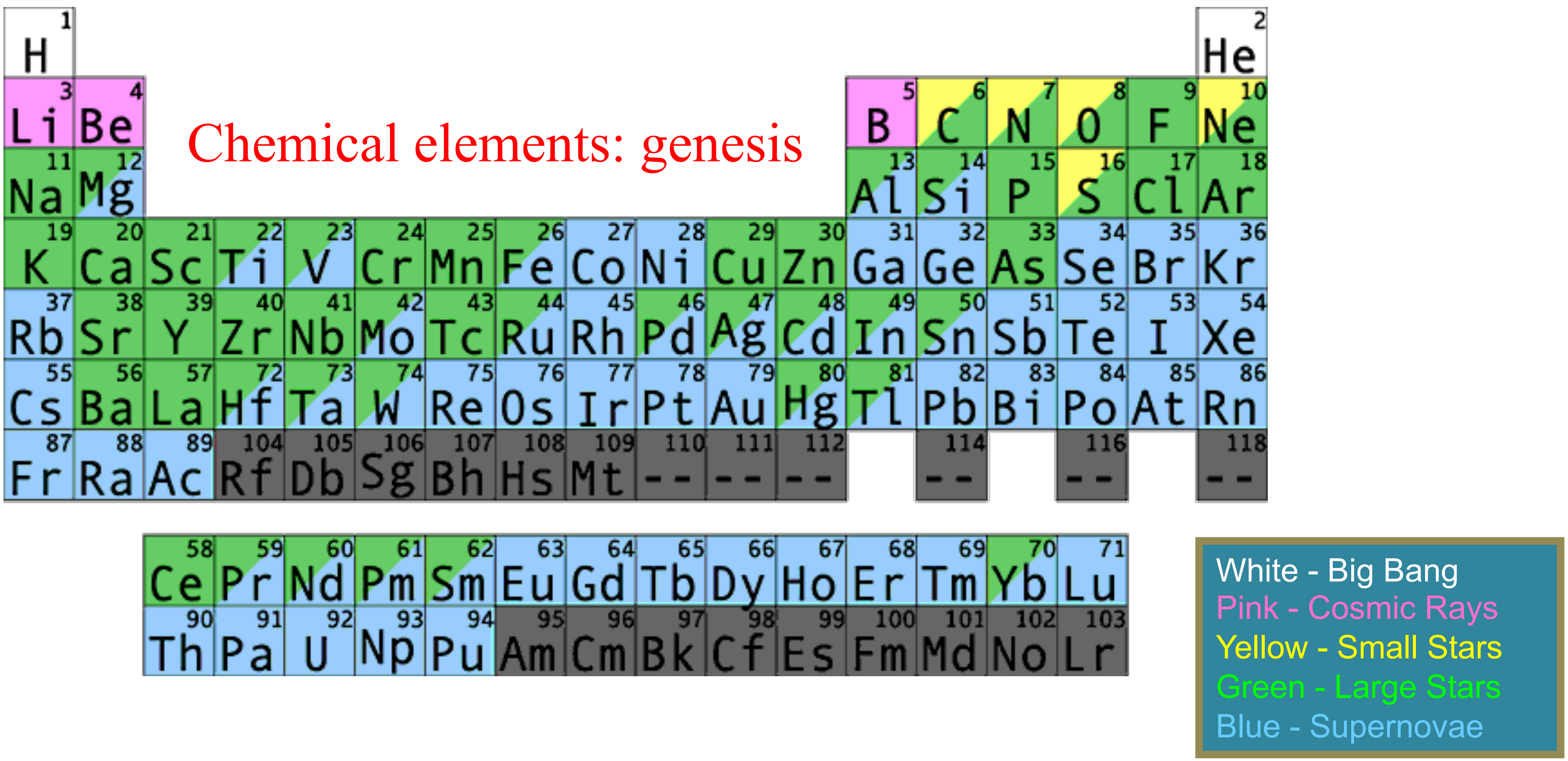
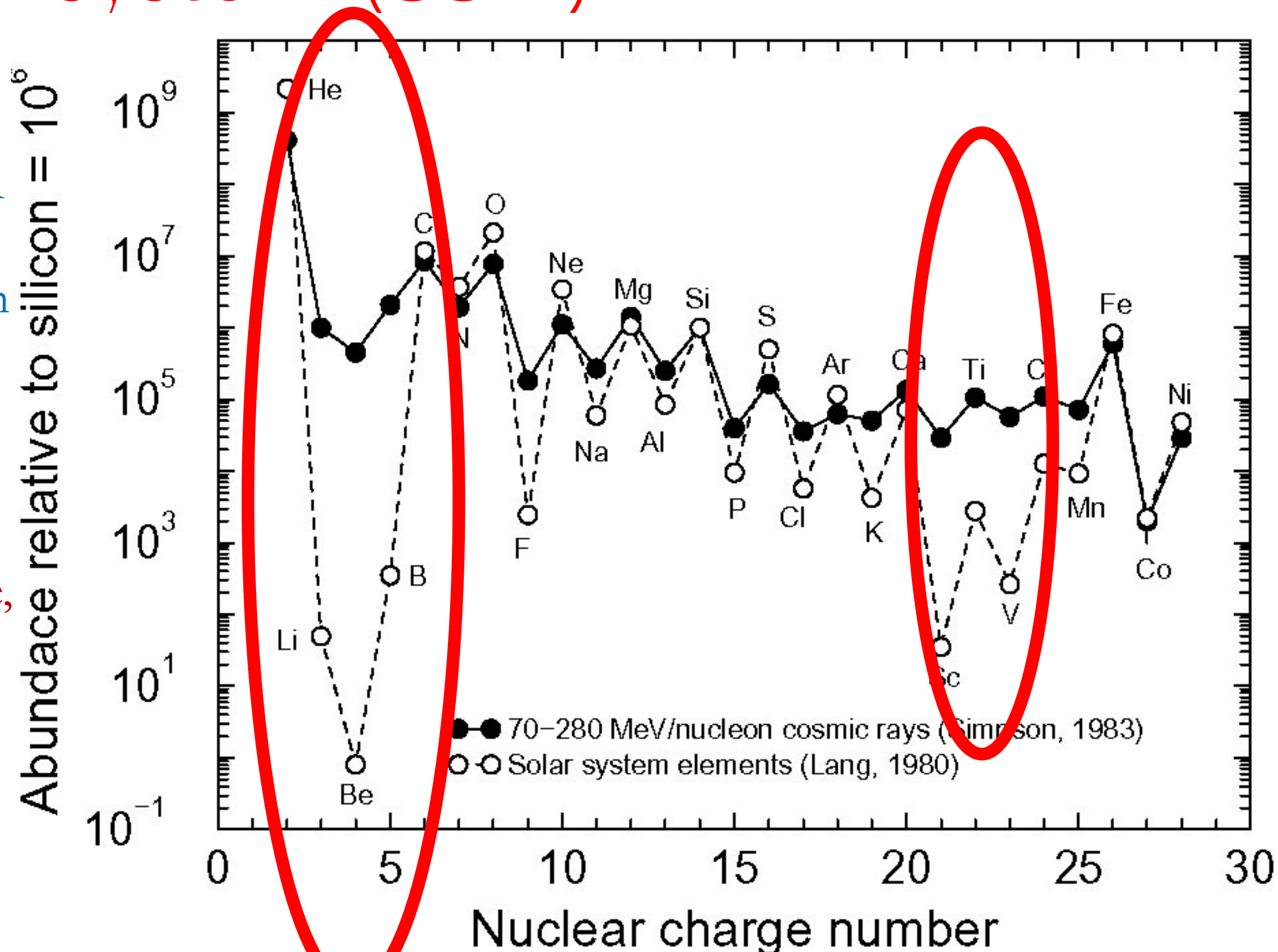
Abundance of elements in the Galaxy

- The "primordial" abundances of the elements are fixed by cosmology: 24% (by mass) of ^{4}He, 76% (by mass) of H
- Nucleosynthesis in stars provides the synthesis of the heavier elements
- Stellar explosions (for $M \gg M_{\odot}$) have a half-life $\ll$ the age of the Universe and provide replenishment of the IG medium
- The percentages of various elements in the Galaxy can be deduced in various ways



Relative abundances of CR and of solar system (SSA)

- H and He are dominant (98%), slightly less in SSA
- Chemical composition of CR shows good agreement with the metallicity relationships in the interplanetary medium [IPM]
- in particular C, O, Mg, Fe
- Great disagreement for Li, Be, B, F, "sub-Fe" (Sc, V)
- Light elements Li, Be, B and those before iron Sc, V are largely abundant in CR wrt SSA
- Too many in CR or too few in IPM?



