TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Dark Matter, Neutrinos and Undergound Physics

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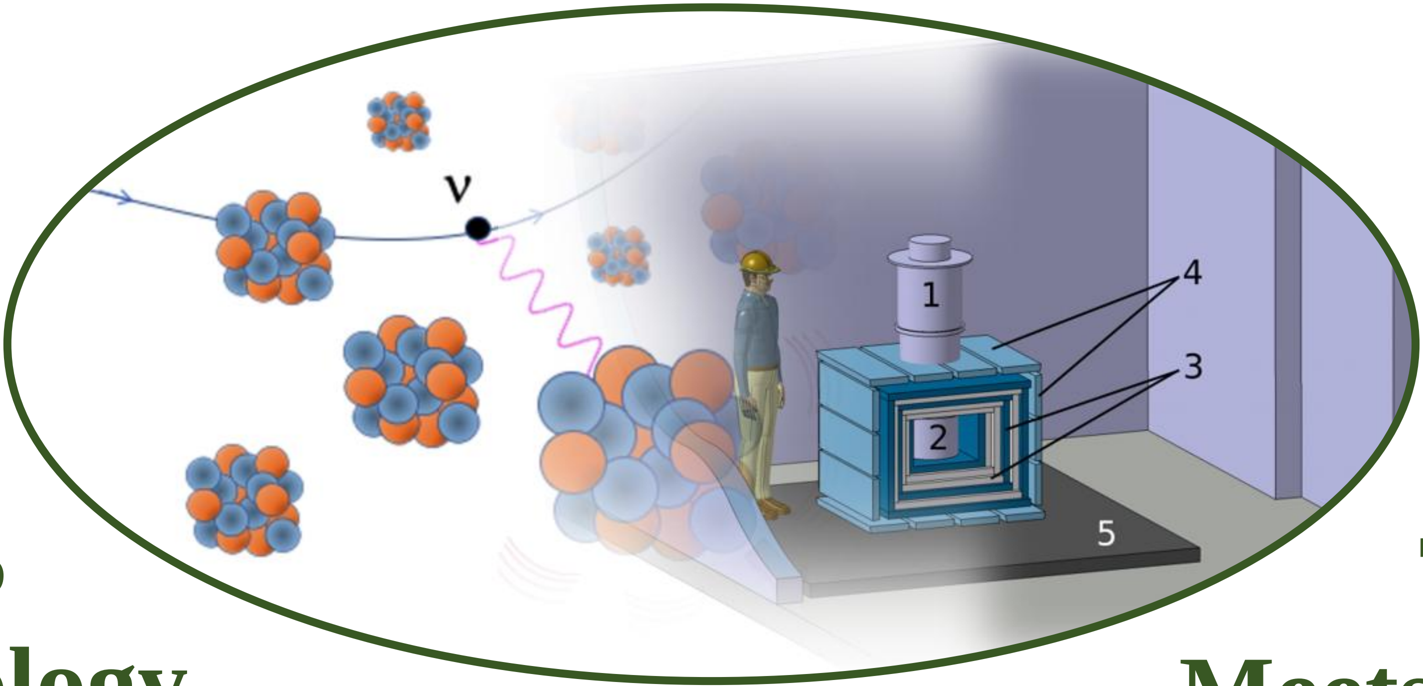
Coherent Elastic Neutrino Nucleus Scattering

From Theory to Discovery

Outlook of This Lecture



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CEνNS Phenomenology

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②

Theory Meets Experiment

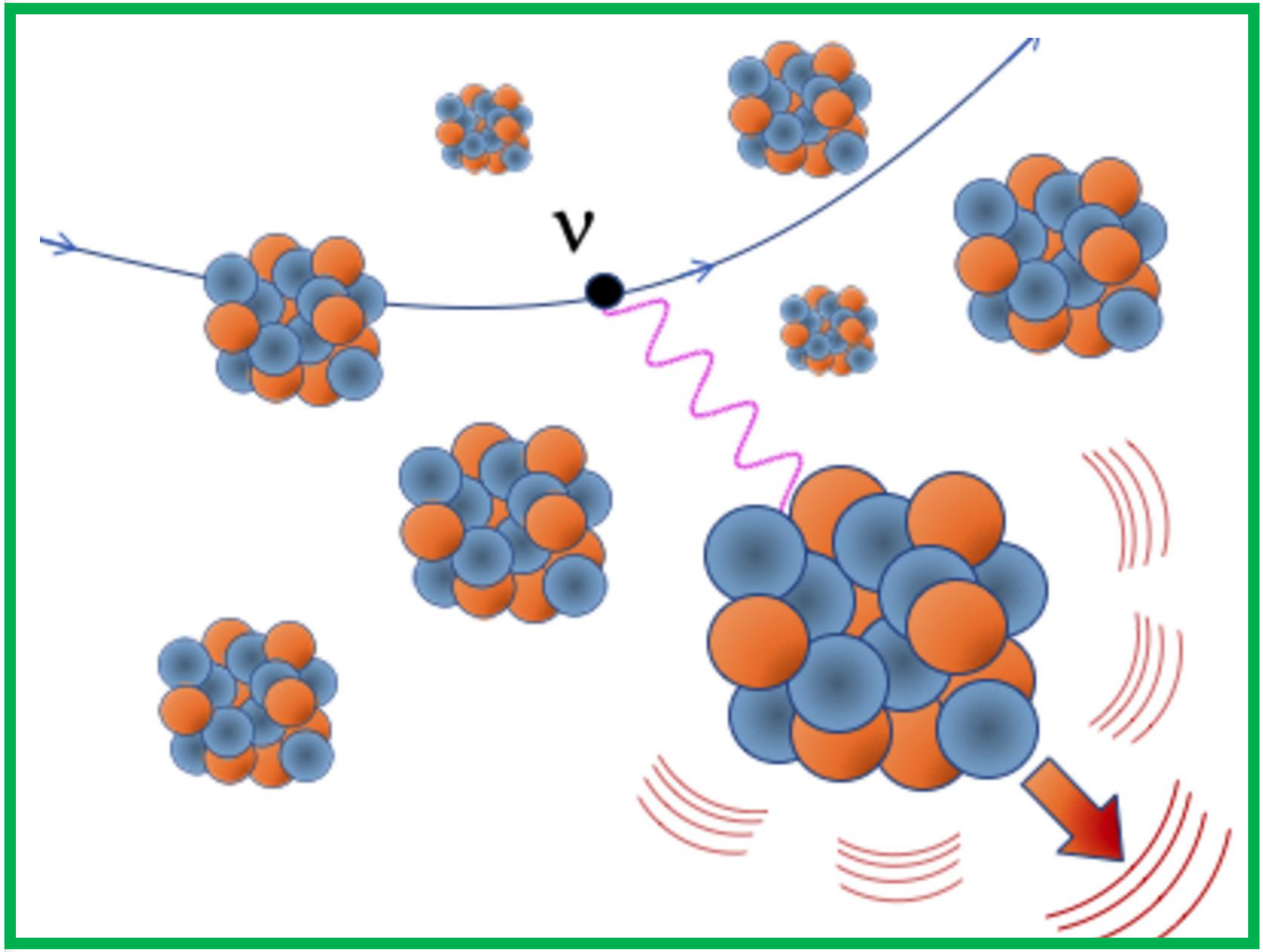
Dr. Riccardo Cerulli

Keywords

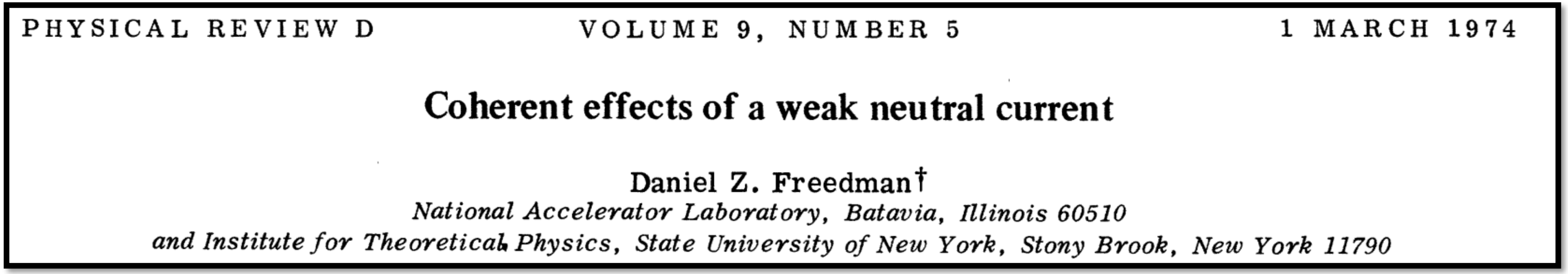
Coherency, Nuclear Recoil, Low-energy Neutrinos, Low-background Experiment, Low-threshold Experiment

CEνNS Phenomenology

i. History
ii. Coherence
iii. Cross Section
iv. Weak Charge of the Nucleus
v. Nuclear Form Factor
vi. Neutrino Sources



History



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Coherent effects of a weak neutral current

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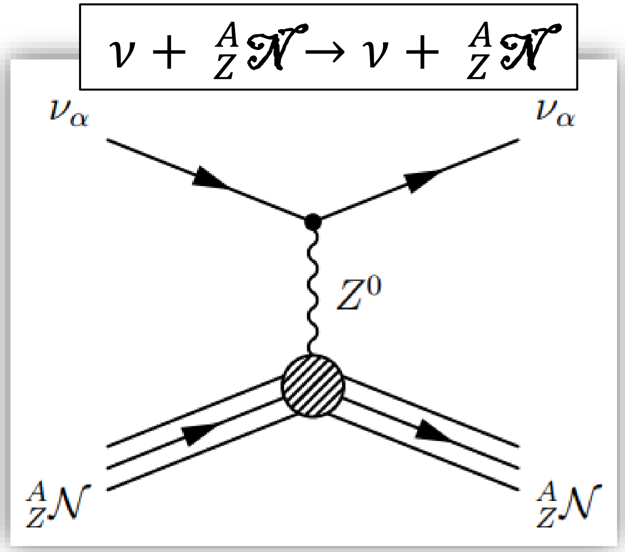
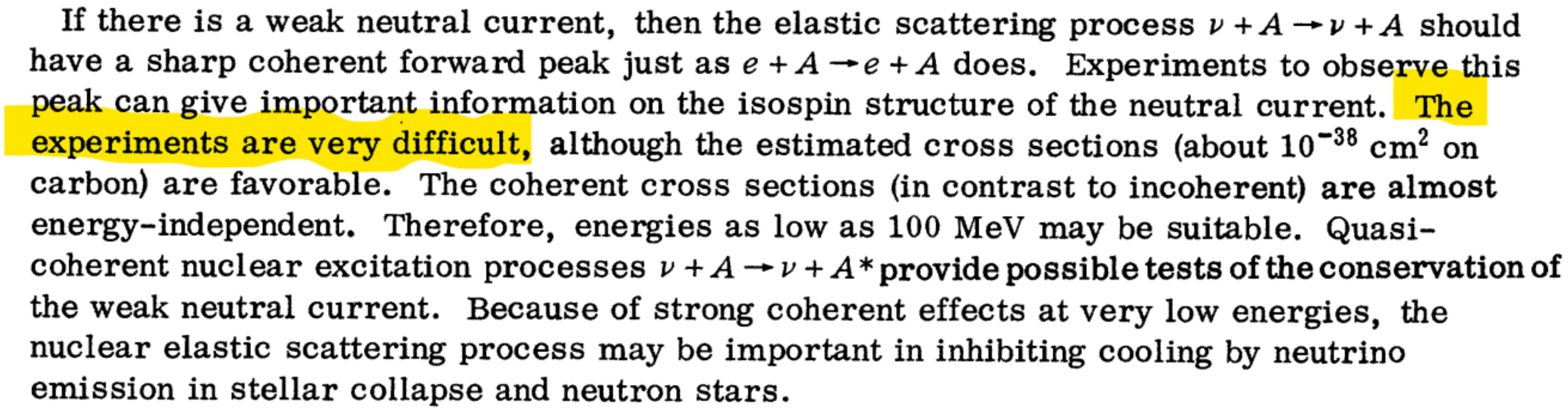
$\nu + {}^{A}_{Z}\mathscr{N} \rightarrow \nu + {}^{A}_{Z}\mathscr{N}$

