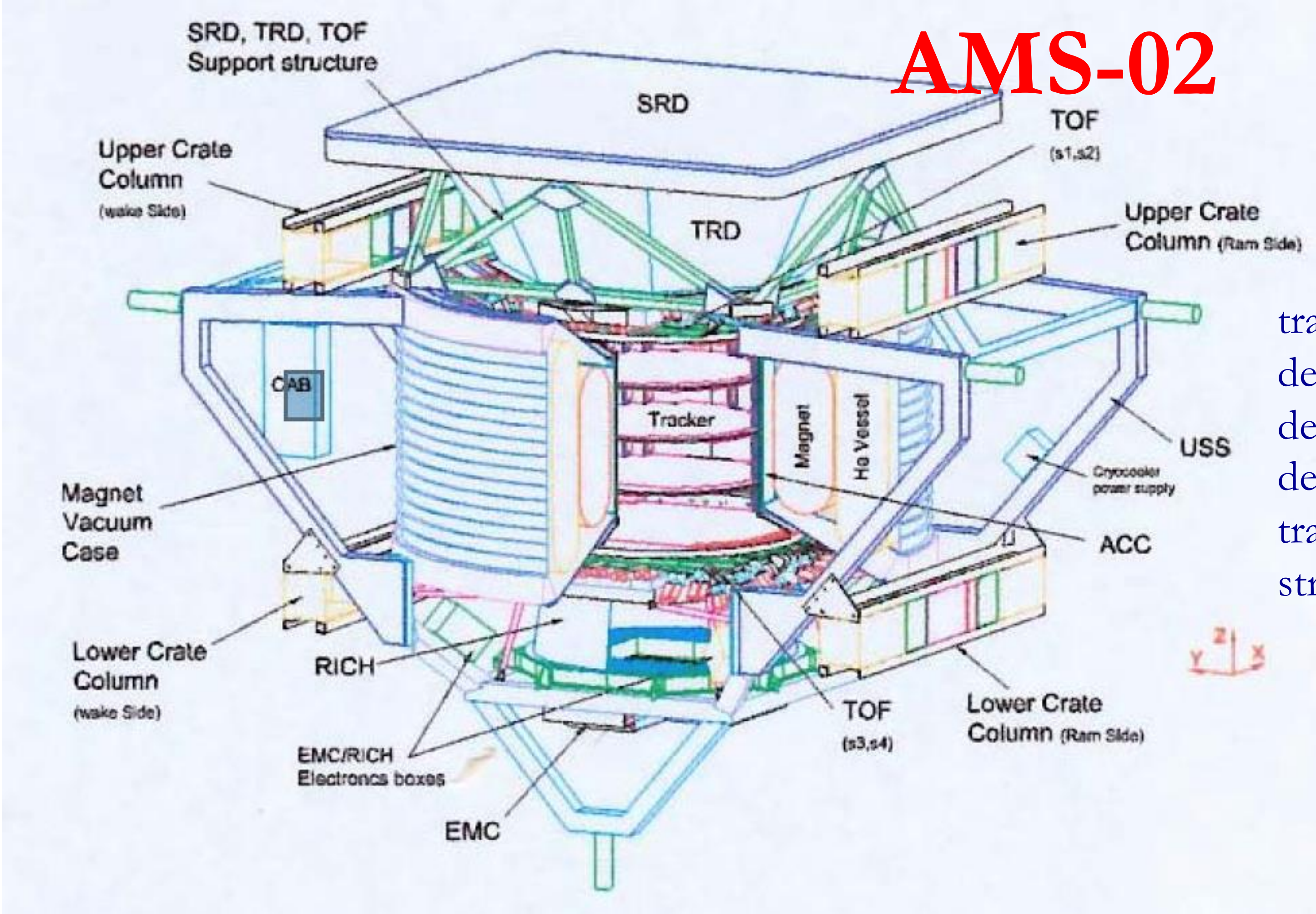
<u>Si Tracker + magnet</u>

- Permanent magnet B=0.4T
- 6 planes double sided Si strips 300 μm thick
- Spatial resolution ~3μm
- MDR = 740 GV/c

<u>Neutron detector</u>

- Extends the energy range for primary protons and electrons up to 10 TeV
- 36 ^{3}He counters in a polyethylene moderator

AMS-02



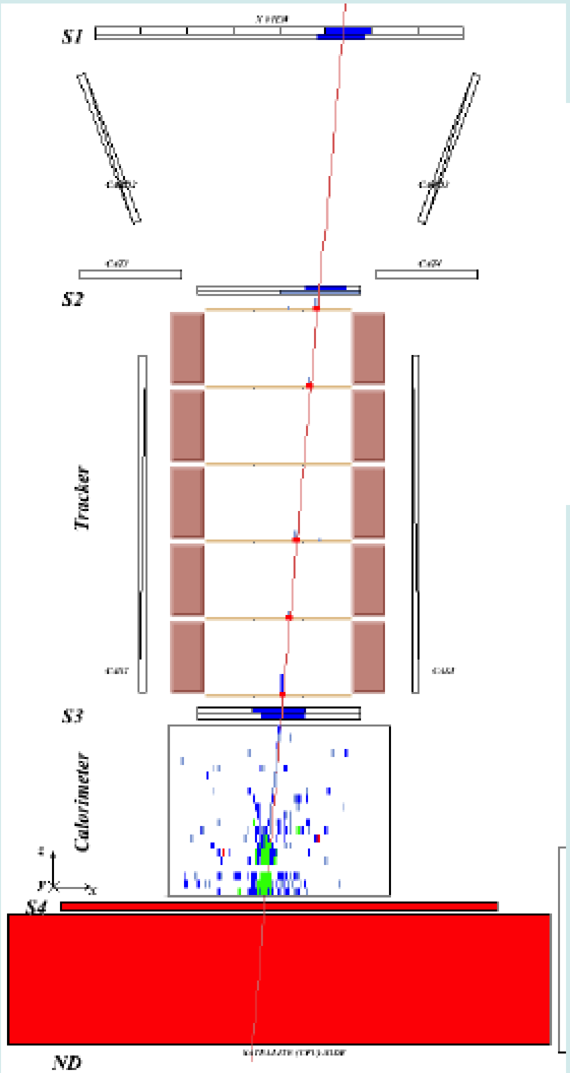
transition radiation detector (TRD) = particle detector using the γ-dependent threshold of transition radiation in a stratified material.

It contains many layers of materials with different indices of refraction.

At each interface between materials, the probability of transition radiation increases with the relativistic γ factor → particles with large γ give off many photons, and small γ give off few.

For a given energy, this allows a discrimination between a lighter particle (which has a high γ and therefore radiates) and a heavier particle (which has a low γ and radiates much less).