L = (Li,Be,B)
M = (C,N,O)

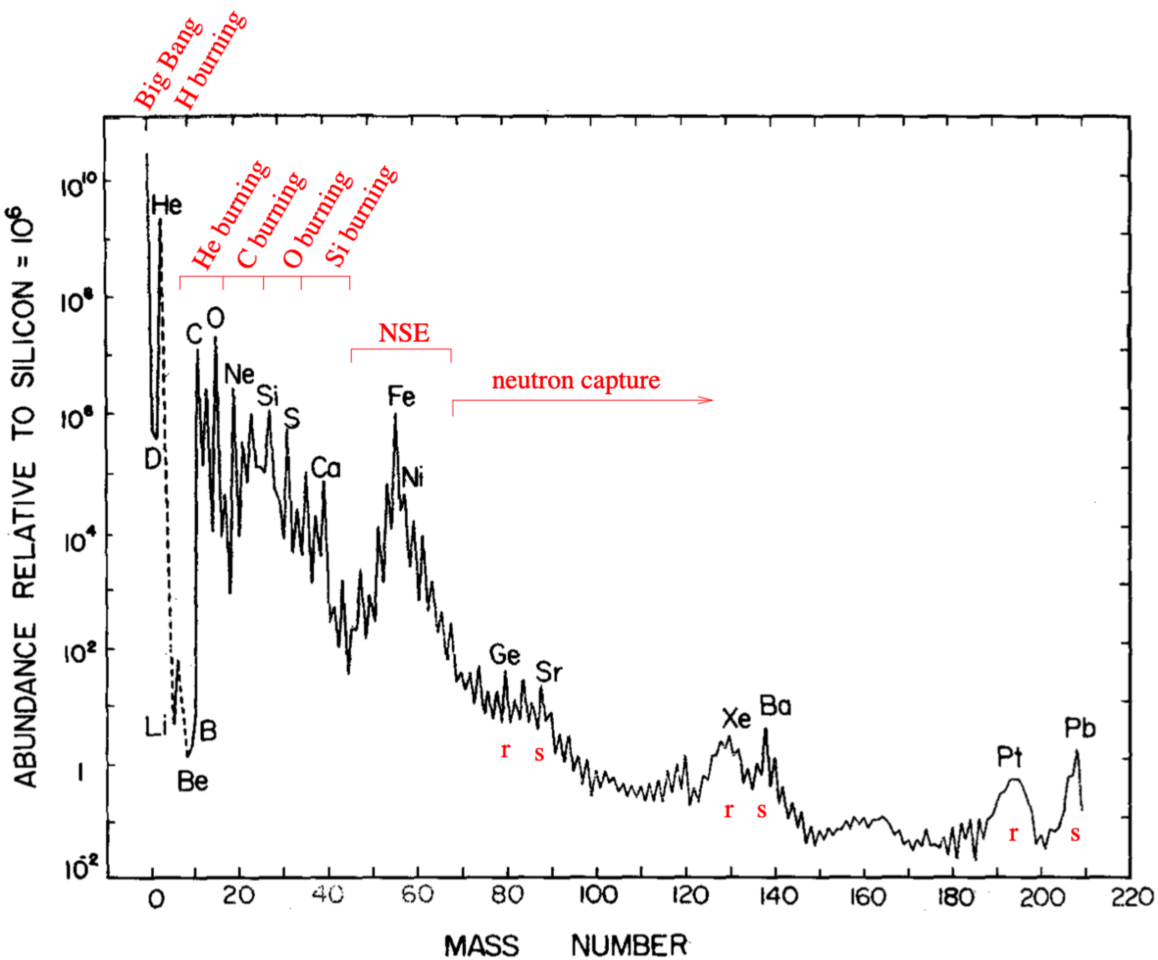
In CR: $R = N_L/N_M \simeq 0.25$

Production of Li, Be, B in CR

- ^{6}Li, Be, B are catalysers of nucleosynthesis reactions. That is they are **not released** at the end of the stellar life. Only ^{7}Li has a small percentage of cosmological origin, while ^{6}Li, Be, B were not produced in the Big Bang.
- Li, Be, B are temporarily produced during the fusion chain, but they are "depleted" during the reactions: the stars burn these elements during their life.
- What is the origin of such rare elements?
- ⇒ Reeves, Fowler & Hoyle (1970) hypothesized their origin as due to the interaction of CR (spallation and fusion of $\alpha + \alpha$) with the interstellar medium (ISM).
- **Mechanism of propagation**. The elements of the group M(=C,N,O) are the elements candidate to produce L(=Li,Be,B) during the propagation.
- The physical process which produces L from M is the spallation, interaction with the protons of interstellar gas.
- How many materials: $\xi = \varrho L$ (g cm^{-2}) the M nuclei must cross to produce the L/M elements in the observed ratio.
- The problem can be set by a system of differential equations.

CRs abundance

The “local galactic” abundance distribution of nuclear species, as a function of mass number A. The abundances are given relative to the Si abundance which is set to 10^6. Peaks due to the r- and s-process



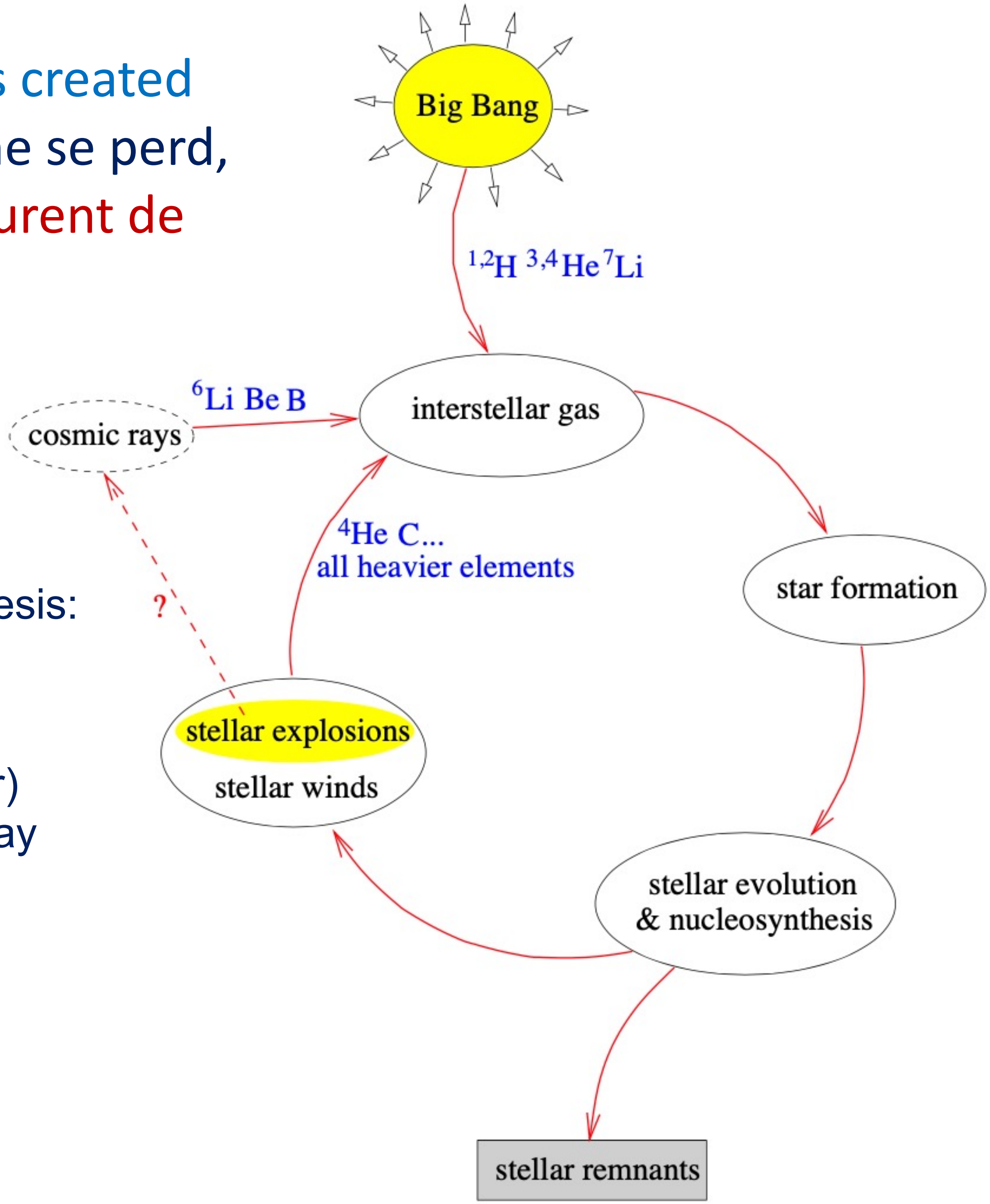
The 25 most abundant nuclei.

rank	Z	element	A	nucleon fraction	source (process)
1	1	H	1	7.057(−1)	BB
2	2	He	4	2.752(−1)	BB, H(CNO, pp)
3	8	O	16	9.592(−3)	He
4	6	C	12	3.032(−3)	He
5	10	Ne	20	1.548(−3)	C
6	26	Fe	56	1.169(−3)	NSE
7	7	N	14	1.105(−3)	H(CNO)
8	14	Si	28	6.530(−4)	O
9	12	Mg	24	5.130(−4)	C, Ne
10	16	S	32	3.958(−4)	O
11	10	Ne	22	2.076(−4)	He
12	12	Mg	26	7.892(−5)	C, Ne
13	18	Ar	36	7.740(−5)	Si, O
14	26	Fe	54	7.158(−5)	NSE, Si
15	12	Mg	25	6.893(−5)	C, Ne
16	20	Ca	40	5.990(−5)	Si, O
17	13	Al	27	5.798(−5)	C, Ne
18	28	Ni	58	4.915(−5)	Si, NSE
19	6	C	13	3.683(−5)	H(CNO)
20	2	He	3	3.453(−5)	BB, H(pp)
21	14	Si	29	3.445(−5)	C, Ne
22	11	Na	23	3.339(−5)	C, H(NeNa)[1]
23	26	Fe	57	2.840(−5)	NSE
24	14	Si	30	2.345(−5)	C, Ne
25	1	H	2	2.317(−5)	BB

[1] H-burning via the NeNa-chain.