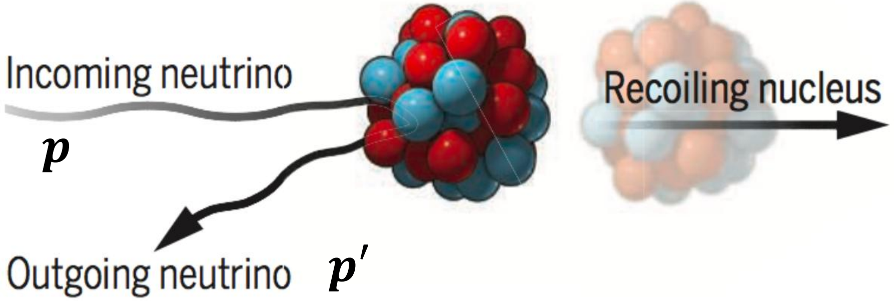




If there is a weak neutral current, then the elastic scattering process $\nu + A \rightarrow \nu + A$ should have a sharp coherent forward peak just as $e + A \rightarrow e + A$ does. Experiments to observe this peak can give important information on the isospin structure of the neutral current. The experiments are very difficult, although the estimated cross sections (about 10^{-38} cm^2 on carbon) are favorable. The coherent cross sections (in contrast to incoherent) are almost energy-independent. Therefore, energies as low as 100 MeV may be suitable. Quasi-coherent nuclear excitation processes $\nu + A \rightarrow \nu + A^*$ provide possible tests of the conservation of the weak neutral current. Because of strong coherent effects at very low energies, the nuclear elastic scattering process may be important in inhibiting cooling by neutrino emission in stellar collapse and neutron stars.

Coherent $\nu - \mathscr{N}$ Interaction

What is a ***nuclear recoil***?



The neutrino interacts with a nucleus containing Z protons and N neutrons, transferring some energy to it. The nucleus, which can be part of a crystalline lattice or free to move (e.g., noble gases), gets perturbed. To give an idea:
$m_N \simeq 100$ GeV; $E_\nu \simeq 10$ MeV $\rightarrow \frac{E_\nu}{m_N} = 10^{-4}$

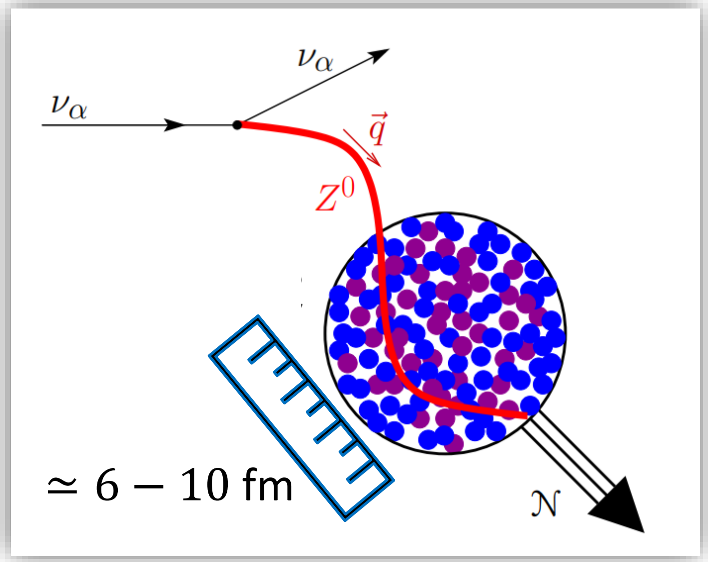
- Qualitatively, the detail of the interaction is governed by the momentum transfer in the process, $q = p' - p$
- This is related to the De Broglie wavelength $\lambda_{Z^0} \simeq \frac{\hbar c}{|q|} \simeq \frac{197 \text{ MeV fm}}{10 \text{ MeV}} \simeq 20 \text{ fm}$ with $q = \sqrt{2m_N T_{\text{nr}}}$

The process is coherent when the de Broglie wavelength is larger than the nuclear dimension

$\nu - \mathscr{N}$ Interactions

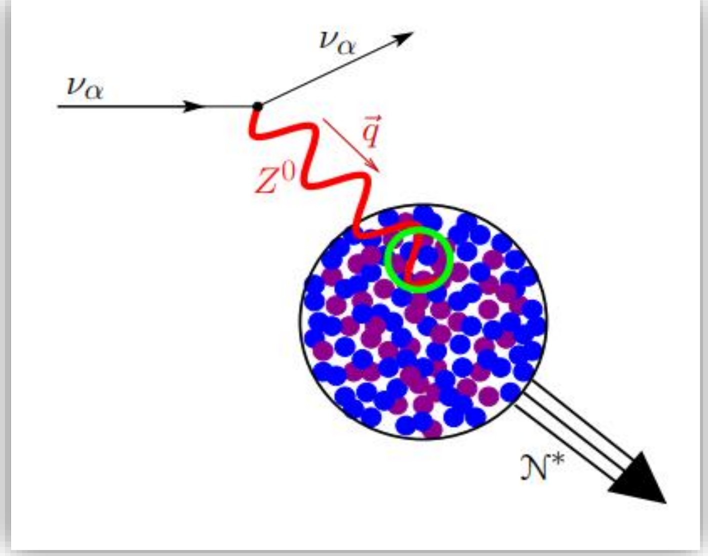
$E_\nu \simeq 10$ MeV $\lambda_{Z^0} > R$

Coherency condition



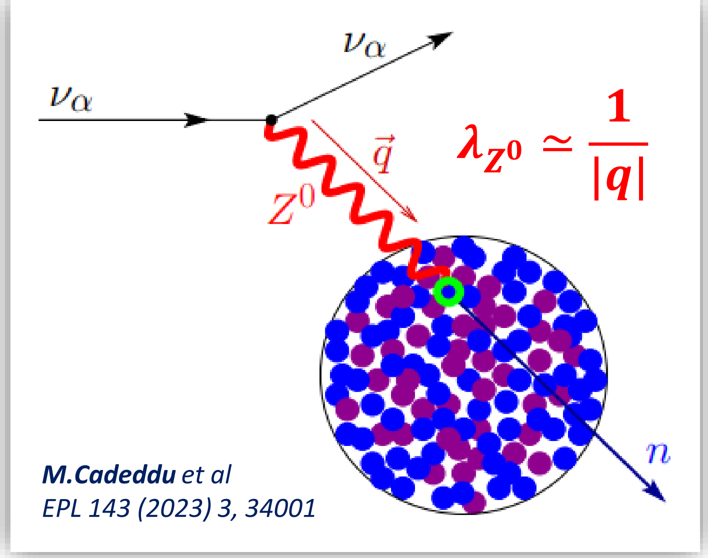
$E_\nu \simeq 100 - 1000$ MeV $\lambda_{Z^0} < R$

Scattering not totally coherent



$E_\nu \simeq 100$ GeV $\lambda_{Z^0} \ll R$

Interaction with individual constituents: inelastic process



***M.Cadeddu** et al*

EPL 143 (2023) 3, 34001

1 MeV 10 MeV 100 MeV 1 GeV 10 GeV 100 GeV E_ν

Coherence occurs when the de Broglie wavelength of the Z^0 exceeds the nuclear size.

Coherence from a Quantum-Mechanical View

Semi-classical treatment

Coherency occurs when an elementary projectile (in this case, the neutrino) elastically scatters off a complex system (for us, the nucleus) made up of A individual constituents (the nucleons). Now, consider the nucleons with specific positions $\vec{x_i}, (i = 1 \ldots . A)$

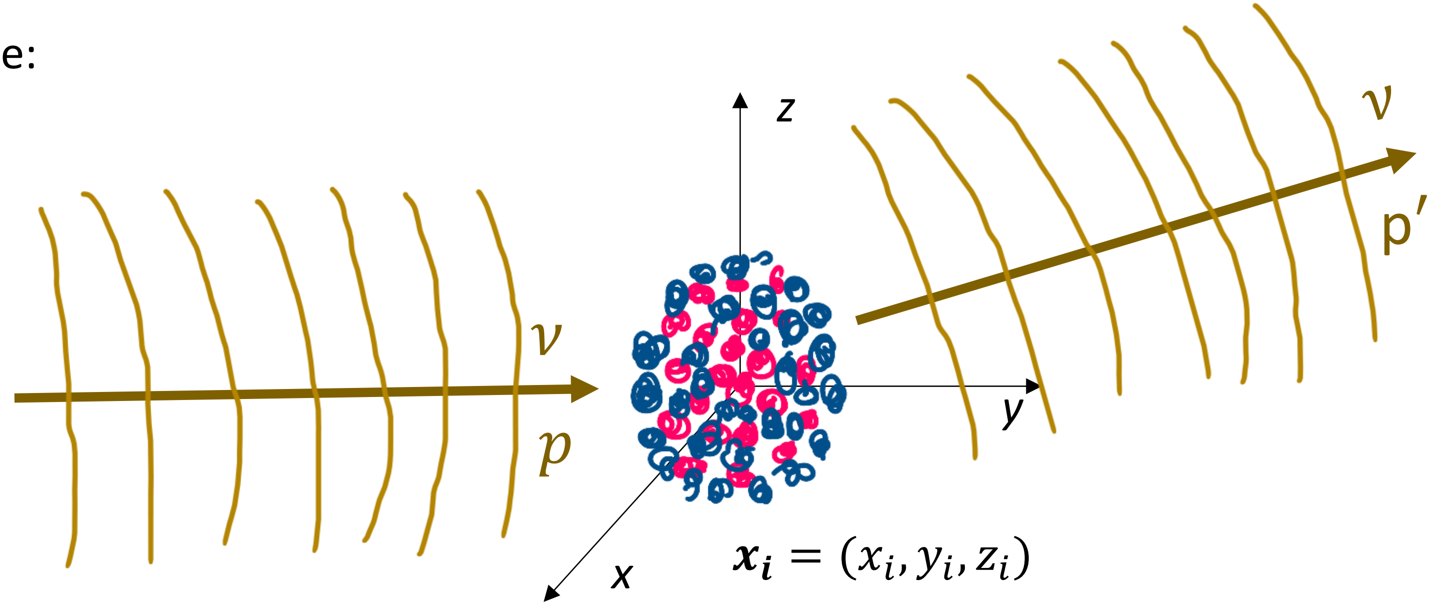
By the principle of superposition, the amplitude of the process $\mathcal{F}(\boldsymbol{p}, \boldsymbol{p}')$ can be written as the sum of the contributions from each individual nucleon $f_i(\boldsymbol{p}, \boldsymbol{p}')$, where the individual amplitudes are weighted by a phase factor that accounts for the relative phase of the wave at the point $\boldsymbol{x_i}$.