Proton / positron selection



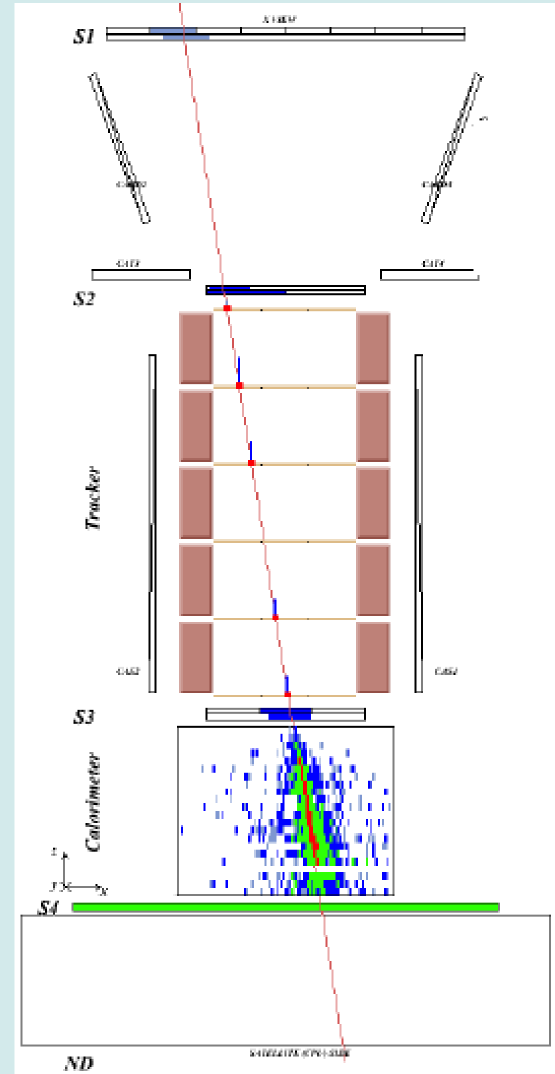
Time-of-flight:
trigger, albedo rejection, mass determination (up to 1 GeV)

Bending in spectrometer:
sign of charge

Ionisation energy loss (dE/dx):
magnitude of charge

Interaction pattern in calorimeter:
electron-like or proton-like, electron energy