The nuclear burning stage, also ‘BB’: Big Bang, ‘NSE’: nuclear statistical equilibrium.

Cosmic recycling: nothing is created nothing is destroyed; rien ne se perd, rien ne se crée; Antoine-Laurent de Lavoisier



Evidence for ongoing nucleosynthesis:

- radioactive material:
  - supernova light curves
  - Tc in AGB stars ($T_{1/2} \leq 4\times10^6$ yr)
  - ^{26}Al decay → γ-rays in Milky Way ($T_{1/2} = 7\times10^5$ yr)
- strong local enrichments:
  - SNRs: C … Fe in X-ray spectra (XMM, ASCA)
  - carbon stars
  - Wolf-Rayet stars

Nucleosynthesis in the early Universe and in the stars

1940: Gamow introduced the theory (with the hope that the relative abundances of all elements could be explained) which credits that the light elements would largely be formed during the first time of rapid expansion of the Universe

- He introduced the fundamental idea of the Big-Bang nucleosynthesis
- When the Universe expanded and cooled down to $T=7.5\ 10^9$ K (650 keV):
- $\rightarrow$ the ratio n/p freezed to the value $\sim 1/7$
- It remained at this value until $T=10^9$ K (90 keV) $\rightarrow$ deuterium and heavier nuclei were formed
- When $T \ll 10^9$ K: all the n were already decayed in p or incorporated in ^{4}He
- The nucleosynthesis through the charged particles stopped because the thermal energies were not enough to allow the interacting nuclei to overcome the coulombian barrier
- We now know that nuclear reactions froze at $T \sim 30$ keV leaving most nuclei in the form of hydrogen and helium. Nucleosynthesis started up again once stars were formed providing “gravitational confinement” for astronomical “fusion reactors.”

The abundance of the elements

In the years 1950-60 the common thinking was that all the chemical elements were produced in the inner of the stars through nuclear processes

Problem

In all the regions of the cosmo, where is possible to measure the abundance of the chemical elements, one observed:

- 75% of the mass is in H
- 23-25% of the mass is in He
- less than 2% in other elements

?

The ratio 3:1 between H and He is ***UNIVERSAL***

Stellar Nucleosynthesis:

H → He → C → N → O → Mg → … → Fe

The mass of the star determines:

- where the chain is interrupted
- the needed time (tens of millions years!)

The primordial nucleosynthesis

- "Light" elements H, He, Li

  They are produced in the primordial Universe a lot of time before the birth of the stars

- "Heavy" elements C, N, O, …, Fe "metals"

  They are produced through the stellar nucleosynthesis

Gamow, Alpher and Herman 1948-50

Short history of the early Universe

Early Universe: T was very high and decreasing with time

Thermal equilibrium between particle-antiparticle pairs and photons: $x + \bar{x} \leftrightarrow 2\gamma$
- as long as kT ≫ $m_x c^2$ ⇒ roughly equal numbers of particles and photons, $N_x \sim N_\gamma$
- When kT < $m_x c^2$, most particle-antiparticle pairs annihilate: $x + \bar{x} \rightarrow 2\gamma$
- Assume a small matter/antimatter asymmetry, $N_x - N_{\bar{x}} \ll N_\gamma$ ⇒ trace amounts of particles 'condense out' and remain...
- e.g. baryons (protons, neutrons), $m_p = 1.67\times10^{-24}$ g: $m_p c^2 = kT$ ⇒ **$T \simeq 10^{13}$ K**

Nucleosynthesis is possible only after: $E_\gamma \lesssim$ binding energy of nuclei ≃ MeV
⇒$T \lesssim 10^{10}$ K⇒$t \gtrsim$ 1s At that time:
- Baryons have condensed out and become non-relativistic; only trace amounts left ⇒ negligible contribution to ρ.
- Universe is dominated by extremely relativistic particles in thermal equilibrium: photons, e-e+ pairs and neutrino, antineutrino pairs (N_ν families, N_ν= 3).

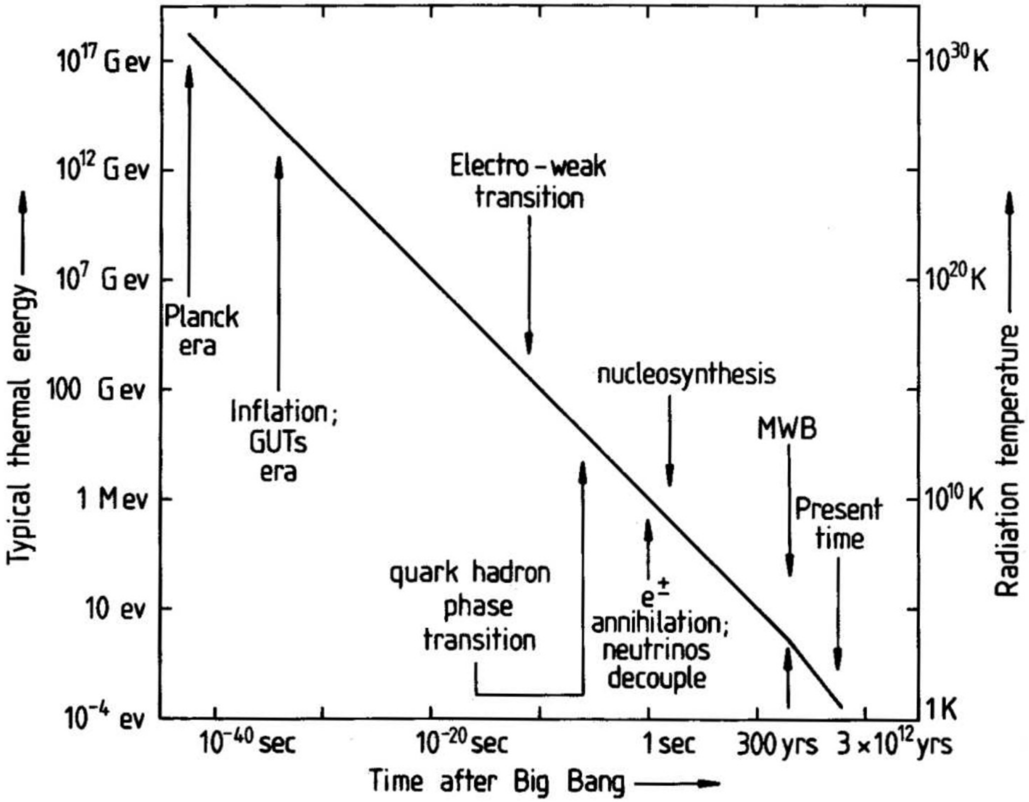
- Hence in the Standard model, both ρ and T are known functions of t ⇒ the outcome of Big Bang nucleosynthesis (BBN) depends only on a single parameter: the **baryon-to-photon ratio, $\eta = \frac{N_B}{N_\gamma} = \frac{n_B}{n_\gamma}$**

Baryon-to-photon ratio, $\eta = \frac{N_B}{N_\gamma} = \frac{n_B}{n_\gamma}$

- **T ≫ 10^{13} K:** $p + \bar{p} \leftrightarrow 2\gamma \Rightarrow \eta \simeq 1$
- **T ~ 10^{13} K:** $p\bar{p}$ pairs annihilate ⇒ $\eta\downarrow\downarrow$ Since then: N_B = constant
- **T ~ 10^{10} K:** $e^{\pm}$ pairs annihilate ⇒ $N_\gamma \uparrow$ Since then ($t \gtrsim 1$s): N_γ = constant

$$\Rightarrow \eta = constant \simeq 10^{-10} - 10^{-9}$$

- Since η is conserved, particularly during BBN, it is a useful measure of the baryon density
- η is related to the baryon density *today*: $\eta = 2.73{\times}10^{-8}\,\Omega_B\,h^2$



- that is value of η at epoch of BBN (t ~ 100 s) should correspond to the measurement of $\Omega_B\,h^2$ from the CMB, when the Universe was 3.8 × 10^5 yrs old: WMAP⇒ $\Omega_B\,h^2$ = 0.023 ± 0.001 ⇒ $\boldsymbol{\eta = (6.1 \pm 0.3)\times10^{-10}}$