When multiple scatterings are negligible, we have:

$$\mathcal{F}(\boldsymbol{p}, \boldsymbol{p}') = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{A}} \boldsymbol{f_i}(\boldsymbol{p}, \boldsymbol{p}') \boldsymbol{e}^{\boldsymbol{i}(\boldsymbol{p}'-\boldsymbol{p})\boldsymbol{x_i}}$$

The cross section is therefore

$$\frac{d\sigma}{d\Omega} \simeq |\mathcal{F}(\boldsymbol{p}, \boldsymbol{p}')|^{2}$$



Coherence from a Quantum-Mechanical View

If $qR \ll 1$, the phase factors will be equal to 1. Therefor

$$\frac{d\sigma}{d\Omega} \simeq A^2 \left|\tilde{f}(\boldsymbol{p},\boldsymbol{p}')\right|^{\mathbf{2}}$$

Here, we introduce the mean ampliture $\tilde{f}$, since the interaction amplitude on protons and neutrons, as we will see, is different.

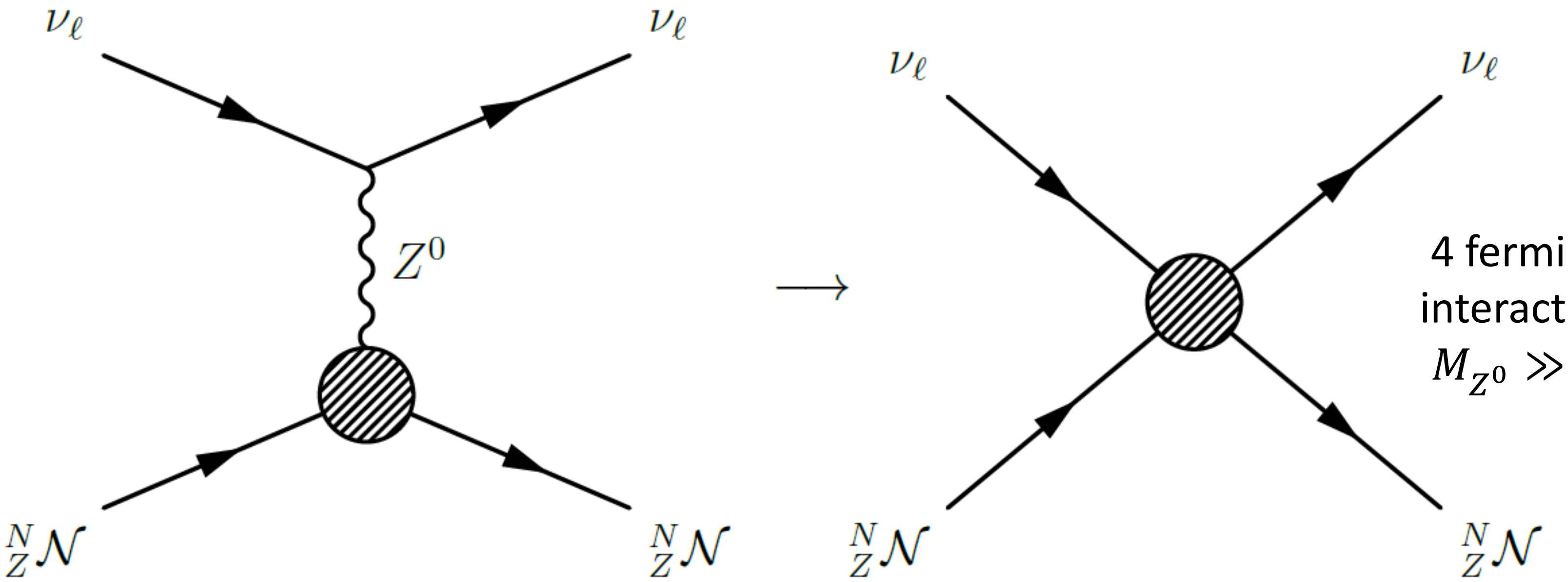
For a quantum system, we cannot define the positions of the constituents of the target. We introduce the spatial density of nucleons $\rho(x)$. In this now realistic case, the amplitude will be:

$$\mathcal{F}(\boldsymbol{p},\boldsymbol{p}') = \tilde{f}(\boldsymbol{p},\boldsymbol{p}') \int \underbrace{d^3x\, \rho(x) e^{iq\cdot x}}$$

It reduces to the previous case for $\rho(x) = \sum_j \delta(x - x_j)$

This quantity is the Fourier transform of the nucleon distribution, and it is what is referred to as the **Form Factor** (FF). It describes the loss of coherence due to the finite structure of the nucleus.

The CEνNS Lagrangian



4 fermion interaction
$M_{Z^0} \gg q$

The CEνNS Lagrangian is

$$\mathcal{L}_{\text{eff}}(\overset{(-)}{\nu}_\ell + \mathcal{N} \to \overset{(-)}{\nu}_\ell + \mathcal{N}) = \frac{G_F}{\sqrt{2}} \sum_q \underbrace{[\bar{\nu}(\gamma^\mu g_V^{\nu_\ell} - g_A^{\nu_\ell}\gamma^5)\nu]}_{\text{Leptonic current}} \underbrace{[\bar{q}(\gamma_\mu g_V^q - g_A^q \gamma^5)q]}_{\text{Hadronic current}}$$

Sum over quarks

The CEνNS Cross Section

The CEνNS cross section is (see Appendix A)

Weak charge of the nucleus

Describes the strength of the neutrino coupling with the nucleus

$$\frac{d\sigma^{\mathrm{SM}}_{\nu_\ell - N}}{dT_{\mathrm{nr}}} = \frac{G_F^2 m_N}{\pi}\left(1-\frac{T_{\mathrm{nr}}}{E_\nu}-\frac{m_N T_{\mathrm{nr}}}{2E_\nu^2}\right) Q_W^2 \quad F_W^2$$

Constants

$G_F = \frac{\sqrt{2}g^2}{8m_W^2}$: *Fermi constant*

$\sigma \propto m_N$: *nuclear mass*

Kinematics

Phase space factor

Nuclear Form Factor

Describes the loss of coherency for increasing neutrino energies

What are the two dominant terms in this expression, in your opinion? As an exercise, try to perform the relativistic kinematics calculation.

?

The Nuclear Weak Charge

Let us focus on a fundamental quantity, the weak charge of the nucleus $\boldsymbol{Q_W}$. This is defined as

$$\boxed{\boldsymbol{Q_W = g_V^p Z + g_V^n N}}$$

- $\boldsymbol{g_V^p}$ and $\boldsymbol{g_V^n}$ are the couplings of neutrinos with individual protons and neutrons. They are well predicted by electroweak theory, and at tree level* they are:

- $g_V^p = \frac{1}{2} - 2\boldsymbol{\sin^2\theta_W} \sim 0.02274$
- $g_V^n = -\frac{1}{2}$

The weak mixing angle $\boldsymbol{\sin^2\theta_W}$ is responsible for the suppression of the neutrino proton coupling

$g_V^p \ll g_V^n$, i.e.

$\sigma \propto N^2$

These couplings are not dependent on the neutrino flavor!

**Radiative corrections* play a fundamental role!

The Weak Mixing Angle

The weak mixing angle, $\boldsymbol{\sin^2\theta_W}$**,** is a key parameter of the electroweak theory $\mathbf{SU(2)_L \otimes U(1)_Y}$

It is responsible for the mixing of the B_μ field ($U(1)_Y$ gauge symmetry), and the $W_\mu^{(3)}$ field associated to the $SU(2)_L$ symmetry.

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & \sin\vartheta_W \\ -\sin\vartheta_W & \cos\vartheta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^{(3)} \end{pmatrix}$$

$e = g\sin\theta_W = g'\cos\theta_W$

- g: $\mathrm{SU(2)_L}$ gauge coupling
- g': $\mathrm{U(1)}_Y$ gauge coupling

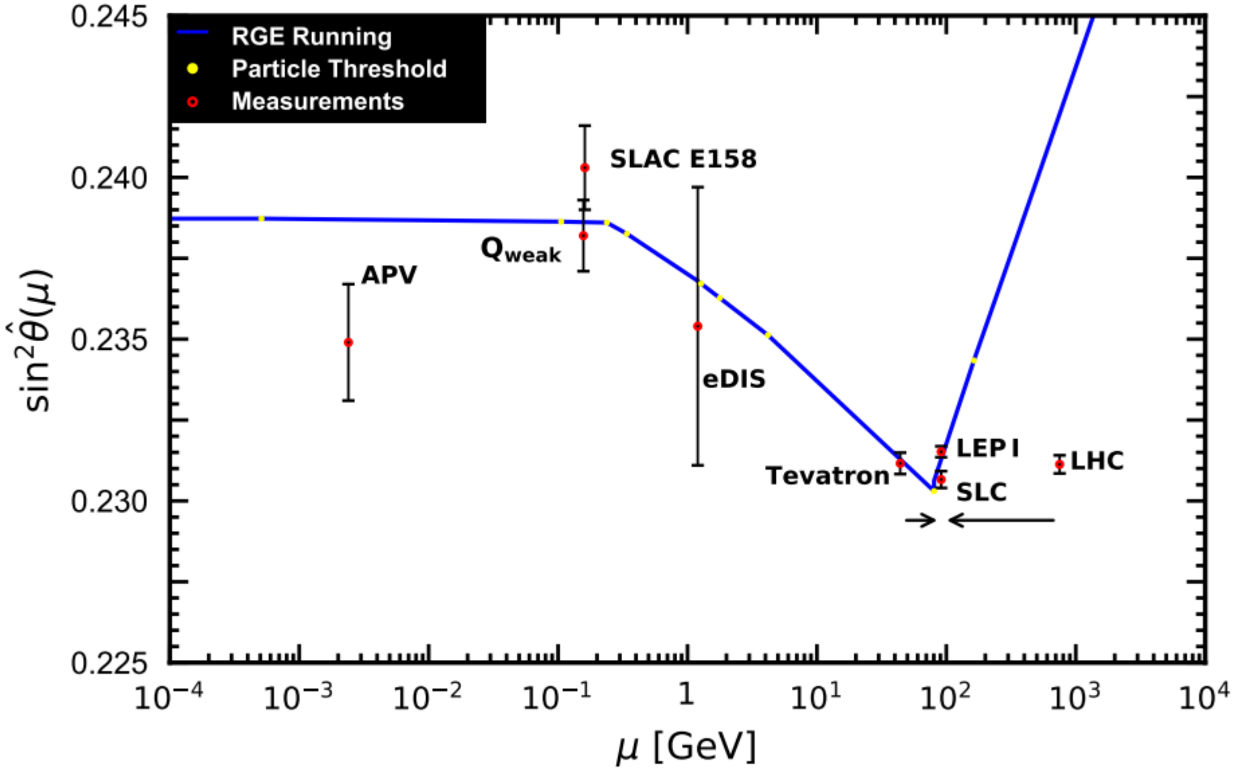
In the $\overline{\mathrm{MS}}$ scheme, it is defined as:

$$\sin^2\hat{\theta}(\mu) = \frac{\hat{g}'^2(\mu)}{\hat{g}^2(\mu)+\hat{g}'^2(\mu)}$$

$\sin^2\theta_W(q^2 = 0) \simeq 0.23873(5)$

The Weak Mixing Angle

Particle data group 2024



The weak mixing angle determines the intensity of the fundamental electroweak couplings.
It is not a constant quantity, but its value changes with energy.