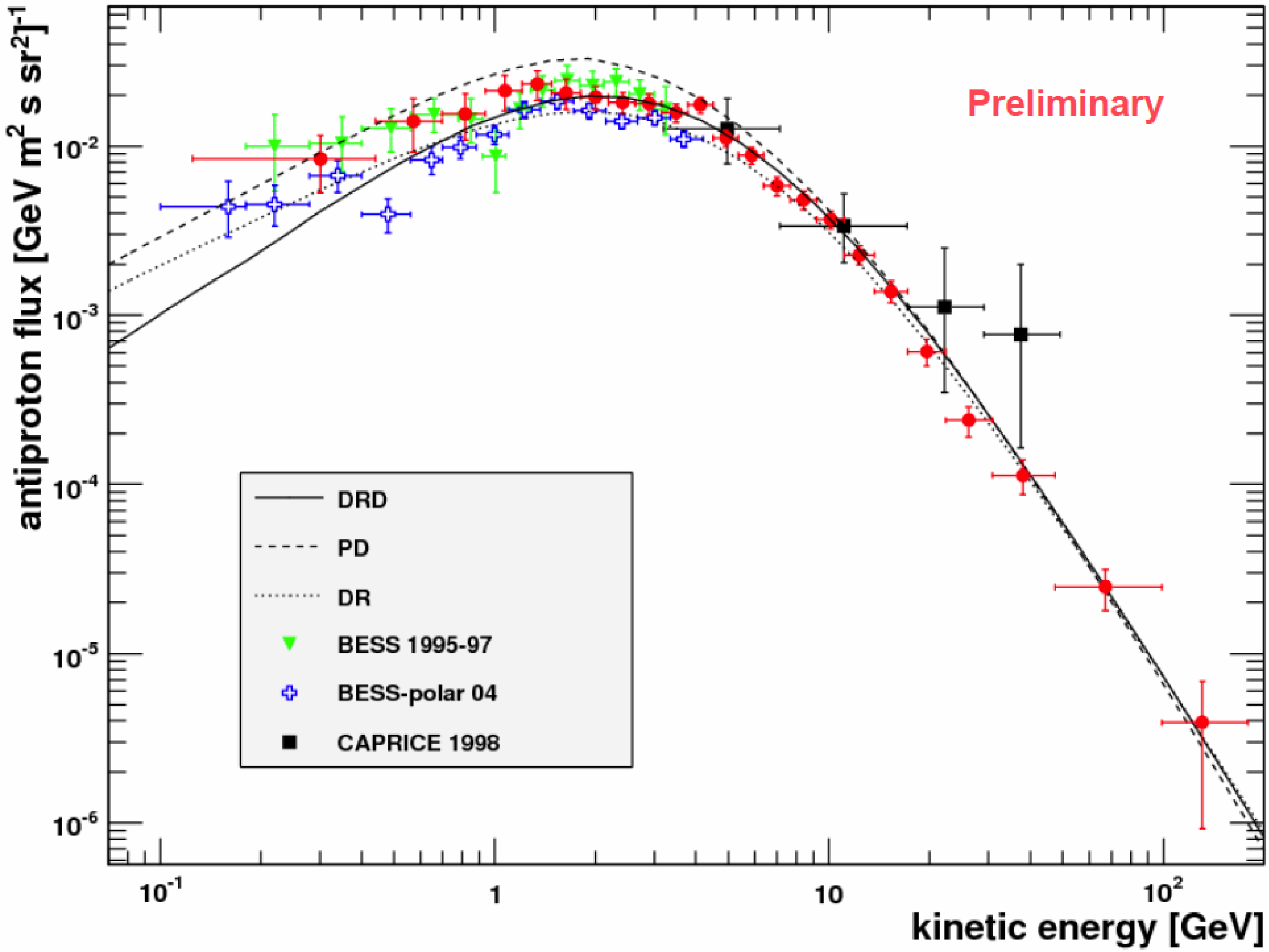
Signal in the Neutron detector



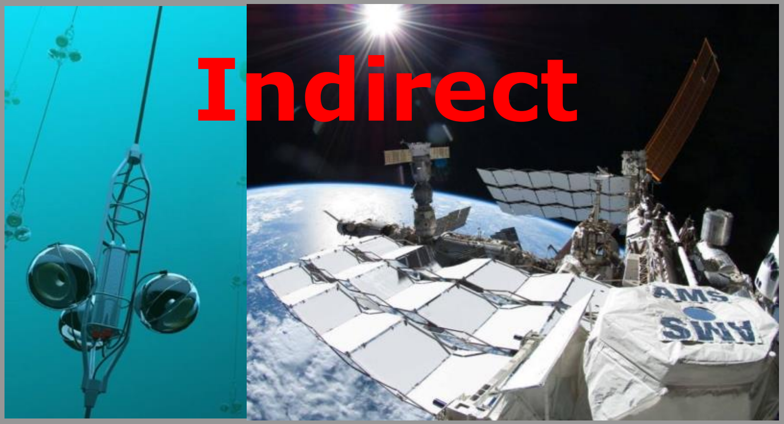
Proton

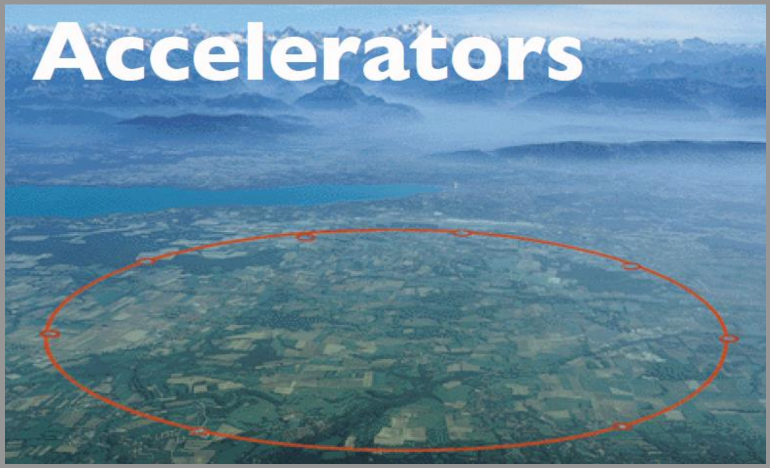
Positron

Not only positrons, but also anti-protons



Hunting the Dark Matter particles

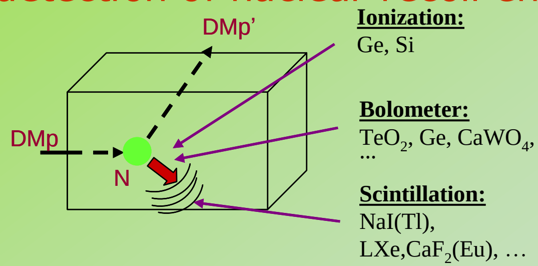




Some direct detection processes:

- Scattering on nuclei

  → detection of nuclear recoil energy



- Inelastic Dark Matter: **W + N → W* + N**

  → W has Two mass states $\chi+$, $\chi-$ with δ mass splitting

  → Kinematical constraint for the inelastic scattering of $\chi-$ on a nucleus

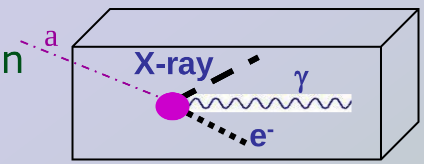
$$\frac{1}{2}\mu v^2 \geq \delta \Leftrightarrow v \geq v_{thr} = \sqrt{\frac{2\delta}{\mu}}$$

- Excitation of bound electrons in scatterings on nuclei

  → detection of recoil nuclei + e.m. radiation

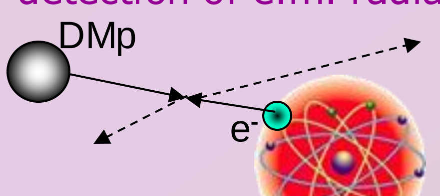
- Conversion of particle into e.m. radiation

  → detection of γ, X-rays, e^-



- Interaction only on atomic electrons

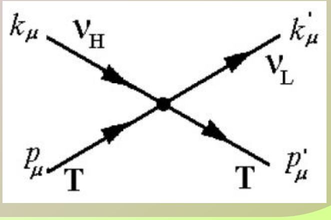
  → detection of e.m. radiation



… even WIMPs

- Interaction of light DMp (LDM) on e^- or nucleus with production of a lighter particle

  → detection of electron/nucleus recoil energy



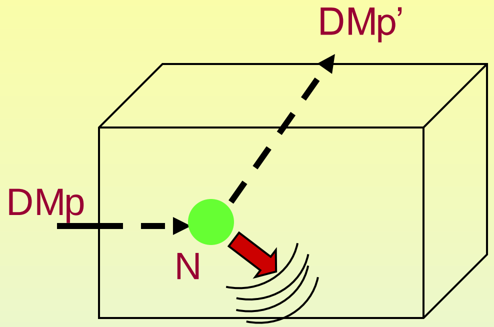
e.g. sterile ν

… also other ideas …

e.g. signals from these candidates are completely lost in experiments based on "rejection procedures" of the e.m. component of their rate

- **… and more**

Just an EXAMPLE: The case of DM particle scatterings on target-nuclei. When just the recoil energy is the detected quantity

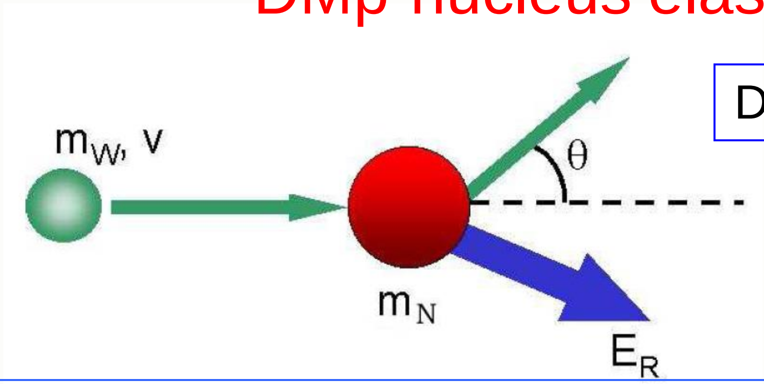


- DM particle-nucleus elastic scattering (SI, SD, SI&SD coupling)
- Preferred inelastic DM particle-nucleus scattering (S_m/S_0 enhanced with respect to the elastic scattering case)

The differential energy distribution depends:

- on the assumed scaling laws, nuclear form factors, spin factors, free parameters ($\rightarrow$ kind of coupling, mixed SI&SD, pure SI, pure SD, pure SD through Z_0 exchange, pure SD with dominant coupling on proton, pure SD with dominant coupling on neutron, preferred inelastic, ...),
- on the assumed astrophysical model (halo model, presence of non-thermalized components, particle velocity distribution, particle density in the halo, ...)
- on instrumental quantities (quenching factors, energy resolution, efficiency, ...)

DMp-nucleus elastic scattering: 1) kinematics



DMp kinetic energy

Angle of scatt. in CM

$$E_R = \frac{1}{2} E \cdot r(1 - \cos\theta^*)$$

$$r = 4 m_W m_N / (m_W + m_N)^2 < 1$$

Minumum velocity to provide recoiling energy E_R:

$$v_{min}(E_R) = \sqrt{2E_R/(r\, m_W)}$$

Maximum recoiling energy given by a DMp with energy E:

$$E_{max} = E \cdot r$$

- Depending on the used target $m_N = 1 – 200$ GeV.
- Tipically in the galactic halo $v \sim 300$ km/s ≈ 10^{-3} c; thus $E \sim 0.5 \times 10^{-6}\, m_W$
- Some numerical examples for $m_N \approx 100$ GeV (typical for the used materials):

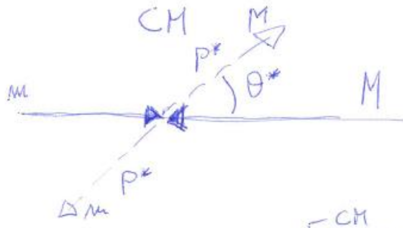
If $m_W << m_N$ one has $r \approx 4\, m_W / m_N$, thus $E_{max} \approx 2 \times 10^{-6}\, m_N\, (m_W / m_N)^2 <<$ 200 keV

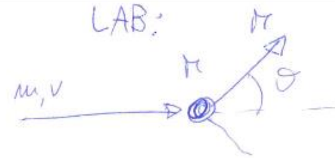
If $m_N = m_W$ one has $r = 1$, thus $E_{max} = E \sim$ 100 keV