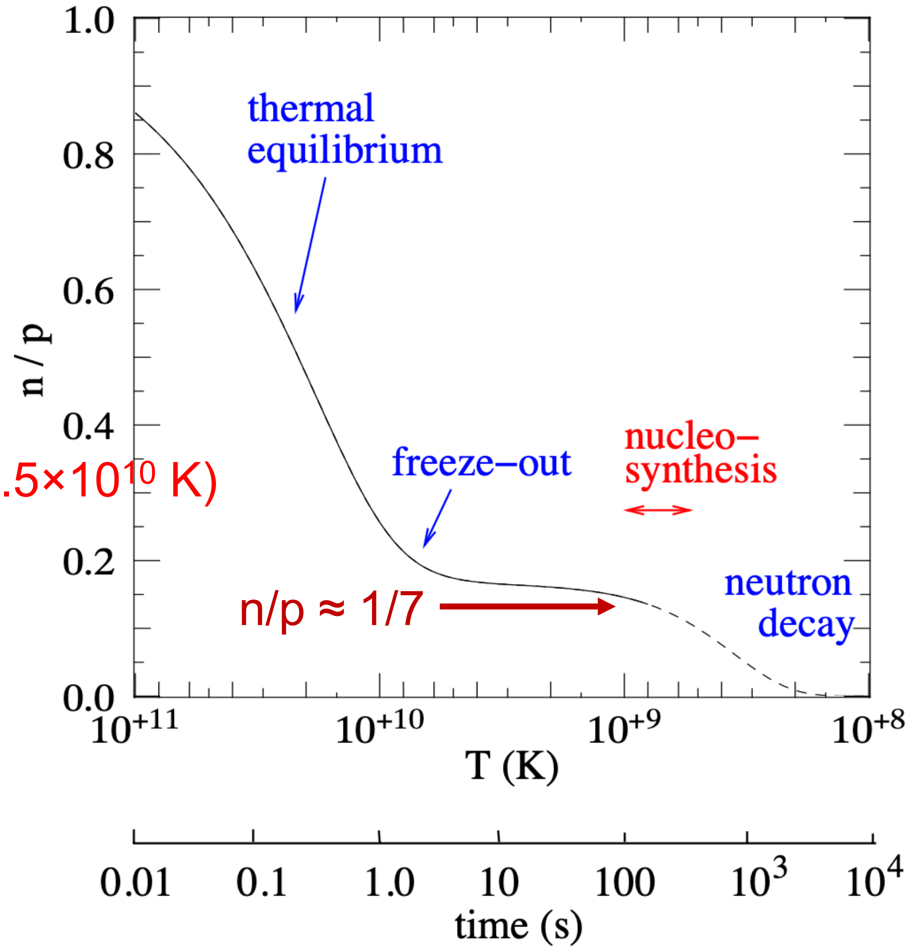
The n/p ratio

- For T high enough, neutrons and protons in equilibrium through **weak interactions**:

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$
$$n + \nu_e \leftrightarrow p + e^-$$

$\Rightarrow$ equilibrium ratio: $\frac{n}{p} = exp\left(-\frac{(m_n - m_p)c^2}{KT}\right)$
$= exp\left(-\frac{1.29\ MeV}{KT}\right)$

(1.29 MeV=1.5×10^{10} K)



- When $T \lesssim T_{crit} (\simeq 10^{10} K)$: $\Rightarrow$ weak interactions become **slower** than the expansion rate $\Rightarrow$ n/p "freezes out" to a constant value: $\frac{n}{p} = exp\left(-\frac{1.29\ MeV}{KT_{crit}}\right)$

- Afterwards, n/p slowly decreases due to neutron decay:

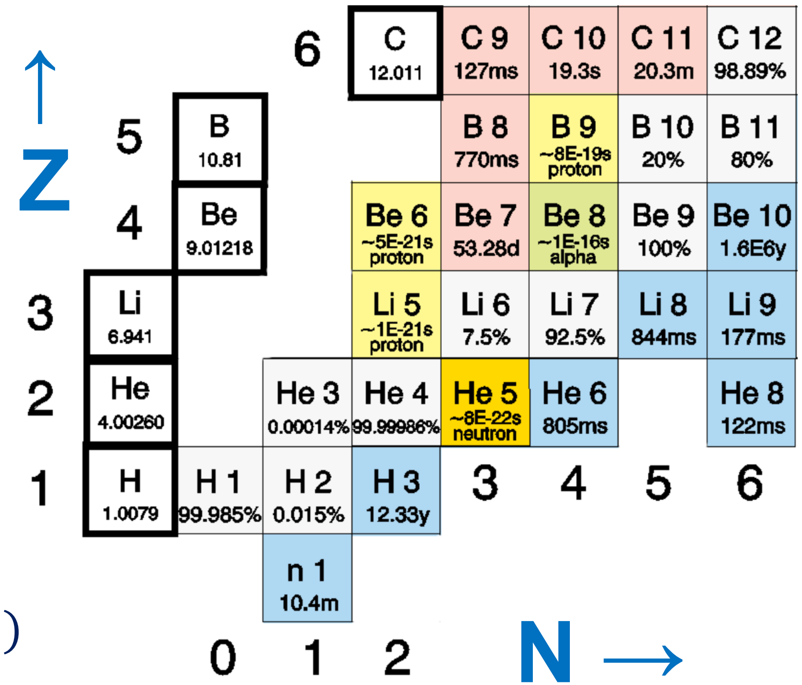
$$n \rightarrow p + e^- + \bar{\nu}_e \quad \text{with } T_{1/2} = 10.3 \text{ min}$$

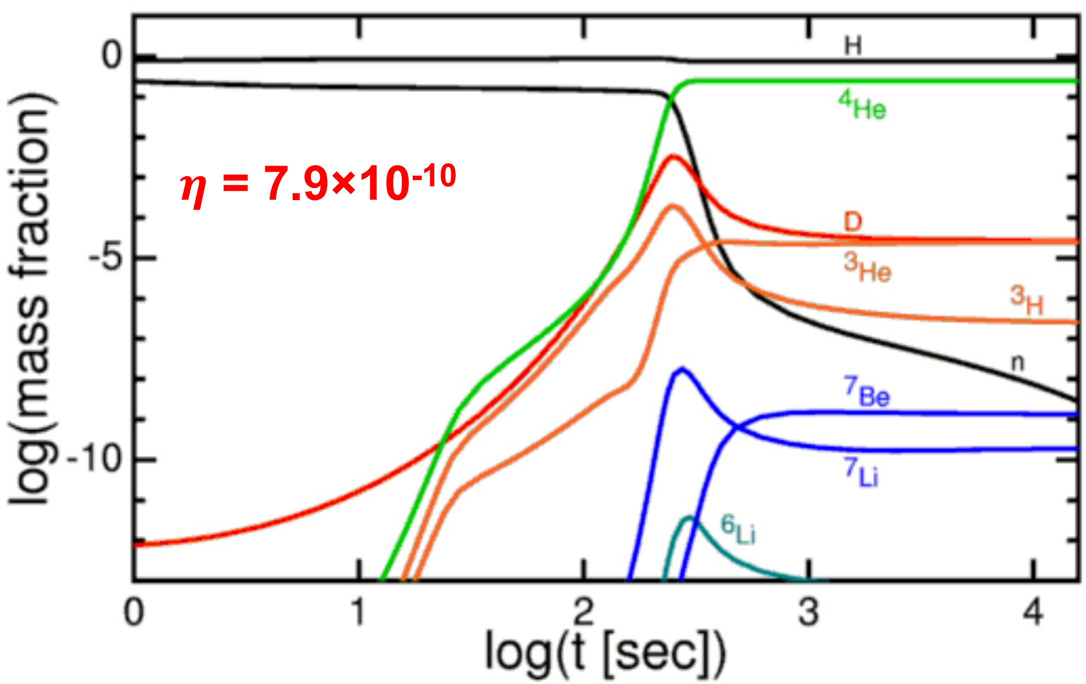
Without further reactions, there would be no neutrons left in the Universe. However, most neutrons are bound by nucleosynthesis before much decay can occur

Time $\lesssim$ 100 s

- **Deuterium**: a bottleneck
- Nucleosynthesis has to go through

  $p + n \rightarrow d + \gamma$ (the alternative $p + p \rightarrow d + e^+ + \nu$ is too slow)
- $T \sim T_{crit}$ synthesis of d is possible
- However, photodisintegration of d:
- $d(\gamma, n)p$ (rate $\propto n_\gamma \exp(-E_B/kT)$) is still much faster than $p(n,\gamma)d$ (rate $\propto n_B$) because $\eta \ll 1$)





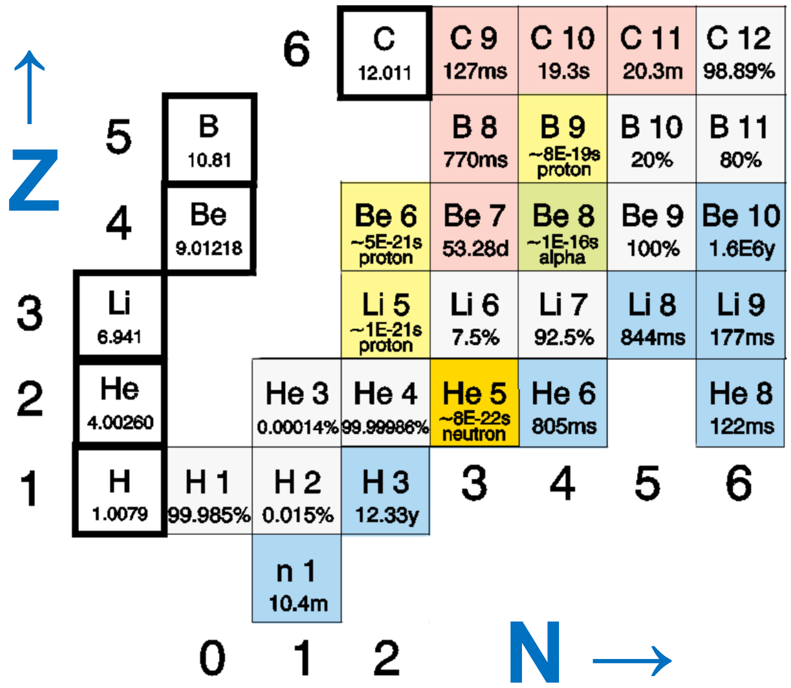
- At **T ≃ 10⁹ K** (t ≃ 100 s) photodisintegrations of deuterium become slow enough.
- Equilibrium d abundance can build up only after **~100 s**,
- nucleosynthesis can really start