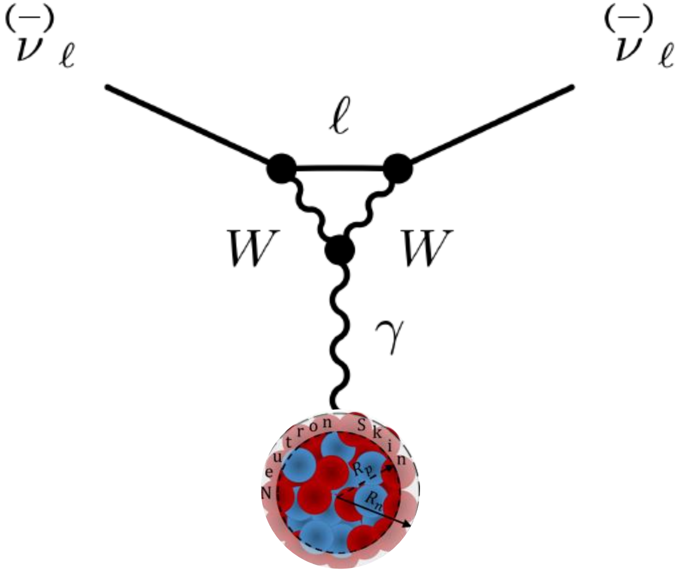
$$\sin^2\theta_W(q^2 \to 0) \simeq 0.23873(5)$$

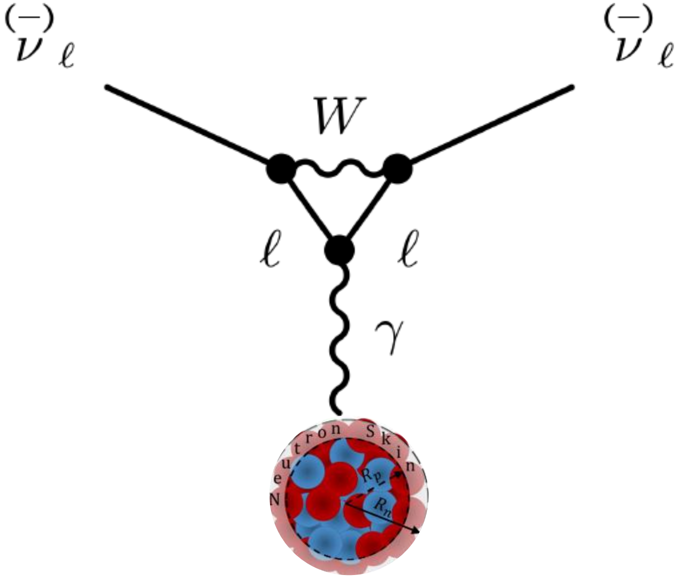
$$g_V^p = \frac{1}{2} - 2\sin^2\theta_W \sim 0.02274$$

The theoretical prediction is very precise.
Few measurements at low energy, CEνNS offers a new opportunity to measure $\sin^2\theta_W(q^2 \to 0)$.

Radiative Corrections

Radiative corrections provide additional contributions to the neutrino-nucleus interaction vertex.





This diagrams are very important. They are they only photon-mediated neutrino interaction in the standard model!

This is the so-called **neutrino charge radius**

There are also other radiative corrections, discussed in Appendix B.

$$\langle r^2_{\nu_e} \rangle \simeq -8.3 \times 10^{-33}\,\mathrm{cm}^2$$

$$\langle r^2_{\nu_\mu} \rangle \simeq -4.8 \times 10^{-33}\,\mathrm{cm}^2$$

$$\langle r^2_{\nu_\tau} \rangle \simeq -3.0 \times 10^{-33}\,\mathrm{cm}^2$$

Neutrino Charge Radius

The neutrino charge radius is therefore a vertex correction to the CEνNS process, responsible for a new contribution in the neutrino-proton coupling through the radiative correction $\boldsymbol{\phi_{\nu_\ell - W}}$, which is connected to $\langle \boldsymbol{r^2_{\nu_\ell}} \rangle$

$$g_V^p \simeq \frac{1}{2} - 2\sin^2\theta_W - 2\boldsymbol{\phi_{\nu_\ell - W}} + \text{other radiative corrections (1 loop)}$$

$$g_V^p \simeq \frac{1}{2} - 2\sin^2\theta_W - \frac{\sqrt{2}\pi\alpha_{EM}}{3G_F}\langle \boldsymbol{r^2_{\nu_\ell}} \rangle + \text{other radiative corrections (1 loop)}$$

Electroweak Couplings (tree level)

$$g_V^p = \frac{1}{2} - 2\boldsymbol{\sin^2\theta_W} \simeq 0.02274$$

$$g_V^n = -\frac{1}{2}$$

Weak mixing angle

With radiative corrections (RC)

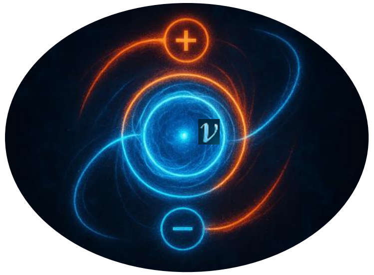
$$g_V^p(\nu_e) \sim 0.0382$$
$$g_V^p(\nu_\mu) \sim 0.0299$$
$$g_V^p(\nu_\tau) \sim 0.0255$$
$$g_V^n = -0.5117$$

***MAC** et al. JHEP 05 (2024) 271*

Fundamental Physics Test

In general, a neutral particle can be described as a superposition of two **charge distributions of opposite signs**, which is non-zero only for non-zero momentum transfers, q^2. This effect is attributed to the series expansion of the charge form factor.

$$\mathbb{f}_Q(q^2) = \mathbb{f}_Q(0) + q^2 \frac{d\mathbb{f}_Q(q^2)}{dq^2}\Big|_{q^2=0} + \dots$$



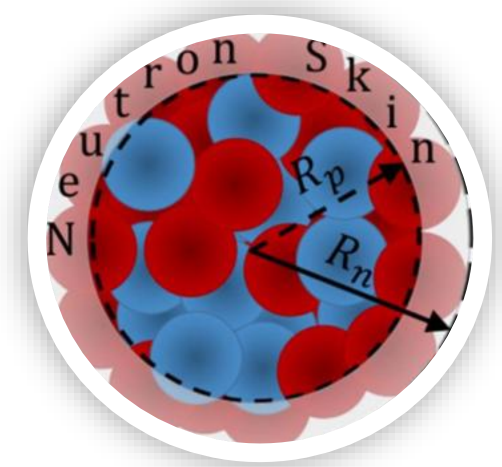
The charge form factor is thus interpreted as the Fourier transform of the charge distribution associated with the neutrino interaction.

$$\mathbb{f}_Q(q^2) = \int \rho(r) e^{-i\vec{q}\cdot\vec{r}} d^3r = \int \rho(r) \frac{\sin(qr)}{qr} d^3r$$

The neutrino charge radius is defined as $\langle r^2 \rangle \equiv 6 \frac{d\mathbb{f}_Q(q^2)}{dq^2}\Big|_{q^2=0}$

We can fit for this data using currently available data

The Nuclear Form Factor



$$\boxed{\mathcal{F}(\boldsymbol{p},\boldsymbol{p}') = \tilde{f}(\boldsymbol{p},\boldsymbol{p}') \int d^3x\, \rho(x) e^{iq\cdot x}}$$

We have already defined the **nuclear form factor (FF)** as the Fourier transform of the nuclear density.
To describe the different nuclear composition in terms of protons and neutrons, the weak form factor is defined as

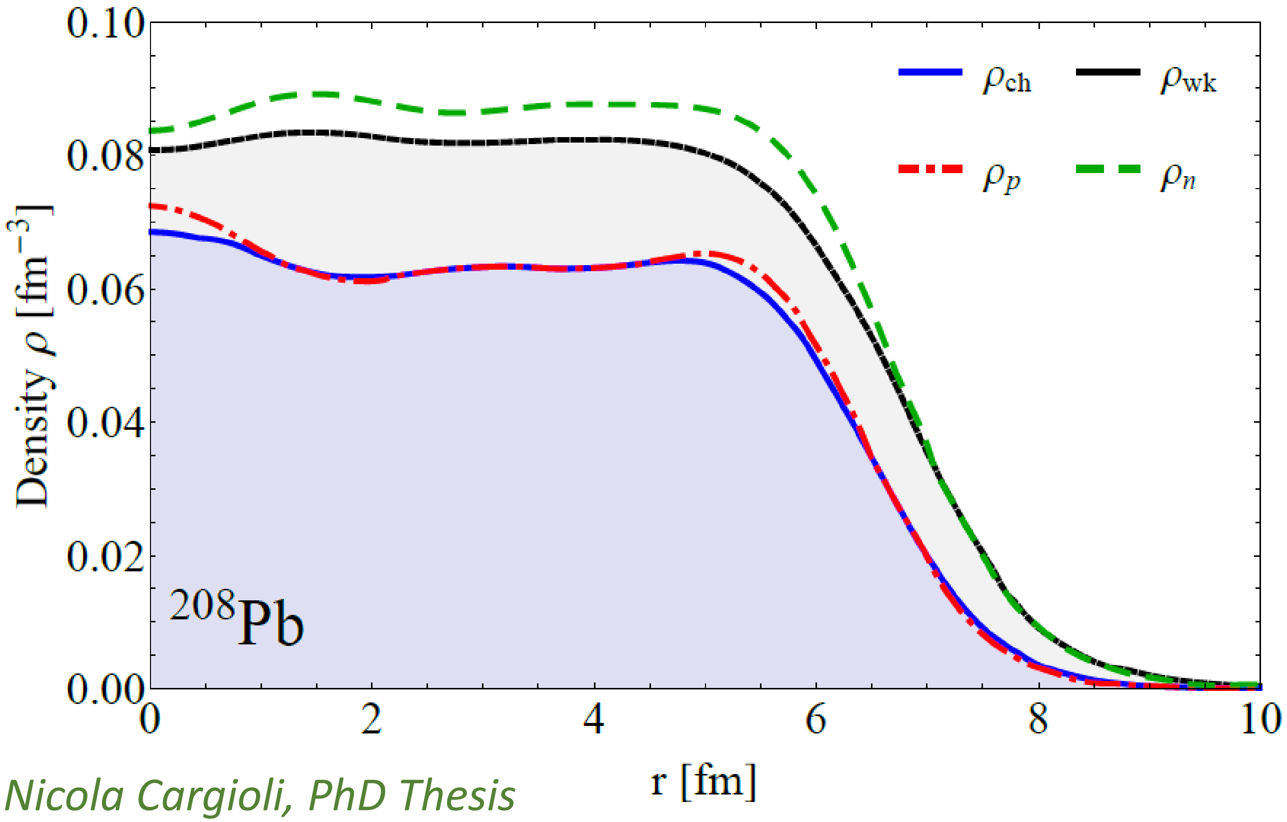
$$F_W = \frac{1}{Q_W}\left(g_V^p Z \boldsymbol{F_p} + g_V^n N \boldsymbol{F_n}\right)$$

This parameterization allows for the separation of the contributions from protons and neutrons.

The proton distribution is generally different from the neutron distribution.
In the case of a large nucleus, this gives rise to the so-called neutron skin, due to the fact that there are more neutrons than protons.

The Nuclear Form Factor

The nuclear density of the **charge distribution** is slightly different from the **proton one**, as the former includes a small contribution from neutrons.



Nicola Cargioli, PhD Thesis

- $\rho_{\rm ch}$: charge distribution
- ρ_p: proton distribution
- ρ_n: neutron distribution
- ρ_{wk}: weak charge distribution

An important quantity is the average rms radius of the distribution

$$R^2_{p,n} = \frac{\int r^2 \rho_{p,n} d^3r}{\int \rho_{p,n} d^3r}$$

The Nuclear Form Factor

The form factor thus takes into account this spatial distribution of nucleons. It can be described analytically using the *Symmetrized 2 parameters Fermi*.

$$F_Z^{\mathrm{SF}}(q^2) = \frac{3}{qc\left[(qc)^2 + (\pi q a)^2\right]}\left[\frac{\pi q a}{\sinh(\pi q a)}\right]\left[\frac{\pi q a \sin(qc)}{\tanh(\pi q a)} - qc\cos(qc)\right]$$

The parameter c is the half density radius, while $a = 4\,t\,/\log(3)$.