If $m_N << m_W$ one has $r \approx 4\, m_N / m_W$, thus $E_{max} \approx 2 \times 10^{-6}\, m_N \sim$ 200 keV

Moreover, the detection of the nuclear recoiling energy depends upon quenching factors => direct detection needs low-energy threshold detectors

Non-relativistic kinematics





$$E_{tot}^{CM} = \frac{1}{2}\mu v^2$$

$$\frac{1}{2}\mu v^2 = \frac{p^{*2}}{2M} + \frac{p^{*2}}{2m} = \frac{p^{*2}}{2\mu} \Rightarrow p^* = \mu v$$

$$\vec{V}_R^{CM} = \frac{\mu v}{M}\begin{pmatrix} \cos\vartheta^* \\ \operatorname{sen}\vartheta^* \end{pmatrix} \qquad \vec{\beta}_{LAB,CM} = \frac{\mu}{M} v \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Q_{LAB} = m v = (m+M)\beta$$

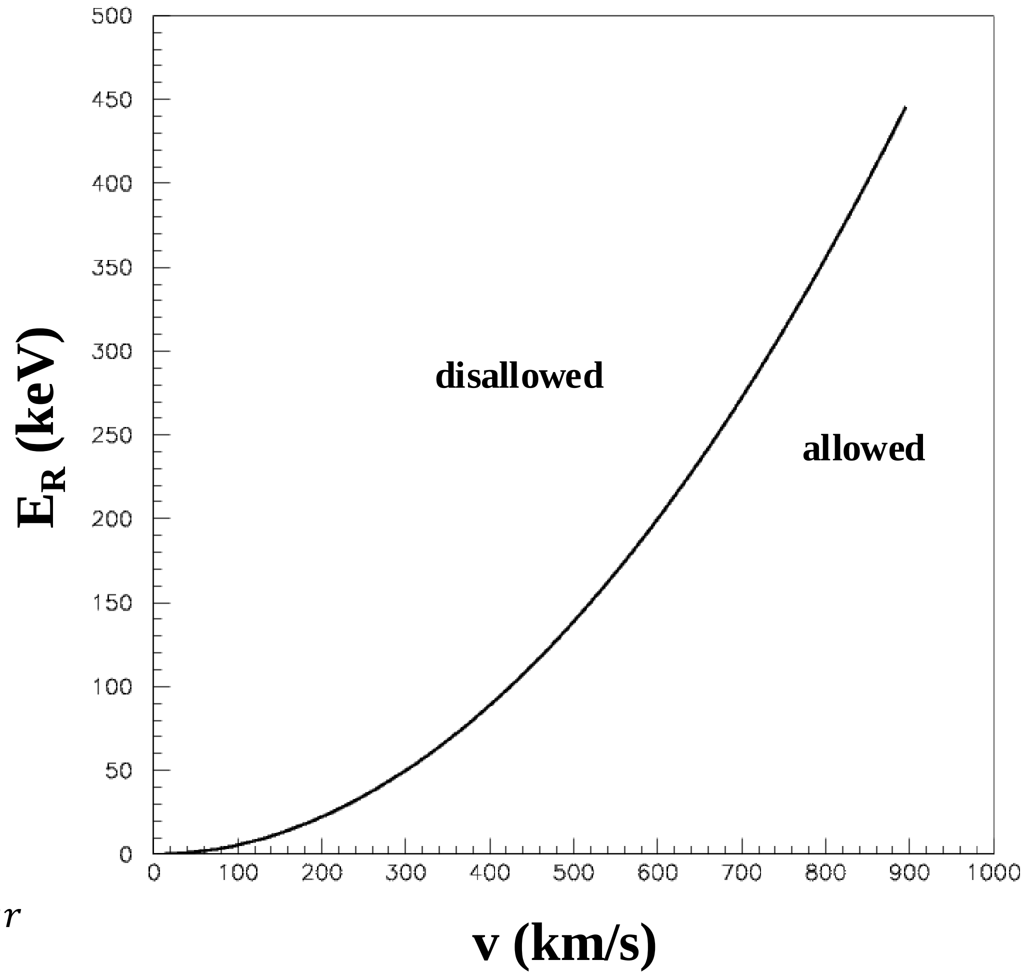
$$\beta = \frac{m v}{m+M} = \frac{\mu}{M} v$$

$$\vec{V}_R^{LAB} = \frac{\mu v}{M}\begin{pmatrix} \cos\vartheta^* - 1 \\ \operatorname{sen}\vartheta^* \end{pmatrix}$$

$$E_R = \frac{1}{2} M {V_R^{LAB}}^2 = \frac{1}{2} M \frac{\mu^2 v^2}{M^2}\left(2 - 2\cos\vartheta^*\right) = \frac{1}{2}\left(\frac{1}{2} m v^2\right)\frac{4 m M}{(m+M)^2}\left(1 - \cos\vartheta^*\right)$$

Assumptions:

- *r=1*
- m_w*=100 GeV*



$$E_R < E_{R,max} = \frac{1}{2} m_w v^2 r$$

$$v > v_{min} = \sqrt{\frac{2E_R}{r m_w}}$$

The Form Factor

Momentum transfer: $\vec{q} = \vec{p_1} - \overrightarrow{p'_1} = \overrightarrow{p'_2} - \vec{p_2}$

Thus $q = \left|\overrightarrow{p'_2}\right| = \sqrt{2M_N E_R}$ and the spatial resolution: $\delta x = \frac{\hbar}{\sqrt{2M_N E_R}}$;

Examples:

- ^{23}Na $M_N = 21.6\ GeV;\ E_R = 10\ keV\ \rightarrow$

  $$\delta x = \frac{197\ MeV}{\sqrt{2 \cdot 21.6\ 10^3 MeV \cdot 10\ 10^{-3} MeV}}\ \text{fm} = 9.5\ \text{fm}$$

  greater than nuclear radius $\approx 1.2\ A^{1/3} = 3.4$ fm

  → the nucleus is seen as point-like → $F^2(q) \approx 1$

- ^{127}I $M_N = 119.4\ GeV;\ E_R = 30\ keV\ \rightarrow$

  $$\delta x = \frac{197\ MeV}{\sqrt{2 \cdot 119.4\ 10^3 MeV \cdot 30\ 10^{-3} MeV}}\ \text{fm} = 2.3\ \text{fm}$$

  lower than nuclear radius $\approx 1.2\ A^{1/3} = 6.0$ fm

  → the nucleus is no longer seen as point-like → $F^2(q) \ll 1$

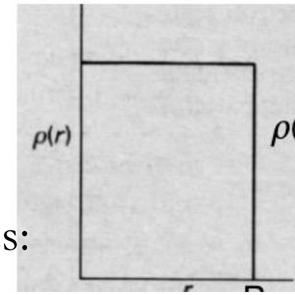
In conclusion, the nucleus can be considered as point-like ($F^2(q) \approx 1$) if $\delta x \geq R_N$,

that is: $$E_R \lesssim 6.7\ keV \left(\frac{100}{A}\right)^{5/3}$$

The Form Factor

In nuclear physics the electric nuclear form factor is the Fourier Transform of the charge density