Nucleosynthesis of Big Bang

After d survives ($T \simeq 10^9$ K, $\rho \simeq 10^{-5}$ g/cm^3, $t \simeq 100$ s) more reactions follow, the most important of which are:

$$D + p \rightarrow {}^3He + \gamma$$
$$D + D \rightarrow {}^3He + n$$
$$D + D \rightarrow {}^3H + p$$
$${}^3He + n \rightarrow {}^3H + p$$
$${}^3He + D \rightarrow {}^4He + p$$
$${}^3H + D \rightarrow {}^4He + n$$

Note: the weak interaction and the reactions involving gammas emission are too slower than those given above, which involve only the strong interaction



Next bottleneck: there is no stable isotope with mass number 5, i.e.

$$\left\{ \begin{array}{l} {}^4He(n,\gamma){}^5He \\ {}^4He(p,\gamma){}^5Li \end{array} \right.$$ are impossible

Small amounts of ^{7}Li, ^{7}Be:

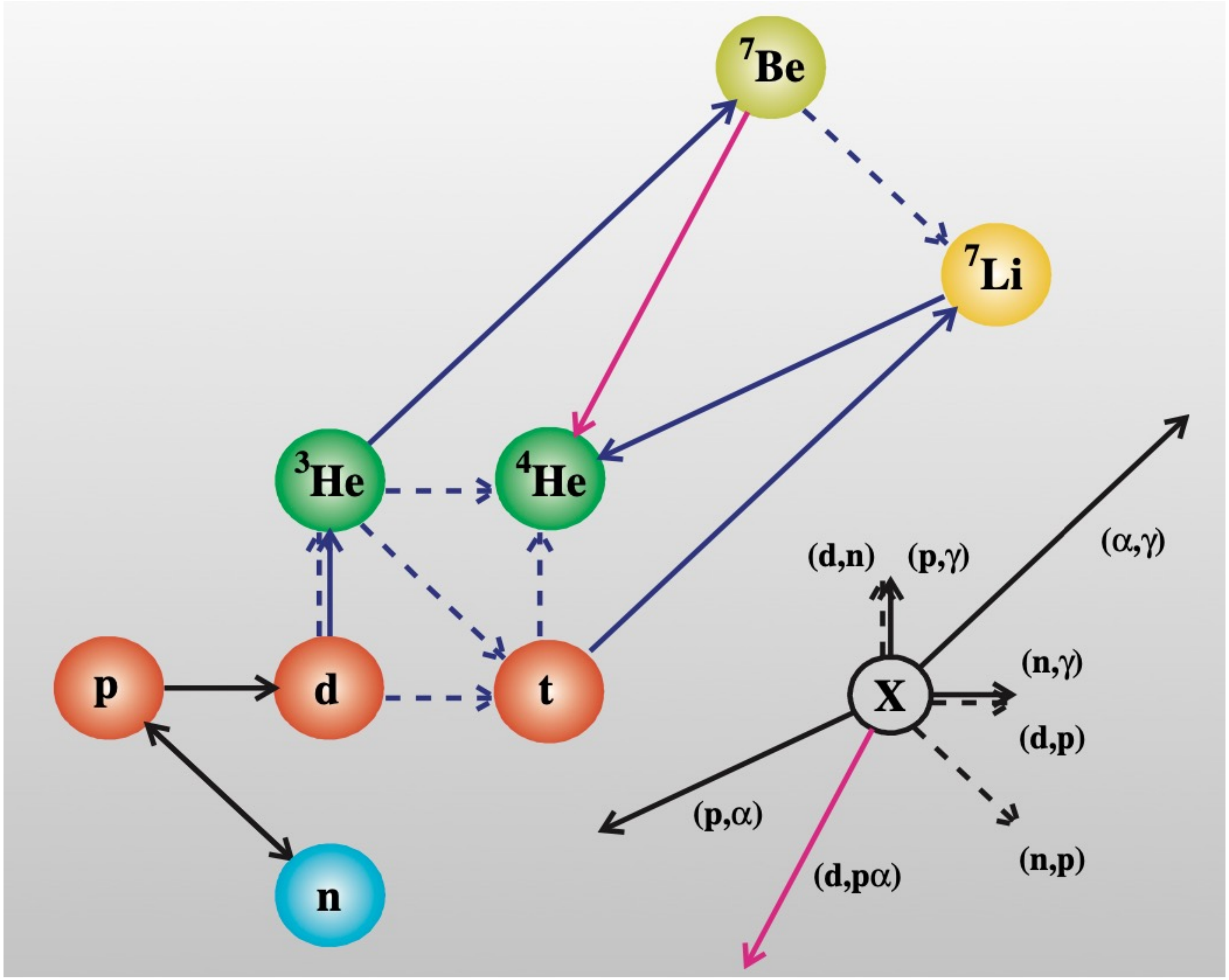
$${}^3He + {}^4He \rightarrow {}^7Be + \gamma$$
$${}^3H + {}^4He \rightarrow {}^7Li + \gamma$$
$${}^7Be + n \rightarrow {}^7Li + p$$
$${}^7Li + p \rightarrow 2\,{}^4He$$

(later followed by ^{7}Be(e$^-$,ν_e)^{7}Li with $T_{1/2}$ = 53 d)

- After ~1000 seconds, T gets so low that Coulomb barriers cause reactions to stop.
- Final mass fractions are determined by competition between the ***expansion rate H*** and the rates of the **nuclear reactions**

→ **effectively all the neutrons go into ^{4}He**

Scheme of the BBN reactions



The first 3 minutes

QUARKS

Baryogenesis

Baryons
protons, neutrons

Leptons
electrons, neutrinos

1 second

Less frequent interactions
1 neutrons every 7 protons

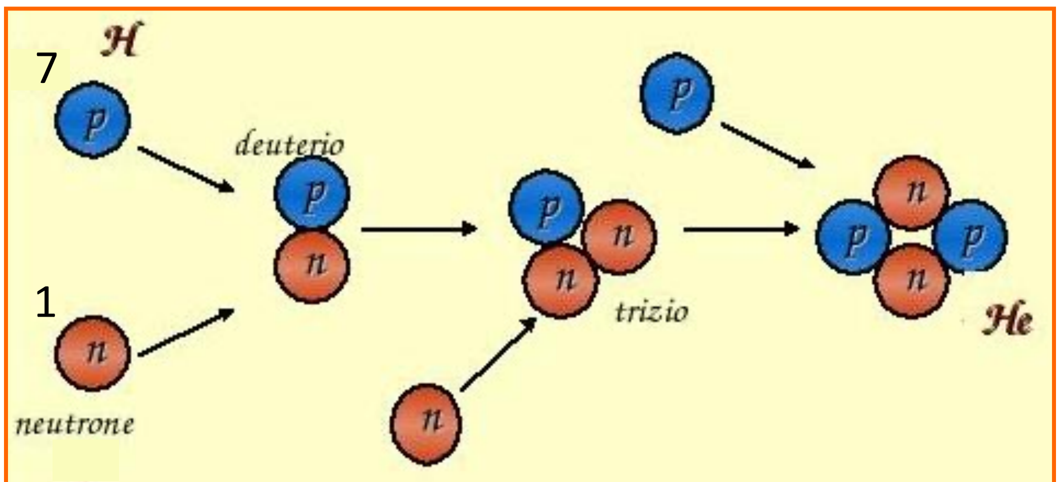
$$\frac{n}{p} \approx \frac{1}{7}$$

100 seconds

Formation of the atomic nuclei

H, He

It will **never** happen in the history of the Universe



Two neutrons produce 1 nucleus of He

25% He   75% H

Mass fraction of ^{4}He:

$$Y = \frac{2n}{p+n} = \frac{2}{\frac{p}{n}+1} \approx \frac{2}{7+1} = 0.25$$

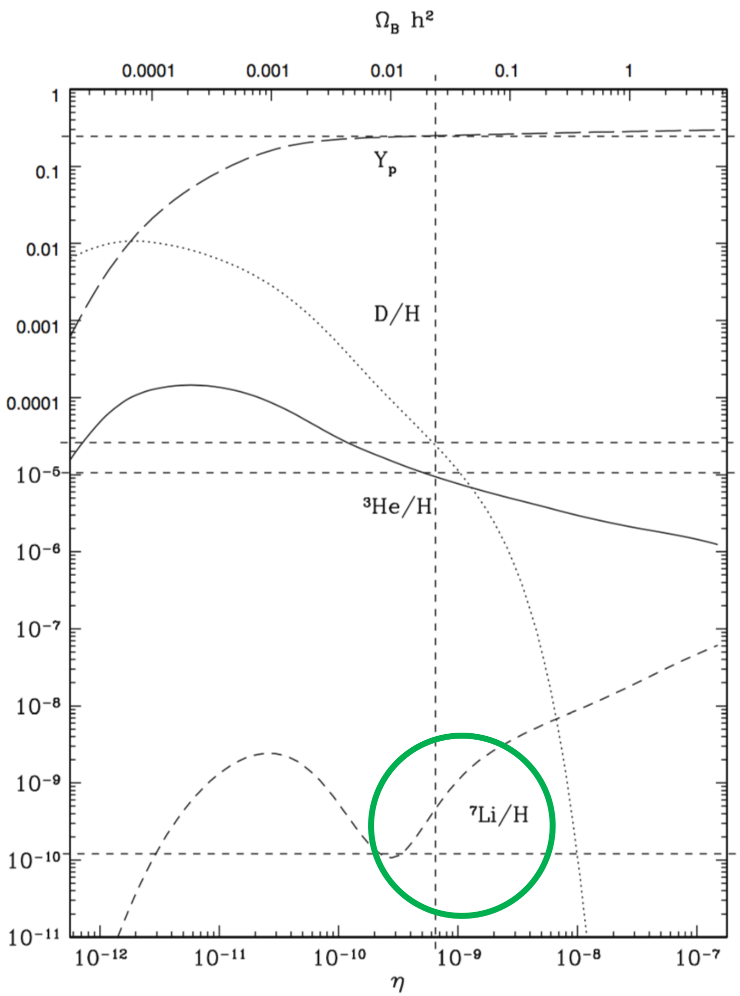
Gamow:

- The Big Bang radiation survives but, because of expansion, cools down
- Gamow prevision of: "relic" radiation = 5 K → -268 °C

Composition of the Universe (baryons)

Density parameters: Ω = density/critical density ← 6 atoms of H/m^3

The cosmological theory of Big Bang foresees, in the most credited scenarios, Ω=1. Indications from anisotropy measurements of CMB



$\Omega = 1$

Baryons (that is protons, nuclei and atoms):

To explain the observations, from nucleosynthesis we know that:

$\Omega_B = 4\%$

Two factors affect BB nucleosynthesis

- The baryon-to-photon ratio, $\eta = \frac{N_B}{N_\gamma} = \frac{n_B}{n_\gamma}$
  - Y(^{4}He) depends only weakly on η because it is mainly determined by the weak interaction rate (⇒ n/p) and not by nuclear reaction rates.
  - Other abundances are very sensitive to η, in particular deuterium ⇒ D provides a sensitive measure of the baryon density
  - (**D is a "baryometer"**).
- The number of ν-families N_ν
  - If N_ν increases
    - ⇒ expansion rate more rapid at a particular T
    - ⇒ T_{crit} increases, n/p freezes out earlier
    - ⇒ Y(^{4}He) increases
    - Hence: Y is sensitive to non-standard particle physics ($N_\nu \neq 3$).
    - The abundances of D, ^{3}He and ^{7}Li are much less sensitive to N_ν.

Gamow thought that subsequently heavier elements have been formed through a succession of n (insensitive to the Coulomb barrier) captures and β decays

The Gamow Universe was likened to a "giant fusion reactor".

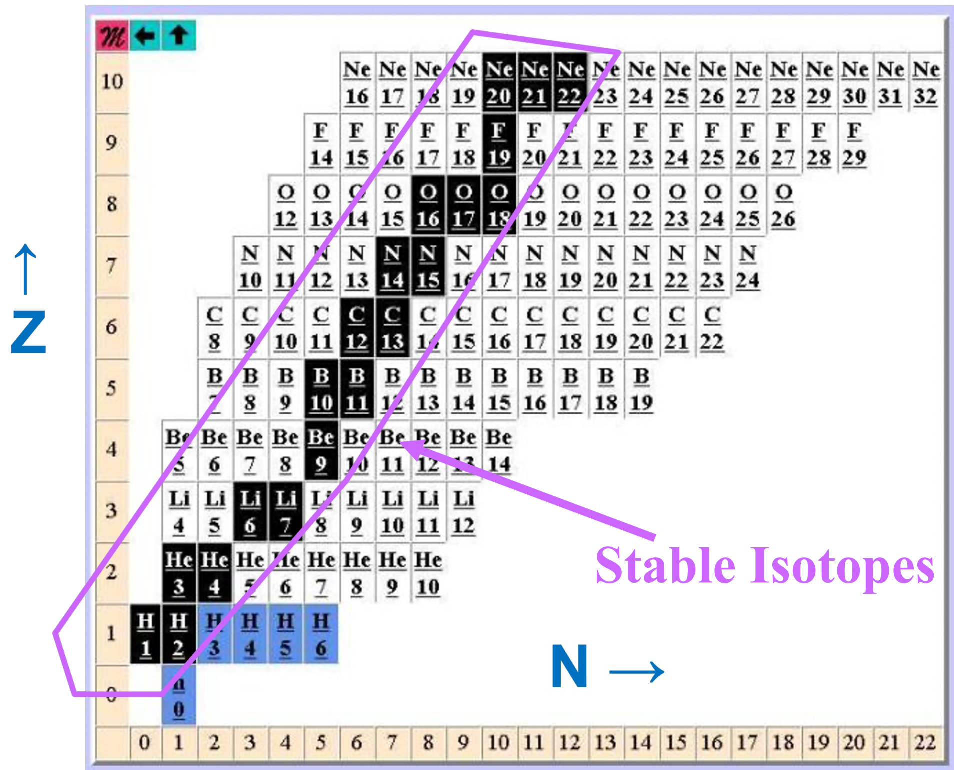
Then:

discovery that in nature there are no stable nuclei with A = 5 and 8

→ implied that the nucleosynthesis of elements heavier than ^{4}He was impossible, this cannot provide stable nuclei either by reacting with the protons (A = 5), or by reacting with other ^{4}He nuclei (A = 8)

1957: Fowler & Hoyle, and Cameron, hypothesized that the nucleosynthesis of elements happens in the stars

Calculations on the abundance of intergalactic ^{4}He confirmed, however, that there was an initial phase of nucleosynthesis in the early Universe, which was stopped with the production of ^{4}He (actually stopped up to ^{7}**Li**).



Important nuclei for nucleosynthesis

- We note the **high** binding energy of ^{4}He in comparison with the other light nuclei, which implies that this species will be the primary product of primordial nucleosynthesis.
- The **absence** of stable nuclei at $A = 5$ or $A = 8$ prevents the production of heavy elements by two-body reactions between ^{1}H and ^{4}He.
- Primordial nucleosynthesis **stops** at $A = 7$.
- The production of heavy elements occurs in stars where the **triple-α reaction** $3\ ^4\text{He} \rightarrow {}^{12}\text{C}$ takes place.
- We note that B / A is a slowly varying function for $A > 12$ with a **broad maximum at ^{56}Fe**, the ultimate product of stellar nucleosynthesis.
- Elements with ***A* > 56** can be produced by "explosive" nucleosynthesis in supernovae

Nucleus	B/A (MeV)	n_x/n_{H} Primordial (observed)	Half-life	Decay mode
p	0	1	$> 10^{32}$ yr	
n	0	0	10.24 min	$n \rightarrow pe^-\bar{\nu}_e$
^{2}H	1.11	$(2.68 \pm 0.27) \times 10^{-5}$		
^{3}H	2.83	0	12.3 yr	$^3\text{H} \rightarrow {}^3\text{He}\ e^-\bar{\nu}_e$
^{3}He	2.57	$(1.1 \pm 0.2) \times 10^{-5}$		
^{4}He	7.07	0.064 ± 0.002		
^{5}Li	5.27	0	3×10^{-22} s	$^5\text{Li} \rightarrow p\ ^4\text{He}$
^{6}Li	5.33	$< 10^{-10}$		
^{7}Li	5.61	$(4.3 \pm 0.1) \times 10^{-10}$		
^{7}Be	5.37	0	53.3 d	$e^-\ ^7\text{Be} \rightarrow \nu_e\ ^7\text{Li}$
^{8}Be	7.06	0	6.7×10^{-17} s	$^8\text{Be} \rightarrow {}^4\text{He}\ ^4\text{He}$
^{12}C	7.6	0		
^{16}O	8.0	0		
^{56}Fe	8.7	0		
^{208}Pb	7.7	0		