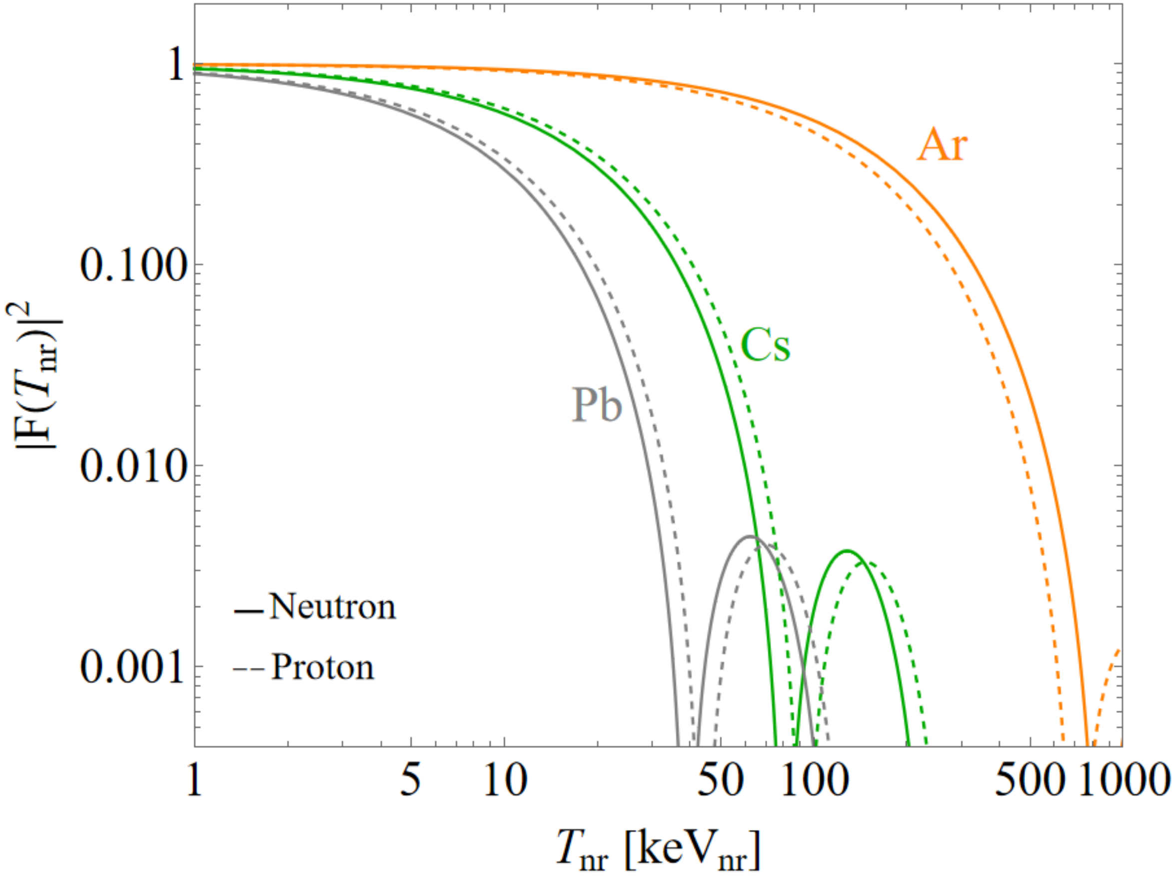
- The thickness is defined as the difference between the radius where the density drops from 90% to 10% of the plateau value ($t \simeq 2.3$ fm)

$$R^2 = 3/5\,c^2 + 7/5\,(\pi a)^2$$

	c [fm]	R_{ch} [fm]	R_p [fm]	R_n [fm]	m_N [GeV]
$^{40}_{18}\mathrm{Ar}$	3.6416	3.426	3.447	3.55	37.216
$^{72}_{32}\mathrm{Ge}$	4.5926	4.0547	4.073	4.22	66.995
$^{127}_{53}\mathrm{I}$	5.5931	4.7492	4.766	5.03	118.221
$^{131}_{54}\mathrm{Xe}$	5.6384	4.761	4.778	5.07	122.296
$^{133}_{55}\mathrm{Cs}$	5.6710	4.804	4.821	5.09	123.801
$^{208}_{82}\mathrm{Pb}$	6.648	5.505	5.521	5.68	192.8

The Nuclear Form Factor

Below is the form factor for protons and neutrons for some nuclei commonly used in CEνNS research.



Again, it is exactly the **Fourier transform** of the nuclear density distribution.

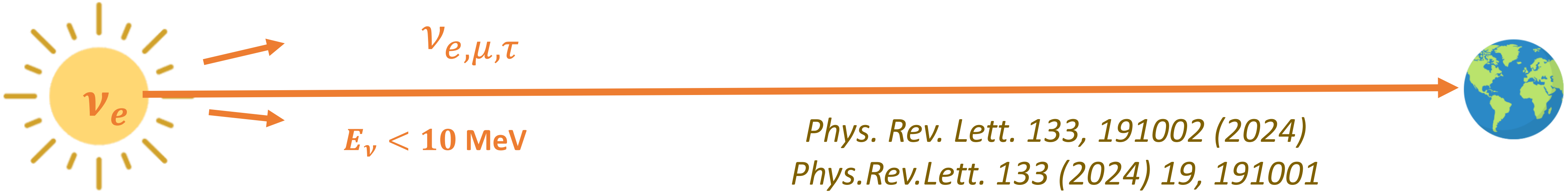
- The form factor is ~1 at low energies, as expected.
- We note the presence of the so-called *diffraction minima*.
- When the form factor becomes small, coherence is lost and the CEνNS cross-section collapses.

- **The form factor *closes* earlier for heavier nuclei.**

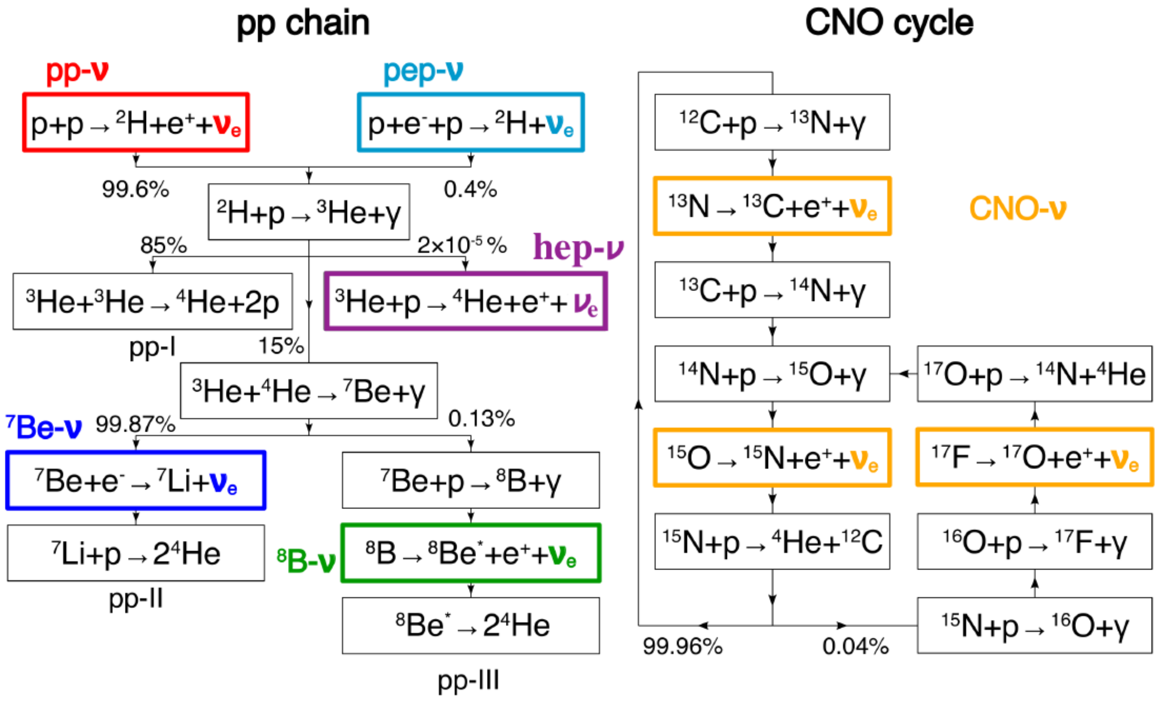
Why does it happen?

Solar Neutrinos

Solar neutrinos: The Sun is the most abundant source of low-energy neutrinos in nature



Phys. Rev. Lett. 133, 191002 (2024)
Phys.Rev.Lett. 133 (2024) 19, 191001



pp chain

pp-ν: $p+p \to {}^2H+e^+ + \nu_e$

pep-ν: $p+e^-+p \to {}^2H+\nu_e$

99.6% 0.4%

${}^2H+p \to {}^3He+\gamma$

85% 2×10^{-5} % hep-ν

${}^3He+{}^3He \to {}^4He+2p$

${}^3He+p \to {}^4He+e^+ + \nu_e$

pp-I 15%

${}^3He+{}^4He \to {}^7Be+\gamma$

⁷Be-ν 99.87% 0.13%

${}^7Be+e^- \to {}^7Li+\nu_e$

${}^7Be+p \to {}^8B+\gamma$

${}^7Li+p \to 2{}^4He$

pp-II

⁸B-ν: ${}^8B \to {}^8Be^*+e^+ + \nu_e$

${}^8Be^* \to 2{}^4He$

pp-III

CNO cycle

${}^{12}C+p \to {}^{13}N+\gamma$

${}^{13}N \to {}^{13}C+e^+ + \nu_e$ CNO-ν

${}^{13}C+p \to {}^{14}N+\gamma$

${}^{14}N+p \to {}^{15}O+\gamma$

${}^{17}O+p \to {}^{14}N+{}^4He$

${}^{15}O \to {}^{15}N+e^+ + \nu_e$

${}^{17}F \to {}^{17}O+e^+ + \nu_e$

${}^{15}N+p \to {}^4He+{}^{12}C$

${}^{16}O+p \to {}^{17}F+\gamma$

${}^{15}N+p \to {}^{16}O+\gamma$

99.96% 0.04%

Neutrinos are produced by the pp chain and the CNO cycle. This results in an extremely intense flux of low-energy neutrinos.