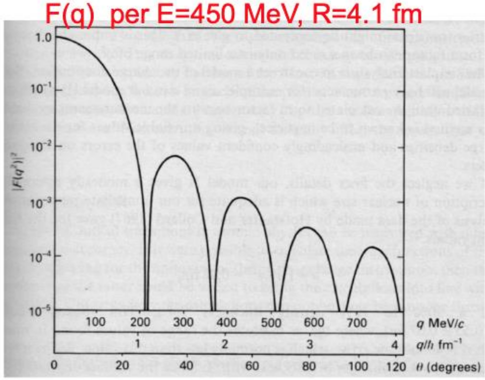
$$F_E(\vec{q}) = \int_0^R \rho(\vec{r}\,')\, e^{i\frac{\vec{q}}{\hbar}\cdot\vec{r}\,'}\, d^3\vec{r}\,'$$



$$\rho(r) = \begin{cases} \dfrac{3}{4\pi R^3} & r \le R \\ 0 & r > R \end{cases}$$

The Form Factor for a spherical charge distribution with sharp edges:

F(q) per E=450 MeV, R=4.1 fm



Charge distribution $f(r)$		Form Factor $F(q^2)$		
point	$\delta(r)/4\pi$	1	constant	electron
exponential	$(a^3/8\pi)\cdot\exp(-ar)$	$(1+q^2/a^2\hbar^2)^{-2}$	dipole	proton
Gaussian	$(a^2/2\pi)^{3/2}\cdot\exp(-a^2r^2/2)$	$\exp(-q^2/2a^2\hbar^2)$	Gaussian	light nuclei
homogeneous sphere	$\begin{cases} 3/4\pi R^3 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$	$3\,\alpha^{-3}(\sin\alpha - \alpha\cos\alpha)$ with $\alpha = \lvert q\rvert R/\hbar$	oscillating	–
homogeneous sphere **with smoothed edges**			oscillating	**Medium/Heavy nuclei**

The Helm Form Factor for a spherical charge distribution with smoothed edges:

$$F(q) = 3\frac{j_1(qr_0)}{qr_0}\exp\left(-\frac{1}{2}s^2q^2\right)$$

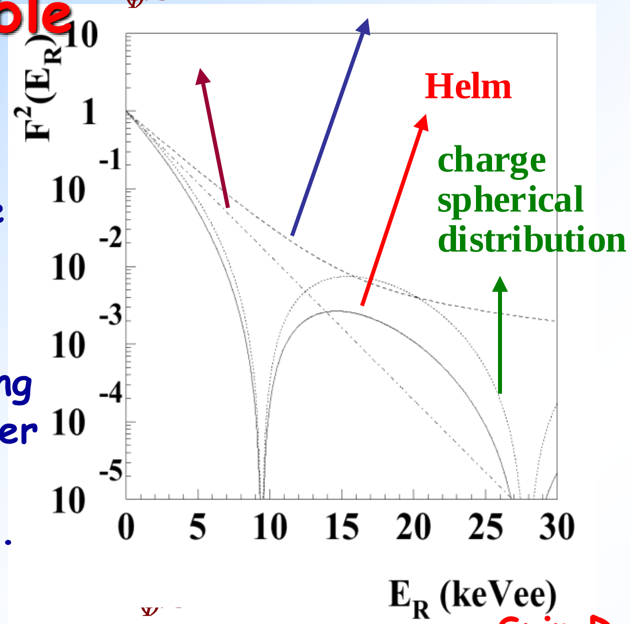
where $s \simeq 1$ fm is the thickness parameter for the nucleus surface, $r_0=(r^2-5s^2)^{1/2}$, $r=1.2A^{1/3}$ fm, and $j_1(qr_0)$ is the spherical Bessel function of index 1 $\left(j_1(z) = \dfrac{\sin z}{z^2} - \dfrac{\cos z}{z}\right)$.

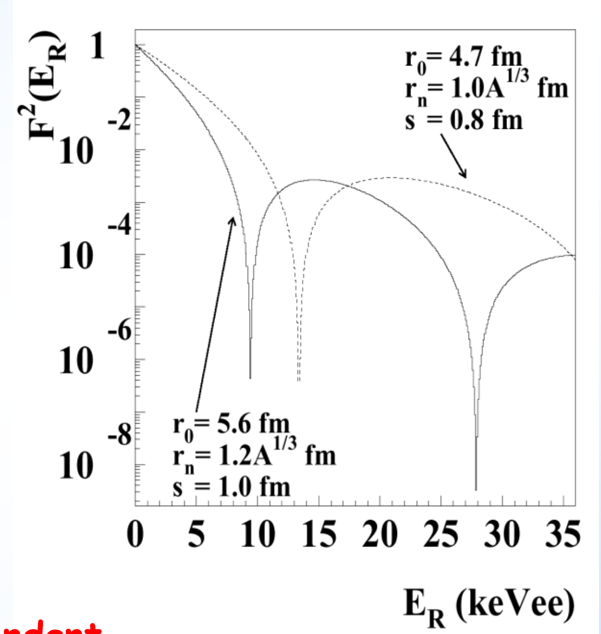
Examples of different Form Factor for ^{127}I available in literature

- Take into account the structure of target nuclei
- In SD form factor: no decoupling between nuclear and Dark Matter particles degrees of freedom; dependence on nuclear potential.

Spin Independent

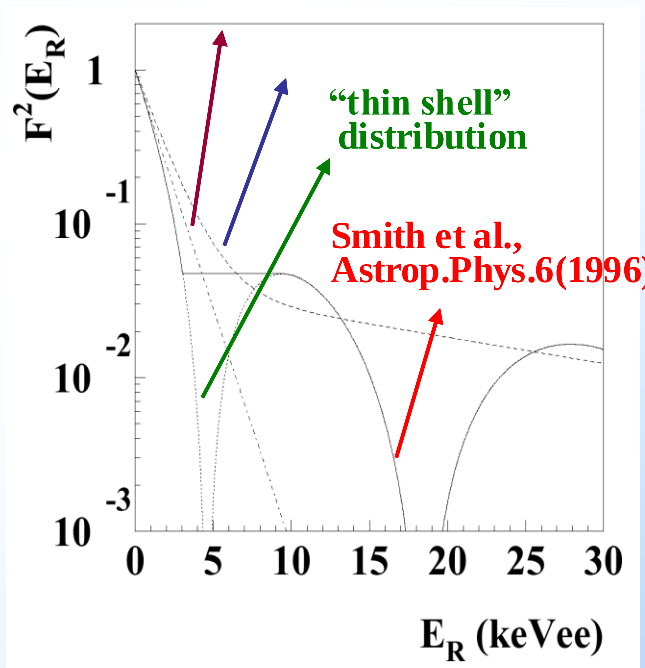
from Helm

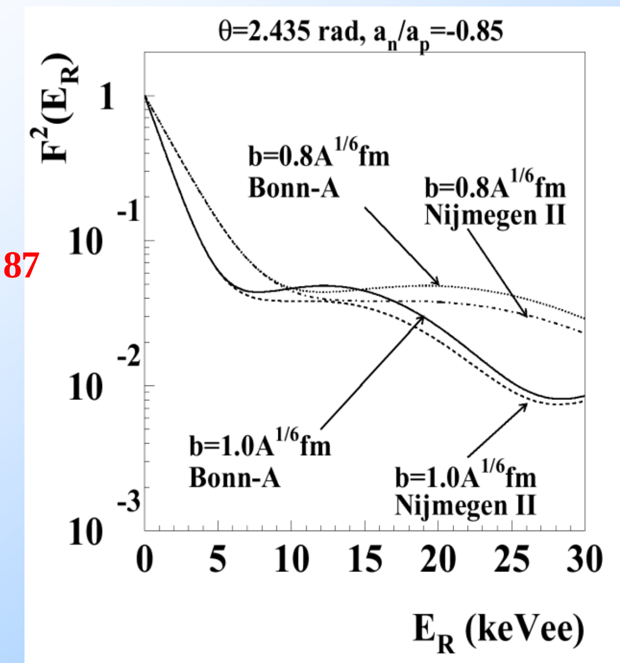




Spin Dependent

from Ressell et al.