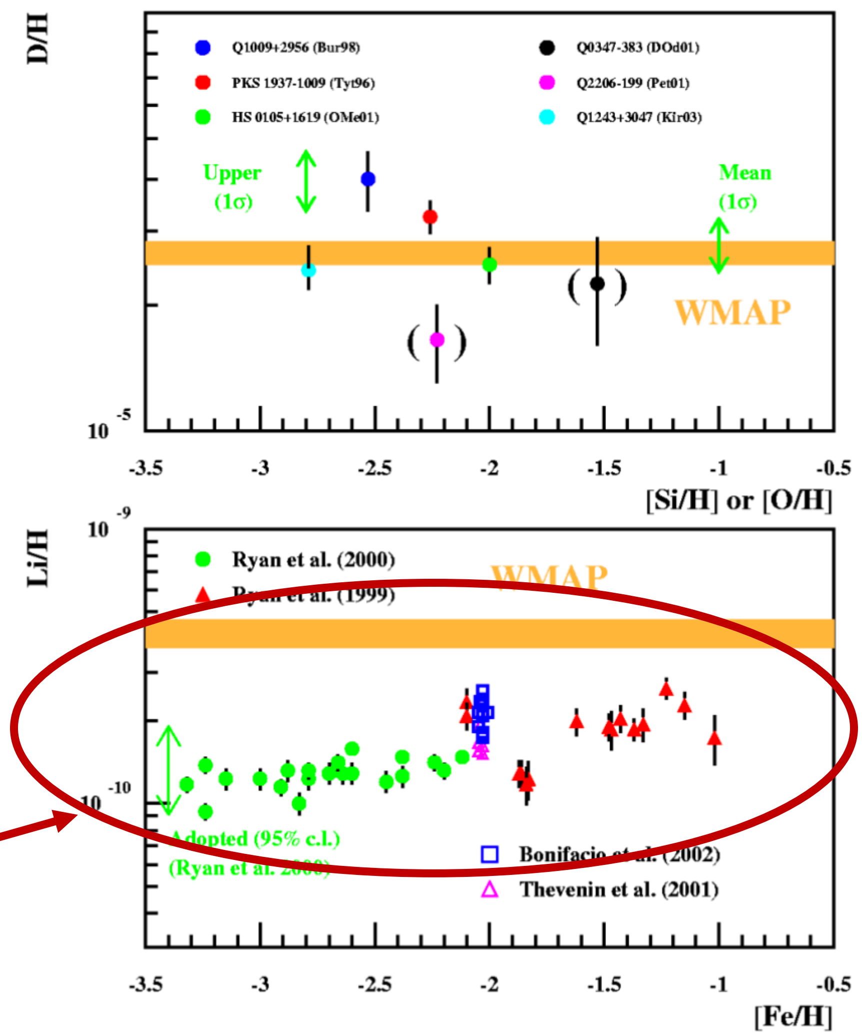
η = baryon/photon ratio

Theoretical and observed primordial abundances relative to ^{1}H. The theoretical abundances are for $\eta = 6.18 \times 10^{-10}$

Nuclei	Theory	Observation
$^2\text{H}:\ 10^5 n_2/n_1$	$2.83(1 \pm 0.03)$	2.68 ± 0.27
$^3\text{He}:\ 10^5 n_3/n_1$	$1.08(1 \pm 0.03)$	1.1 ± 0.2
$^4\text{He}:\ \rho_4/\rho_1$	0.2486 ± 0.0005	0.240 ± 0.006
$^7\text{Li}:\ 10^{10} n_7/n_1$	$4.6(1 \pm 0.1)$	1.25 ± 0.3

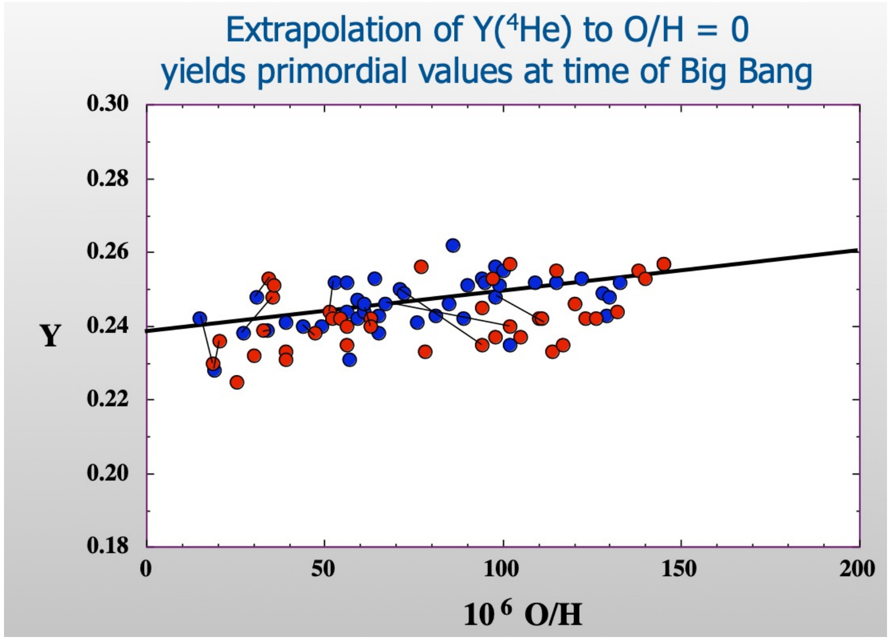
BBN results

- Primordial abundances of H, D, ^{3}He, ^{4}He, ^{7}Li do not contradict standard BBN: all abundances fit (almost) a single value of the parameter η
- The implied value of η during BBN corresponds to the value derived from the CMB: $\boldsymbol{\eta \simeq 6\times 10^{-10}}$
- Hence, the Universe at t≃100s (T ~10^9 K) and at t≃3.8×10^5 yr (T ~10^4 K) **agree**!
- Normal, baryonic matter makes up only ≈ 5% of the critical density of the Universe
- Apparent discrepancy for ^{7}Li, possible explanations:
  - ✓ destruction of ^{7}Li in stars, due to extra (e.g. rotational) mixing?
  - ✓ systematic errors in the derived abundance?
  - ✓ unknown rate of some of the involved nuclear reactions?

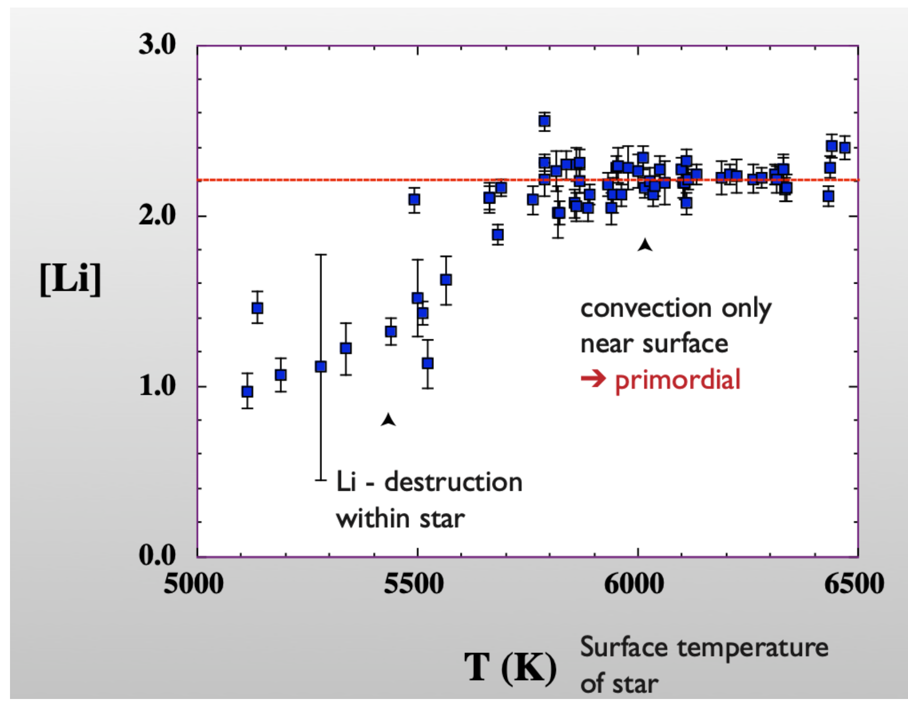


^{7}Li problem!