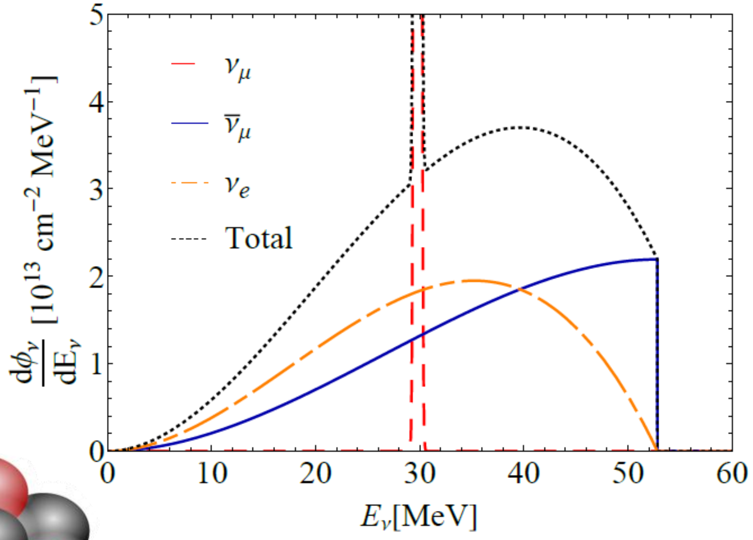
Universe 7.7 (2021)
Rev. Mod. Phys. 92(2020), p. 45006

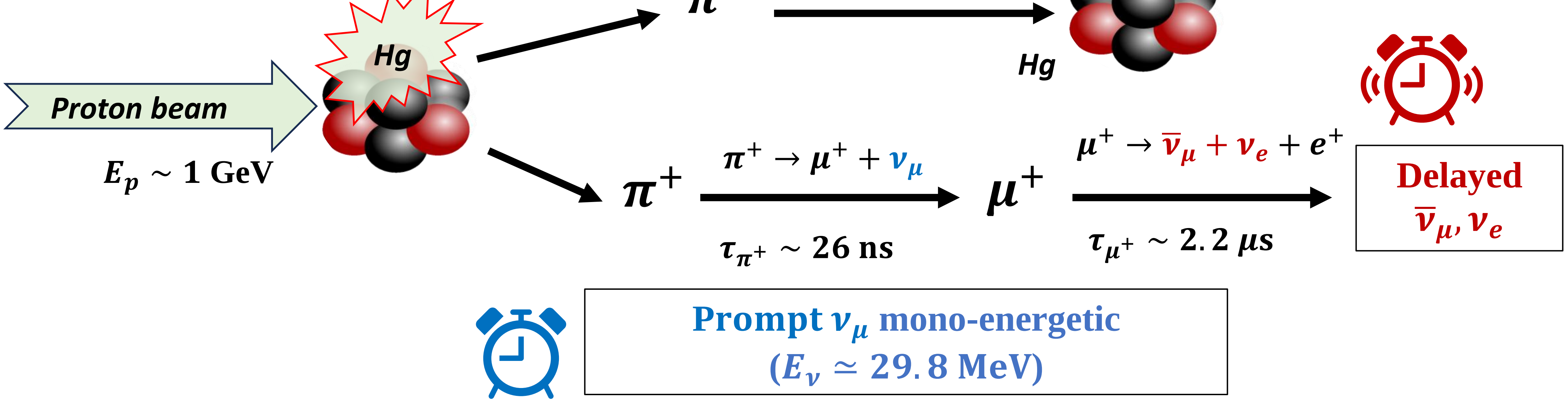
π-DAR Neutrinos

Pion Decay at Rest: π-DAR.

A pulsed proton beam strikes a mercury target → Pions are produced following the nuclear cascade.

The π^+ decay produce a prompt ν_μ emission, followed by a delayed $\nu_e, \bar{\nu}_\mu$ emission





$E_p \sim 1$ GeV

π^-

Captured 99%

π^+

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$\tau_{\pi^+} \sim 26$ ns

μ^+

$\mu^+ \rightarrow \bar{\nu}_\mu + \nu_e + e^+$

$\tau_{\mu^+} \sim 2.2\ \mu s$

Delayed
$\bar{\nu}_\mu, \nu_e$

Prompt ν_μ mono-energetic
$(E_\nu \simeq 29.8 \text{ MeV})$

Neutrinos from Nuclear Reactors

$\bar{\nu}_e$

$E_\nu \sim 3$ MeV

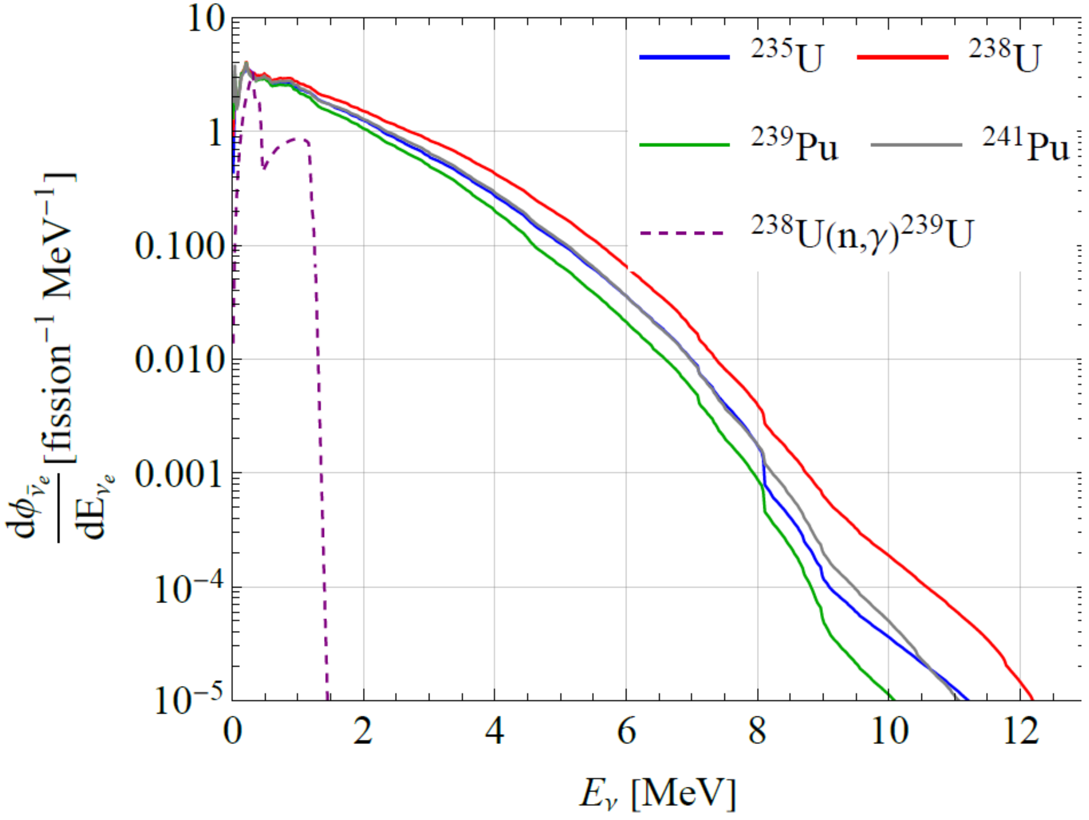
There are many nuclear reactors in the world, about 400. There is also a global movement with many experiments searching for CEνNS at reactors.

Neutrinos are produced through the β decay of fission fuels $^{235}\mathrm{U}$, $^{239}\mathrm{Pu}$, $^{241}\mathrm{Pu}$, $^{238}\mathrm{U}$.

Additionally, the neutrons produced can be absorbed by the fuel i or by the reactor walls, triggering further reaction chains.

We are dealing with a very intense $\bar{\nu}_e$ flux :~ $\mathbf{10^{13}}\ \boldsymbol{\nu/[s\ cm^2]}$

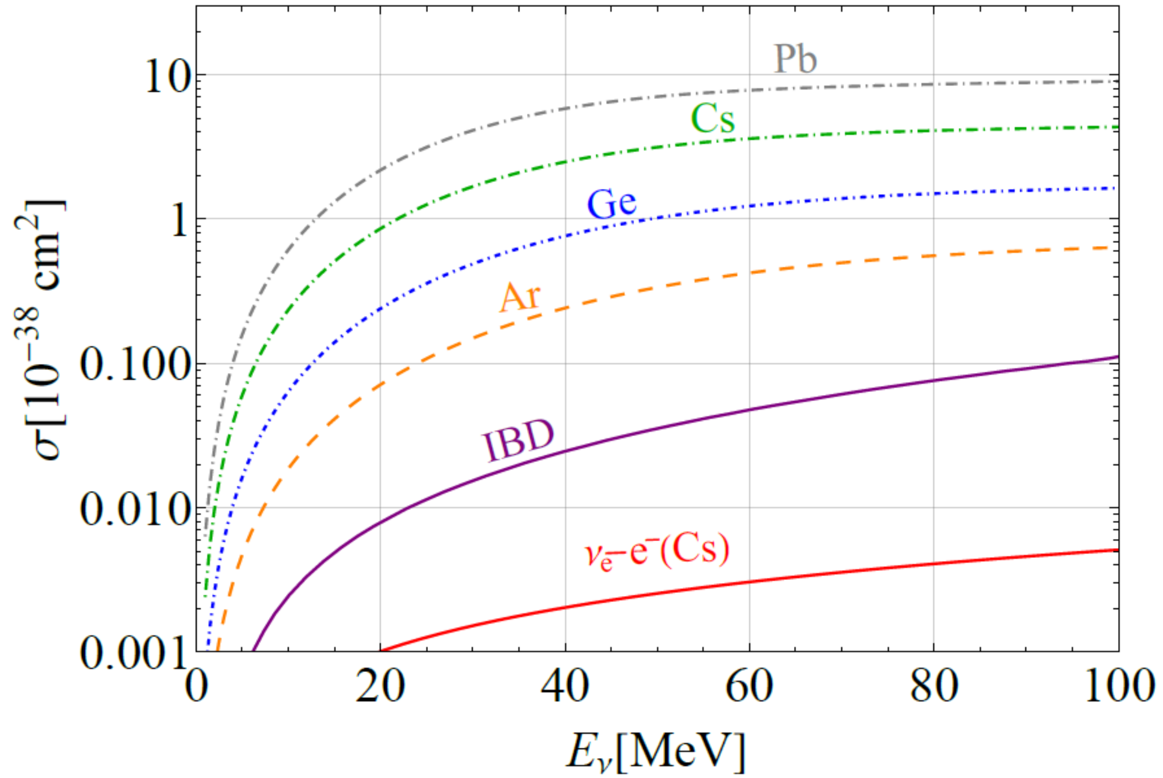
Reactor Flux - Phys.Rev.C 108 (2023) 5, 055501



The CEνNS Cross Section

Considering all the elements, the cross-section takes the form:

$$\frac{d\sigma^{\mathrm{SM}}_{\nu_\ell - N}}{dT_{\mathrm{nr}}} = \frac{G_F^2 M}{\pi}\left(1 - \frac{M T_{\mathrm{nr}}}{2E_\nu^2}\right)\left[g_V^p(\nu_\ell)\, Z F_Z(|\bar{q}|^2) + g_V^n N F_N(|\bar{q}|^2)\right]^2$$



Couplings:

$g_V^p(\nu_e) \sim 0.0382$

$g_V^p(\nu_\mu) \sim 0.0299$

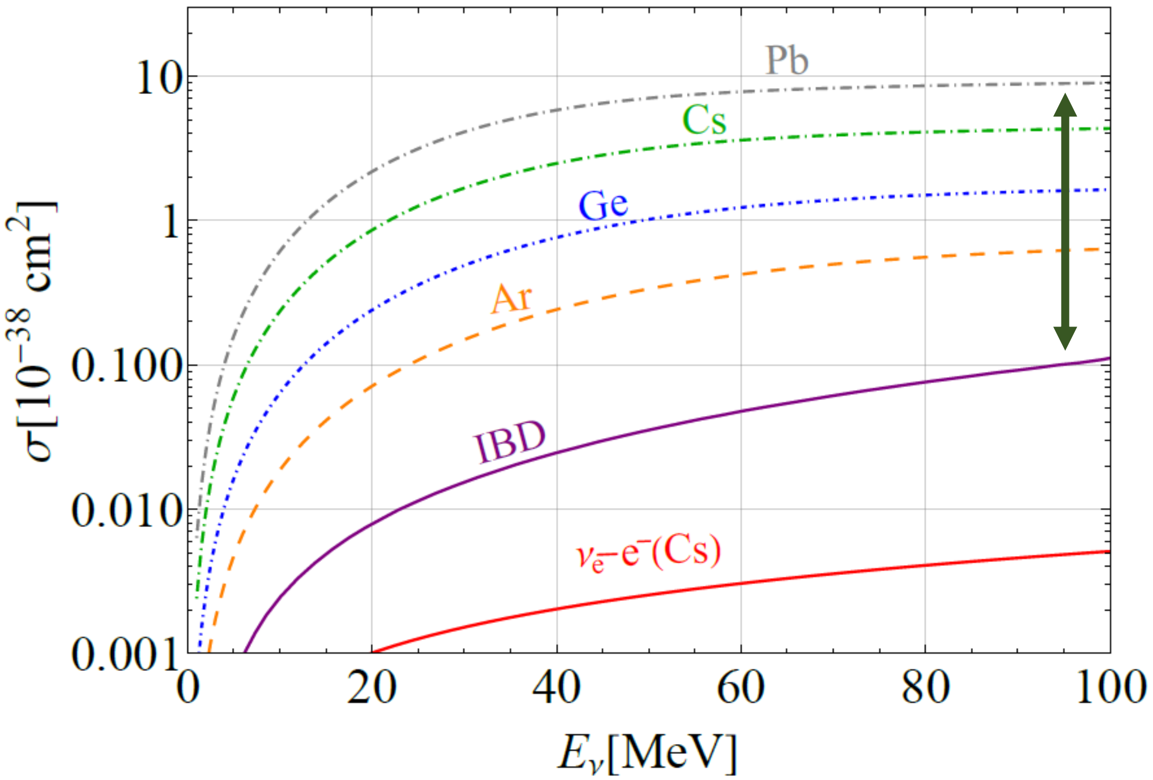
$g_V^p(\nu_e) \sim 0.0255$

$g_V^n = -0.5117$

We now evaluate $\sigma = \int_0^{T_{\mathrm{nr}}^{\max}} dT_{\mathrm{nr}} \frac{d\sigma}{dT_{\mathrm{nr}}}$

$$T_{\mathrm{nr}}^{\max} = \frac{2E_\nu^2}{m_N + 2E_\nu} \simeq \frac{2E_\nu^2}{m_N}$$