Similar situation for all the target nuclei considered in the field

WIMP case - SI

$$E_R = \frac{\mu_N^2 v^2}{M_N c^2}(1-\cos\theta^*)$$

$$dE_R = \frac{\mu_N^2 v^2}{M_N c^2}\, d\cos\theta^* \,\frac{d\varphi^*}{2\pi}$$

$$\frac{d\Omega}{dE_R} = \frac{2\pi M_N c^2}{\mu_N^2 v^2}$$

$$\frac{d\sigma}{dE_R} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{dE_R} = \frac{\sigma_n}{4\pi}\frac{\mu_N^2}{\mu_p^2} A^2 \frac{d\Omega}{dE_R}$$

$$\mu_N = \frac{M_N M_W}{M_N + M_W}$$

$$\mu_p = \frac{m_p \cdot M_W}{M_W + m_p}$$

Scaling law of cross sections:
$\sigma_N = \sigma_n\,(\mu_N/\mu_p)^2\,A^2$

$$\frac{d\sigma}{dE_R} = \frac{1}{2}\sigma_n A^2 \frac{M_N c^2}{\mu_p^2 v^2}$$

DMp-nucleus elastic scattering: 2) cross section

Considering the typical expected energies:

Nuclear Form Factor

$$\frac{d\sigma}{dE_R}(v, E_R) = \frac{d\sigma}{dE_R}(v, 0) \cdot F^2(E_R)$$

$$\frac{d\sigma}{dE_R}(v, 0) = \frac{d\sigma}{d\Omega^*}\frac{d\Omega^*}{dE_R}$$

- Assuming isotropic distribution in CM: $\frac{d\sigma}{d\Omega^*} = \frac{\sigma}{4\pi}$
- $E_R = \frac{1}{2}E \cdot r(1 - \cos\theta^*)$ ➡ $dE_R = \frac{1}{4\pi}E \cdot r d\Omega^*$

➡ $$\frac{d\sigma}{dE_R} = \frac{\sigma}{E_{max}}$$ ← point-like

$$E_{max} = E \cdot r$$

$$\frac{dN}{dE_R} = N_T \cdot \frac{\rho V}{M_W} \cdot \frac{d\sigma}{dE_R} \cdot f(v)\,dv \cdot F(E_R)$$

$$\frac{d\sigma}{dE_R} = \frac{1}{2}\sigma_m A^2 \frac{M_N c^2}{\mu_p^2 v^2}$$

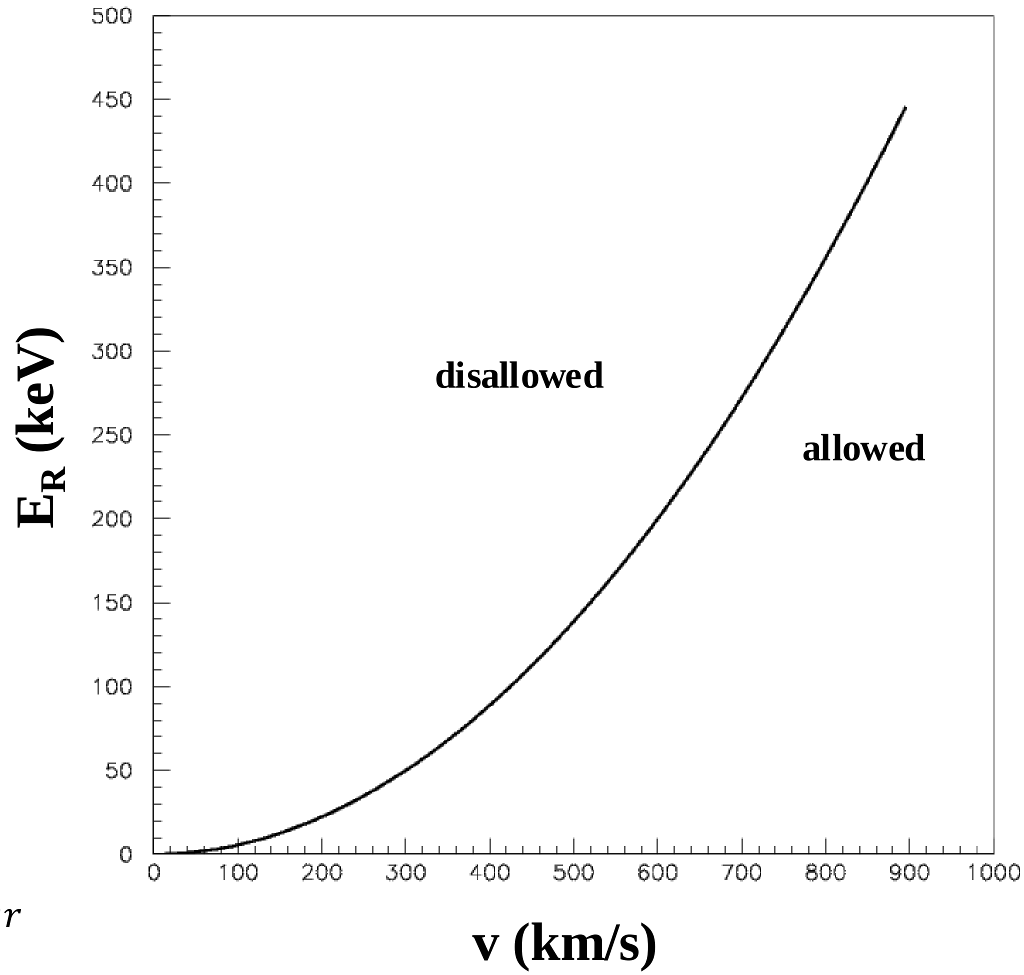
$$\frac{dN}{dE_R} = N_T \frac{\rho}{M_W} \frac{1}{2}\sigma_m A^2 \frac{M_N c^2}{\mu_p^2} \int_{v_{min}}^{\infty} \frac{f(v)}{v} dv \cdot F(E_R)$$

For Na and Iodine:
$N_T = 4.015\ 10^{24}\ \mathrm{kg}^{-1}$

$$I(v_{min}) = \int_{v_{min}}^{\infty} \frac{f(v)}{v} dv \qquad [I] = \frac{s}{km}$$

Assumptions:

- $r=1$
- $m_w=100\ GeV$



$$E_R < E_{R,max} = \frac{1}{2} m_w v^2 r$$

$$v > v_{min} = \sqrt{\frac{2E_R}{r m_w}}$$

DM particle-nucleus elastic scattering

Differential energy distribution:

Case of flux of monoenergetic particles:

$$R = N_B \, n_W \, \sigma \, v$$

Dark Matter in the galactic halo has a velocity distribution f(v) :

$$R = N_B \, n_W \int v \, f(v) \, \sigma \, dv$$

$$n_W = \frac{\rho_W}{m_W} = \frac{\xi \rho_0}{m_W}$$

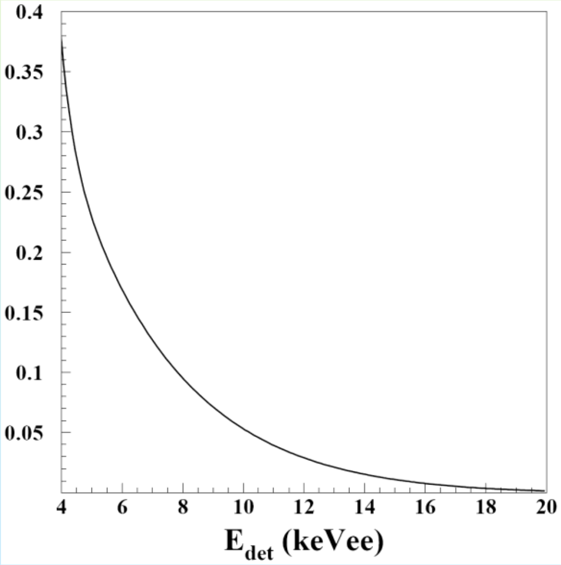
ξ is the halo fraction; ρ_0 is the halo density

$$\frac{dR}{dE_R} = N_T \frac{\rho_W}{m_W} \int_{v_{\min}(E_R)}^{v_{\max}} \frac{d\sigma}{dE_R}(v, E_R) v f(v) dv = N_T \frac{\rho_W m_N}{2 m_W m_{Wp}^2} \cdot \Sigma(E_R) \cdot I(E_R)$$

$$I(E_R) = \int_{v_{\min}(E_R)}^{v_{\max}} \frac{f(v)}{v} dv$$

$$v_{\min} = \sqrt{\frac{m_N E_R}{2 m_{WN}^2}}$$

minimal velocity providing E_R recoil energy



$N_{T,B}$: number of target nuclei

f(v): DM particle velocity distribution in the Earth frame (**it depends on v_e**)

$v_e = v_{sun} + v_{orb} \cos\omega t$

v_{max}: maximal DM particle velocity in the Earth frame

Example:

- **m_W=20 GeV;**
- **pure SI;**
- **NFW halo;**
- **v_0=200 km/s;**
- **NaI(Tl)**

A toy model on the velocity distribution

$$I(E_R) = \int_{v_{min}(E_R)}^{\infty} \frac{f(v)}{v} dv$$

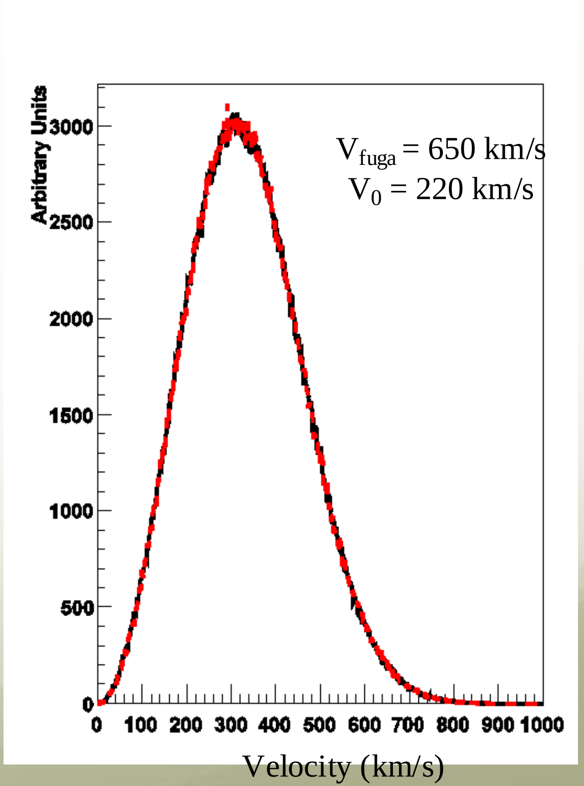
The simplest velocity distribution for the galactic halo is given by the iso-thermal sphere where **f(v)** is a **Maxwellian** distribution

$$f(v)dv = \int_{\Omega} g(\vec{v})\, d^3v \quad \Rightarrow \quad g(\vec{w}) = \frac{1}{\pi^{3/2} v_0^3} e^{-\frac{w^2}{v_0^2}} \quad (\vec{w} = \vec{v} + \vec{v_E})$$

$$f(v) = \frac{2v^2}{\sqrt{\pi} v_0^3} e^{-\frac{v^2+v_E^2}{v_0^2}} \int_{-1}^{1} e^{-\frac{2vv_E cos\vartheta}{v_0^2}} dcos\vartheta =$$

$$= \frac{v}{\sqrt{\pi} v_0 v_E} e^{-\frac{v^2+v_E^2}{v_0^2}} \left(e^{\frac{2vv_E}{v_0^2}} - e^{-\frac{2vv_E}{v_0^2}} \right)$$

$$= \frac{v}{\sqrt{\pi} v_0 v_E} \left(e^{-\frac{(v-v_E)^2}{v_0^2}} - e^{-\frac{(v+v_E)^2}{v_0^2}} \right)$$



BUT: v_E is dependent on time (then, annual modulation of the signal)

DM particle-nucleus elastic scattering

Differential energy distribution:

$$\frac{dR}{dE_R} = N_T \frac{\rho_W}{m_W} \int_{v_{\min}}^{v_{\max}} \frac{d\sigma}{dE_R}(v, E_R)\, v f(v)\, dv$$

SI+SD differential cross sections:

$$\frac{d\sigma}{dE_R}(v,E_R) = \left(\frac{d\sigma}{dE_R}\right)_{SI} + \left(\frac{d\sigma}{dE_R}\right)_{SD} =$$

$$\frac{2G_F^2 m_N}{\pi v^2}\left\{\left[Zg_p + (A-Z)g_n\right]^2 F_{SI}^2(E_R) + 8\frac{J+1}{J}\left[a_p\langle S_p\rangle + a_n\langle S_n\rangle\right]^2 F_{SD}^2(E_R)\right\}$$