Comparison with IBD and Other Processes



Up to this point, we are able to understand why CEνNS is so interesting

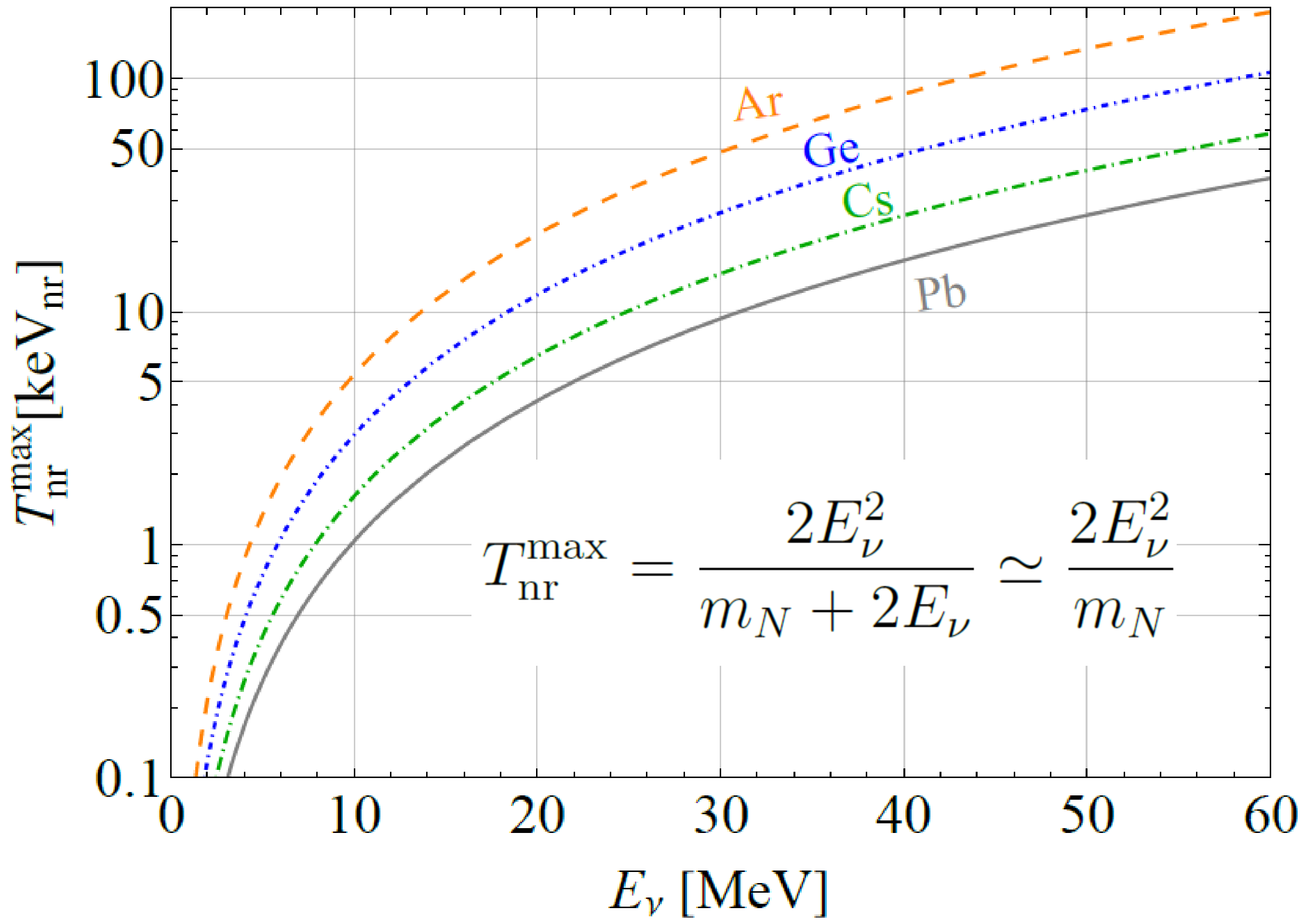
- The cross-section is very large, and scales with N^2
- For ^{208}Pb, it is about 2 orders of magnitude larger than inverse beta decay (IBD).
- It is about 4 orders of magnitude larger than the neutrino cross-section on a cesium atom.

IBD: $\bar{\nu} + p \to n + e^+$

$$\sigma^0_{\mathrm{IBD}} = \frac{G_F^2 \cos^2\theta_C}{\pi}\left(f^2 + 3g^2\right) E_e p_e$$
$$E_e = E_\nu - (M_N - M_p)$$
$$E_\nu > 1.806\ \mathrm{MeV}$$
$$E_e = E_\nu - (M_N - M_p)$$

Why is it so difficult to observe?

CEνNS: Main Challange



$$T_{\rm nr}^{\rm max} = \frac{2E_\nu^2}{m_N + 2E_\nu} \simeq \frac{2E_\nu^2}{m_N}$$

The answer comes from kinematics

- **Small nucles:** for the same E_ν the nucleus recoils more, but the cross section is smaller
- **Big nucleus:** the nucleus recoils less, but the cross section is bigger

In general, a detector with a very low threshold, ~keV, with a low background, and typically a large mass, is required.

End of the 1° Part

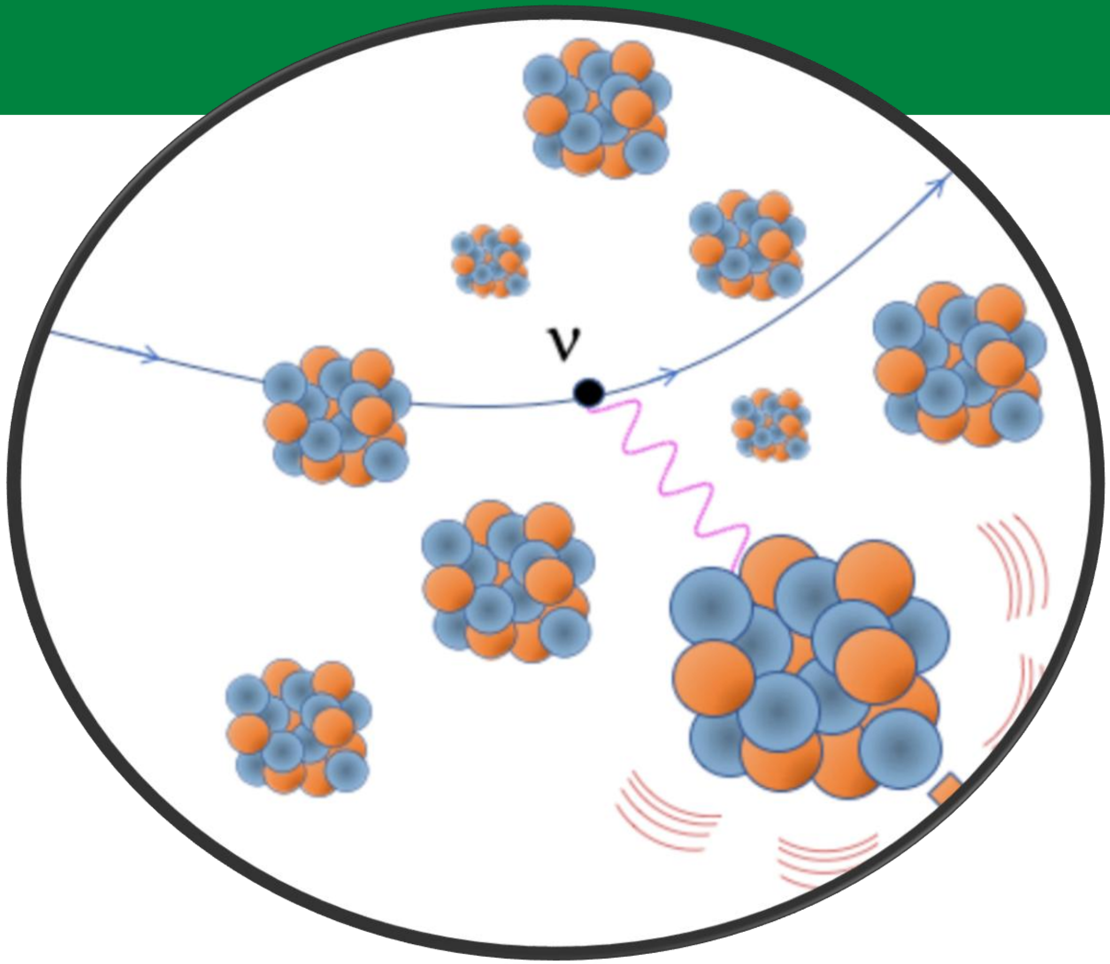
Questions?

2° Part:

Dr. Riccardo Cerulli will tell you how we made it! (… 43 years after the theoretical prediction)

Supplemental Material

1. Appendix A: Evaluation of the Cross Section
2. Appendix B: Radiative Corrections



CEνNS Cross Section

- **Appendix A**
- Appendix B

We remind here the Lagrangian for the interaction

$$\mathcal{L}_{\text{eff}}(\overset{(-)}{\nu}_\ell + \mathcal{N} \to \overset{(-)}{\nu}_\ell + \mathcal{N}) = \frac{G_F}{\sqrt{2}} \sum_q [\bar{\nu}(\gamma^\mu g_V^{\nu_\ell} - g_A^{\nu_\ell}\gamma^5)\nu][\bar{q}(\gamma_\mu g_V^q - g_A^q\gamma^5)q]$$

Where g_V, g_A are the electrowek couplings with the Z^0 boson.