$g_{p,n}$($a_{p,n}$) effective DM particle-nucleon couplings

$\langle S_{p,n}\rangle$ nucleon spin in the nucleus

$F^2(E_R)$ nuclear form factors

m_{Wp} reduced DM particle-nucleon mass

Generalized SI/SD DM particle-nucleon cross sections:

$$\sigma_{SI} = \frac{4}{\pi} G_F^2 m_{Wp}^2 g^2 \qquad \sigma_{SD} = \frac{32}{\pi}\frac{3}{4} G_F^2 m_{Wp}^2 \bar{a}^2$$

where:

$$g = \frac{g_p + g_n}{2} \cdot \left[1 - \frac{g_p - g_n}{g_p + g_n}\left(1 - \frac{2Z}{A}\right)\right]$$

$$\bar{a} = \sqrt{a_p^2 + a_n^2} \qquad tg\theta = \frac{a_n}{a_p}$$

Used – but not universal – scaling laws for nuclear cross sections:

$$\sigma_{SI}^{nucleus} = \sigma_{SI} A^2 \left(\frac{m_{WN}}{m_{Wp}}\right)^2 \qquad \sigma_{SD}^{nucleus} = \sigma_{SD} \frac{F_{spin}^N}{F_{spin}^p}\left(\frac{m_{WN}}{m_{Wp}}\right)^2$$

spin factors:

$$F_{spin}^N = \Lambda^2 J(J+1)$$

$$F_{spin}^p = \frac{3}{4}$$

Examples of uncertainties in models and scenarios

see for some details e.g.:
Riv.N.Cim.26 n.1 (2003) 1, IJMPD13(2004)2127, EPJC47 (2006)263, IJMPA21 (2006)1445

Nature of the candidate and couplings

- WIMP class particles (neutrino, sneutrino, etc.): SI, SD, mixed SI&SD, preferred inelastic + e.m. contribution in the detection
- Light bosonic particles
- Kaluza-Klein particles
- Mirror dark matter
- Heavy Exotic candidate
- …etc. etc.

Halo models & Astrophysical scenario

- Isothermal sphere ⇒ very simple but unphysical halo model
- Many consistent halo models with different density and velocity distribution profiles can be considered with their own specific parameters (see e.g. PRD61(2000)023512)
- Caustic halo model
- Presence of non-thermalized DM particle components
- Streams due e.g. to satellite galaxies of the Milky Way (such as the Sagittarius Dwarf)
- Multi-component DM halo
- Clumpiness at small or large scale
- Solar Wakes
- …etc. …

Instrumental quantities

- Energy resolution
- Efficiencies
- Quenching factors
- Channeling effects
- Their dependence on energy
- …

Scaling laws
of cross sections for the case of recoiling nuclei

- Different scaling laws for different DM particle:

$\sigma_A \propto \mu^2 A^2 (1+\varepsilon_A)$

$\varepsilon_A = 0$ *generally assumed*

$\varepsilon_A \approx \pm 1$ *in some nuclei? even* for neutralino candidate in MSSM (see Prezeau, Kamionkowski, Vogel et al., PRL91(2003)231301)

Form Factors
for the case of recoiling nuclei

- Many different profiles available in literature for each isotope
- Parameters to fix for the considered profiles
- Dependence on particle-nucleus interaction
- In SD form factors: no decoupling between nuclear and Dark Matter particles degrees of freedom + dependence on nuclear potential

Spin Factors
for the case of recoiling nuclei

- Calculations in different models give very different values also for the same isotope
- Depend on the nuclear potential models
- Large differences in the measured counting rate can be expected using:

  either SD not-sensitive isotopes

  or SD sensitive isotopes depending on the unpaired nucleon (compare e.g. odd spin isotopes of Xe, Te, Ge, Si, W with the ^{23}Na and ^{127}I cases).

Quenching Factor

- differences are present in different experimental determinations of *q* for the same nuclei in the same kind of detector depending on its specific features (e.g. *q* depends on dopant and on the impurities; in liquid noble gas e.g.on trace impurities, on presence of degassing/releasing materials, on thermodynamical conditions, on possibly applied electric field, etc); assumed 1 in bolometers
- channeling effects possible increase at low energy in scintillators (dL/dx)
- possible larger values of *q* (AstropPhys33 (2010) 40)

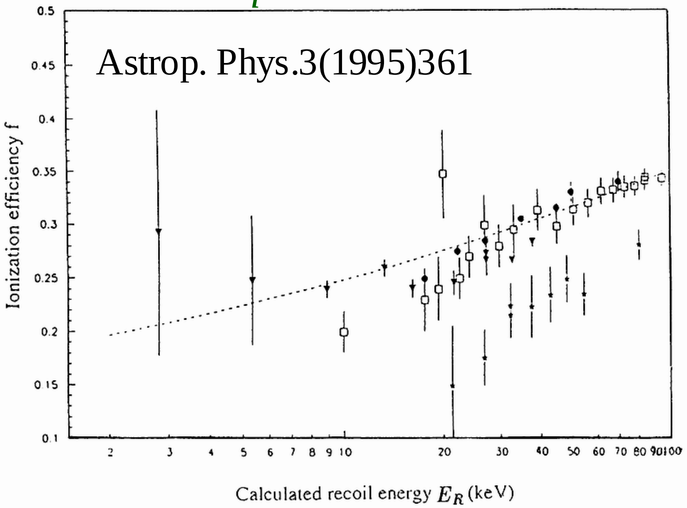
→ energy dependence

… and more …

Quenching factor

Quenching factors, q, measured by neutron sources or by neutron beams for some detectors and nuclei

Ex. of different q determinations for Ge



- differences are often present in different experimental determinations of q for the same nuclei in the same kind of detector
- e.g. in doped scintillators q depends on dopant and on the impurities/trace contaminants; in LXe e.g.on trace impurities, on initial UHV, on presence of degassing/releasing materials in the Xe, on thermodynamical conditions, on possibly applied electric field, etc.
- Some time increases at low energy in scintillators (dL/dx)

… and more

recoil/electron response ratio measured with a neutron source or at a neutron generator

Nucleus/Detector	Recoil Energy (keV)	q	Reference
NaI(Tl)	(6.5-97)	(0.30 ± 0.01) for Na	[46]
	(22-330)	(0.09 ± 0.01) for I	[46]
	(20-80)	(0.25 ± 0.03) for Na	[119]
	(40-100)	(0.08 ± 0.02) for I	[119]
	(4-252)	(0.275 ± 0.018) for Na	[120]
	(10-71)	(0.086 ± 0.007) for I	[120]
	(5-100)	(0.4 ± 0.2) for Na	[121]
	(40-300)	(0.05 ± 0.02) for I	[121]
CaF_2(Eu)	(30-100)	(0.06-0.11) for Ca	[120]
	(10-100)	(0.08-0.17) for F	[120]
	(90-130)	(0.049 ± 0.005) for Ca	[45]
	(75-270)	(0.069 ± 0.005) for F	[45]
	(53-192)	(0.11-0.20) for F	[122]
	(25-91)	(0.09-0.23) for Ca	[122]
CsI(Tl)	(25-150)	(0.15-0.07)	[123]
	(10-65)	(0.17-0.12)	[124]
	(10-65)	(0.22-0.12)	[125]
CsI(Na)	(10-40)	(0.10-0.07)	[125]
Ge	(3-18)	(0.29-0.23)	[126]
	(21-50)	(0.14-0.24)	[127]
	(10-80)	(0.18-0.34)	[128]
	(20-70)	(0.24-0.33)	[129]
Si	(5-22)	(0.23-0.42)	[130]
	22	(0.32 ± 0.10)	[131]
Liquid Xe	(30-70)	(0.46 ± 0.10)	[72]
	(40-70)	(0.18 ± 0.03)	[132]
	(40-70)	(0.22 ± 0.01)	[133]
Bolometers	-	**assumed** 1 (see also NIMA507(2003)643))	