For neutrinos $g_{V,A}^{\nu_\ell} = 1/2$

For quarks $g_V^q = T_3^q - Q_q \sin^2\vartheta_W$, i.e.

$$g_V^q = \frac{1}{2} - \frac{4}{3}\sin^2\vartheta_W\,, \qquad q = u, c, t$$

$$g_V^q = -\frac{1}{2} + \frac{2}{3}\sin^2\vartheta_W\,, \qquad q = d, s, b$$

Quarks Number Operator

Here we introduce the operator *number of quarks*, $N_q^{p(n)}$ which projects the quark vector current into nucleon states and returns the number of quarks inside the proton or neutron:

$$\begin{aligned}\langle n|\bar{q}\gamma^\mu q|n\rangle &= N_q^n \bar{n}\gamma^\mu n,\\ \langle p|\bar{q}\gamma^\mu q|p\rangle &= N_q^p \bar{p}\gamma^\mu p,\\ \langle p|\bar{q}\gamma^\mu q|n\rangle &= 0.\end{aligned}$$

Defining the vectorial current of quarks as $J^\mu = \sum_{q=u,d} \bar{q}\gamma^\mu g_V^q q$ we get

$$\begin{aligned}\left(\langle n| + \langle p|\right) J^\mu \left((|n\rangle + |p\rangle\right) &= g_V^u N_u^n \bar{n}\gamma^\mu n + g_V^d N_d^n \bar{n}\gamma^\mu n + g_V^u N_u^p \bar{p}\gamma^\mu p + g_V^d N_d^p \bar{p}\gamma^\mu p\\ &= (2g_V^u + g_V^d)\bar{p}\gamma^\mu p + (g_V^u + g_V^d)\bar{n}\gamma^\mu n\\ &= g_V^p \bar{p}\gamma^\mu p + g_V^n \bar{n}\gamma^\mu n.\end{aligned}$$

$$\begin{aligned}g_V^p &= 2g_V^u + g_V^d = \frac{1}{2} - 2\sin^2\vartheta_W \simeq 0.0227\\ g_V^n &= g_V^u + 2g_V^d = -\frac{1}{2}.\end{aligned}$$

Nucleon Number Operator

The formalism described so far does not describe the interaction with the entire nucleus, but rather with the individual protons and neutrons. To proceed, we introduce the nucleon number operator, Z and N, which returns the number of protons and neutrons within the nucleus when applied to the initial nuclear state. $|\mathcal{N}\rangle$.

Defining $W^\mu = \sum_q \bar{q}(\gamma_\mu g_V^q - g_A^q \gamma^5)q$, we have $\langle\mathcal{N}|W^\mu|\mathcal{N}\rangle = (g_V^p Z + g_V^n N)\bar{\mathcal{N}}\gamma^\mu\mathcal{N}$.

We rewrite the lagrangian as

$$\mathcal{L}_{\text{eff}}(\overset{(-)}{\nu}_\ell + \mathcal{N} \to \overset{(-)}{\nu}_\ell + \mathcal{N}) = \frac{G_F}{\sqrt{2}} \sum_q [\bar{\nu}\gamma_\mu \mathbb{P}_L \nu][\mathcal{Q}_W \bar{\mathcal{N}}\gamma^\mu \mathcal{N}]$$

The matrix element in terms of the spinors, with spin $ss'rr'$, is

$$M^{ss'rr'} = \frac{G_F}{\sqrt{2}} \mathcal{Q}_W [\overline{u}^{s'}(p')\gamma^\mu P_L u(p)^s][\overline{u}^{r'}(k')\gamma_\mu u^r(k)]$$

The matrix element weighted over the spin is

$$\langle|M|^2\rangle = \sum_{s,s'} \frac{1}{2} \sum_{r,r'} |M^{ss'rr'}|^2$$

CEνNS Cross Section

- **Appendix A**
- Appendix B

The cross section will be therefore

$$\frac{d\sigma_{\nu_\ell-\mathcal{N}}}{dt} = \frac{1}{16\pi}\frac{1}{(s-m_N^2)^2}\langle|M|^2\rangle$$

Where *s, t* are the mandelstam variables

$$t = -2m_N T_{\rm nr}\ ,$$

$$s = m_N^2 + 2E_\nu m_N\,.$$

The observable is the nuclear recoil energy:

$$\frac{d\sigma_{\nu-\mathcal{N}}}{dT_{\rm nr}} = \frac{G_F^2}{128\pi}\frac{\mathcal{Q}_W^2}{E_\nu^2 m_N}L^{\mu\nu}W_{\mu\nu}$$

CEνNS Cross Section

- **Appendix A**
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The weighted matrix element is, in fact $\langle |M|^2 \rangle = \frac{G_F^2}{2} \mathcal{Q}_W^2 L^{\mu\nu} W_{\mu\nu}$

$$L^{\mu\nu} = \sum_{s,s'} [\overline{u}^{s'}(p')\gamma^\mu P_L u^s(p)][\overline{u}^s(p)\gamma^\nu P_L u^{s'}(p')] \, ,$$

$$W_{\mu\nu} = \sum_{r,r'} [\overline{u}^{r'}(k')\gamma_\mu u^r(k)][\overline{u}^{r'}(k)\gamma_\nu u^{r'}(k)] \, .$$

Recalling that $\sum_s u^s(p)\overline{u}^s(p) = \not{p} + m$ and $\not{p} = \gamma^\mu p_\mu$

Here, one *only* has to use the trace rules to obtain

$$L^{\mu\nu} W_{\mu\nu} \simeq 128 E_\nu^2 m_N^2 \left(1 - \frac{T_{\text{nr}}}{E_\nu} - \frac{m_N T_{\text{nr}}}{2E_\nu^2}\right)$$

CEνNS Cross Section

- **Appendix A**
- Appendix B

Therefore we have

$$\left(\frac{d\sigma_{\nu-\mathcal{N}}}{dT_{\mathrm{nr}}}\right)_{q^2\to 0} \simeq \frac{G_F^2}{\pi} m_N \left(1 - \frac{T_{\mathrm{nr}}}{E_\nu} - \frac{m_N T_{\mathrm{nr}}}{2E_\nu^2}\right)\left[Z g_V^p + N g_V^n\right]^2$$

Including the form factors, we get our expression.

$$\frac{d\sigma_{\nu-\mathcal{N}}}{dT_{\mathrm{nr}}} \simeq \frac{G_F^2}{\pi} m_N \left(1 - \frac{m_N T_{\mathrm{nr}}}{2E_\nu^2}\right)\left[Z g_V^p F_p(q^2) + N g_V^n F_n(q^2)\right]^2$$

Radiative Corrections

- Appendix A
- **Appendix B**

Our formalism for radiative corrections is based on the Particle data group (PDG) report, which refers to *Erler et al.* [Prog.Part.Nucl.Phys. 71 (2013) 119-149](). We evaluate the left/right neutrino couplings with fermions at the one loop level

Weak mixing angle

Neutrino charge radius

$$g_{LL}^{\nu_\ell f} = \rho \left[\frac{1}{2} - Q_f \hat{s}_0^2 + \boxtimes_{ZZ} \right] - Q_f \varnothing_{\nu_\ell W} + \text{⧖}_{WW} \qquad (f = u),$$

$$g_{LL}^{\nu_\ell f} = \rho \left[-\frac{1}{2} - Q_f \hat{s}_0^2 + \boxtimes_{ZZ} \right] - Q_f \varnothing_{\nu_\ell W} + \Box_{WW} \qquad (f = d, e),$$

$$g_{LR}^{\nu_\ell f} = -\rho \left[Q_f \hat{s}_0^2 + \boxtimes_{ZZ} \right] - Q_f \varnothing_{\nu_\ell W} \qquad (f = u, d, e),$$

Radiative Corrections

$$\varnothing_{\nu_\ell W} = -\frac{\alpha}{6\pi}\left(\ln\frac{M_W^2}{m_\ell^2}+\frac{3}{2}\right)$$

WW-box

$$\square_{WW} = -\frac{\hat{\alpha}_Z}{2\pi\hat{s}_Z^2}\left[1-\frac{\hat{\alpha}_s(M_W)}{2\pi}\right]$$

WW crossed-box

$$\text{⧖}_{WW} = \frac{\hat{\alpha}_Z}{8\pi\hat{s}_Z^2}\left[1+\frac{\hat{\alpha}_s(M_W)}{\pi}\right]$$

ZZ-box

$$\boxtimes_{ZZ}^{fX} = -\frac{3\hat{\alpha}_Z}{8\pi\hat{s}_Z^2\hat{c}_Z^2}\left(g_{LX}^{\nu_\ell f}\right)^2\left[1-\frac{\hat{\alpha}_s(M_Z)}{\pi}\right]$$

$$\begin{cases} g_{LL}^{\nu_\ell e} = -\frac{1}{2}+\hat{s}_Z^2 \\ g_{LR}^{\nu_\ell e} = \hat{s}_Z^2 \\ g_{LL}^{\nu_\ell u} = \frac{1}{2}-\frac{2}{3}\hat{s}_Z^2 \\ g_{LR}^{\nu_\ell u} = -\frac{2}{3}\hat{s}_Z^2 \\ g_{LL}^{\nu_\ell d} = -\frac{1}{2}+\frac{1}{3}\hat{s}_Z^2 \\ g_{LR}^{\nu_\ell d} = \frac{1}{3}\hat{s}_Z^2. \end{cases}$$

Those are the ***tree-level couplings,*** *with* $\sin^2\theta_W(M_Z)$ *insted of* $\sin^2\theta_W(0)$.

Numerically:

$\rho = 1.00063$, $\alpha_Z = \alpha_{EM}(M_Z) = 127.952^{-1}$, $\alpha_s(M_W) = 123^{-1}$, $s_0^2 = \sin^2\theta_W(0) = 0.23863$,
$s_Z^2 = \sin^2\theta_W(M_Z) = 0.23121$, $c_Z^2 = 1-\sin^2\theta_W$

Radiative Corrections

For protons

$$g_L^{\nu_\ell\, p} = 2g_L^{\nu_\ell\, u} + g_L^{\nu_\ell\, d} = \frac{\rho}{2} - \rho\hat{s}_0^2 - \varnothing_{\nu_\ell W} + 2\,\text{⧖}_{WW} + \Box_{WW} + 2\rho\,\boxtimes_{ZZ}^{uL} + \rho\boxtimes_{ZZ}^{dL},$$

$$g_R^{\nu_\ell\, p} = 2g_R^{\nu_\ell\, u} + g_R^{\nu_\ell\, d} = -\rho\hat{s}_0^2 - \varnothing_{\nu_\ell W} - 2\rho\,\boxtimes_{ZZ}^{uR} - \rho\,\boxtimes_{ZZ}^{dR}\,.$$

$$g_V^{\nu_\ell\, p} = \rho\left(\frac{1}{2} - 2\hat{s}_0^2\right) + 2\,\text{⧖}_{WW} + \Box_{WW} - 2\varnothing_{\nu_\ell W} + \rho(2\boxtimes_{ZZ}^{uL} + \boxtimes_{ZZ}^{dL} - 2\boxtimes_{ZZ}^{uR} - \boxtimes_{ZZ}^{dR})$$

Flavor dependence

Radiative Corrections

For neutrons

The neutron coupling does not depend on the flavor

$$g_L^{\nu_\ell\, n} = g_L^{\nu_\ell\, u} + 2 g_L^{\nu_\ell\, d} = -\frac{\rho}{2} + \rho(\boxtimes_{ZZ}^{uL} + 2\boxtimes_{ZZ}^{dL}) + \hourglass_{WW} + 2\Box_{WW},$$

$$g_R^{\nu_\ell\, n} = g_R^{\nu_\ell\, u} + 2 g_R^{\nu_\ell\, d} = -\rho(\boxtimes_{ZZ}^{uR} + 2\boxtimes_{ZZ}^{dR}).$$

Finally

$$g_V^{\nu\, n} = -\frac{\rho}{2} + 2\Box_{WW} + \hourglass_{WW} + \rho(2\,\boxtimes_{ZZ}^{dL} + \boxtimes_{ZZ}^{uL} - 2\,\boxtimes_{ZZ}^{dR} - \boxtimes_{ZZ}^{uR})$$