^{4}He abundancies

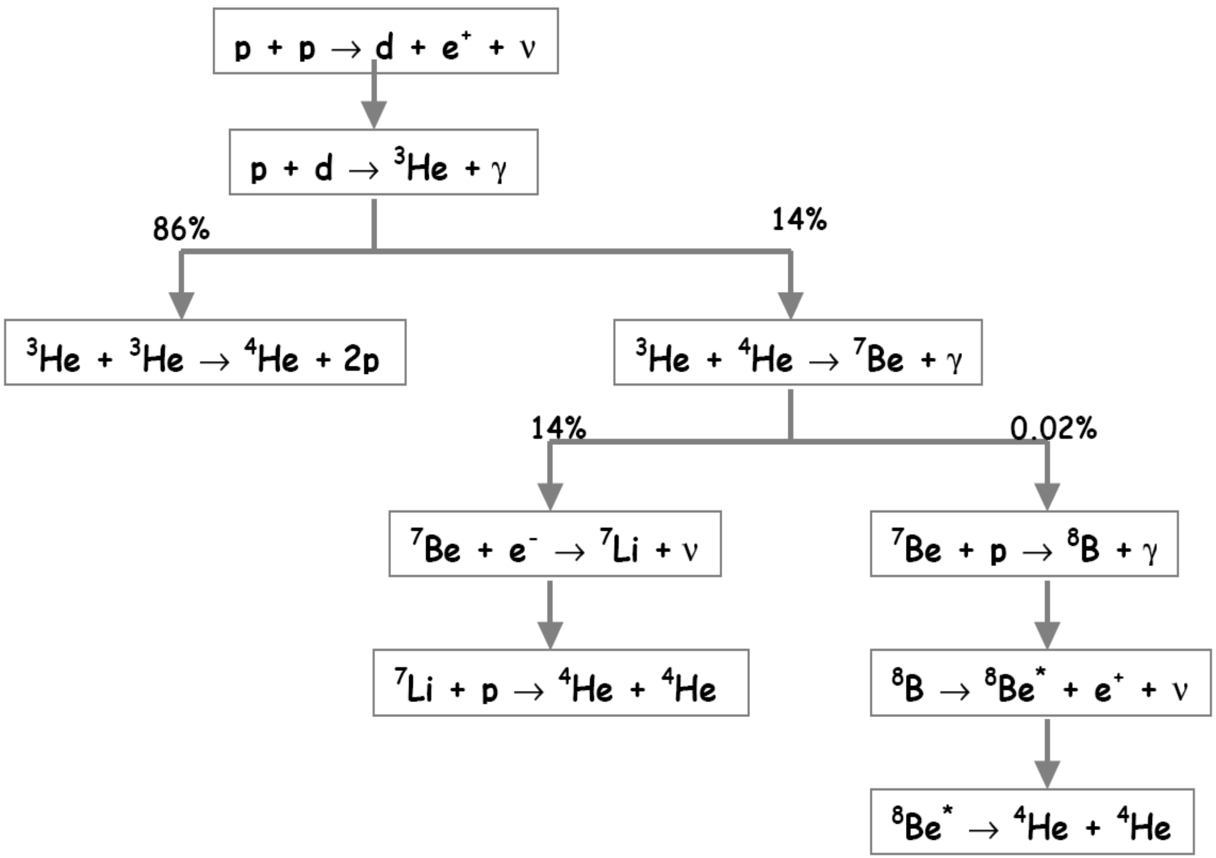


^{7}Li abundancies from old Halo Stars ...with low Fe-fraction



In the stars of the main sequence (the most common, like our Sun), the energy production mechanism occurs through the transformation of 4p in a nucleus of ^{4}He, according to the process

$$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu \qquad (Q=26.73\ MeV).$$



p-p chain

The process occurs through a serie of two-body reactions, which form the p-p chain

1938 -- Thermonuclear reaction chains

Hans Bethe (Nobel prize 1967)

1938: no ν in the nuclear reactions ...

MARCH 1, 1939 PHYSICAL REVIEW VOLUME 55

Energy Production in Stars*

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(Received September 7, 1938)

It is shown that the *most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons*. These reactions form a cycle in which the original nucleus is reproduced, *viz.* $C^{12}+H=N^{13}$, $N^{13}=C^{13}+\epsilon^+$, $C^{13}+H=N^{14}$, $N^{14}+H=O^{15}$, $O^{15}=N^{15}+\epsilon^+$, $N^{15}+H=C^{12}+He^4$. Thus carbon and nitrogen merely serve as catalysts for the combination of four protons (and two electrons) into an α-particle (§7).

The carbon-nitrogen reactions are unique in their cyclical character (§8). For all nuclei lighter than carbon, reaction with protons will lead to the emission of an α-particle so that the original nucleus is permanently destroyed. For all nuclei heavier than fluorine, only radiative capture of the protons occurs, also destroying the original nucleus. Oxygen and fluorine reactions mostly lead back to nitrogen. Besides, these heavier nuclei react much more slowly than C and N and are therefore unimportant for the energy production.

The agreement of the carbon-nitrogen reactions with observational data (§7, 9) is excellent. In order to give the correct energy evolution in the sun, the central temperature of the sun would have to be 18.5 million degrees while integration of the Eddington equations gives 19. For the brilliant star Y Cygni the corresponding figures are 30 and 32. This good agreement holds for all bright stars of the main sequence, but, of course, not for giants.

For fainter stars, with lower central temperatures, the reaction $H+H=D+\epsilon^+$ and the reactions following it, are believed to be mainly responsible for the energy production. (§10)

It is shown further (§5–6) that *no elements heavier than He^4 can be built up in ordinary stars*. This is due to the fact, mentioned above, that all elements up to boron are disintegrated by proton bombardment (α-emission!) rather than built up (by radiative capture). The instability of Be^8 reduces the formation of heavier elements still further. The production of neutrons in stars is likewise negligible. The heavier elements found in stars must therefore have existed already when the star was formed.

Finally, the suggested mechanism of energy production is used to draw conclusions about astrophysical problems, such as the mass-luminosity relation (§10), the stability against temperature changes (§11), and stellar evolution (§12).

§1. INTRODUCTION

THE progress of nuclear physics in the last few years makes it possible to decide rather definitely which processes can and which cannot occur in the interior of stars. Such decisions will be attempted in the present paper, the discussion being restricted primarily to main sequence stars. The results will be at variance with some current hypotheses.

The first main result is that, under present conditions, no elements heavier than helium can be built up to any appreciable extent. Therefore we must assume that the heavier elements were built up *before* the stars reached their present state of temperature and density. No attempt will be made at speculations about this previous state of stellar matter.

The energy production of stars is then due entirely to the combination of four protons and two electrons into an α-particle. This simplifies the discussion of stellar evolution inasmuch as the amount of heavy matter, and therefore the opacity, does not change with time.

The combination of four protons and two electrons can occur essentially only in two ways. The first mechanism starts with the combination of two protons to form a deuteron with positron emission, *viz.*

$$H+H=D+\epsilon^+. \quad (1)$$

The deuteron is then transformed into He^4 by further capture of protons; these captures occur very rapidly compared with process (1). The second mechanism uses carbon and nitrogen as catalysts, according to the chain reaction

$$\begin{aligned} &C^{12}+H=N^{13}+\gamma, \qquad N^{13}=C^{13}+\epsilon^+\\ &C^{13}+H=N^{14}+\gamma,\\ &N^{14}+H=O^{15}+\gamma, \qquad O^{15}=N^{15}+\epsilon^+\\ &N^{15}+H=C^{12}+He^4. \end{aligned} \quad (2)$$

The catalyst C^{12} is reproduced in all cases except about one in 10,000, therefore the abundance of carbon and nitrogen remains practically unchanged (in comparison with the change of the number of protons). The two reactions (1) and

* Awarded an A. Cressy Morrison Prize in 1938, by the New York Academy of Sciences.

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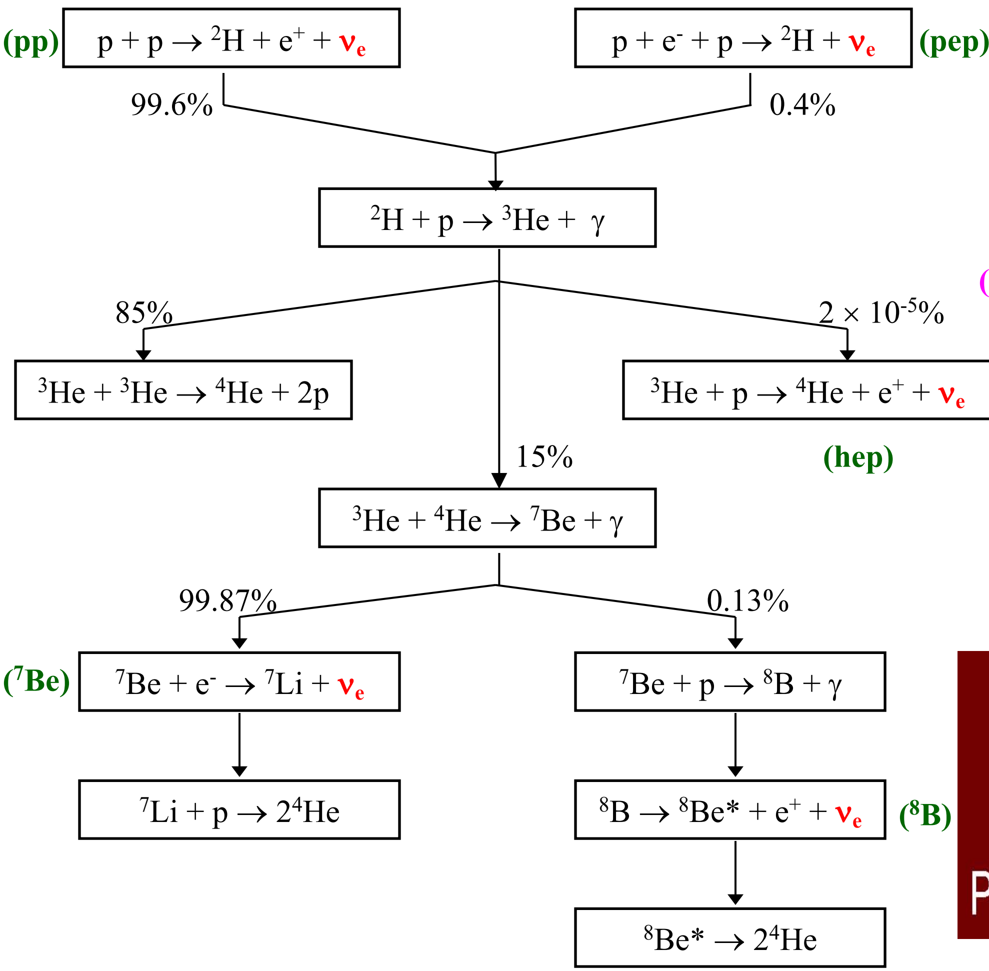
First evidences of anomalies in the neutrino field arrived from **solar neutrinos** (end of '60 and '70)

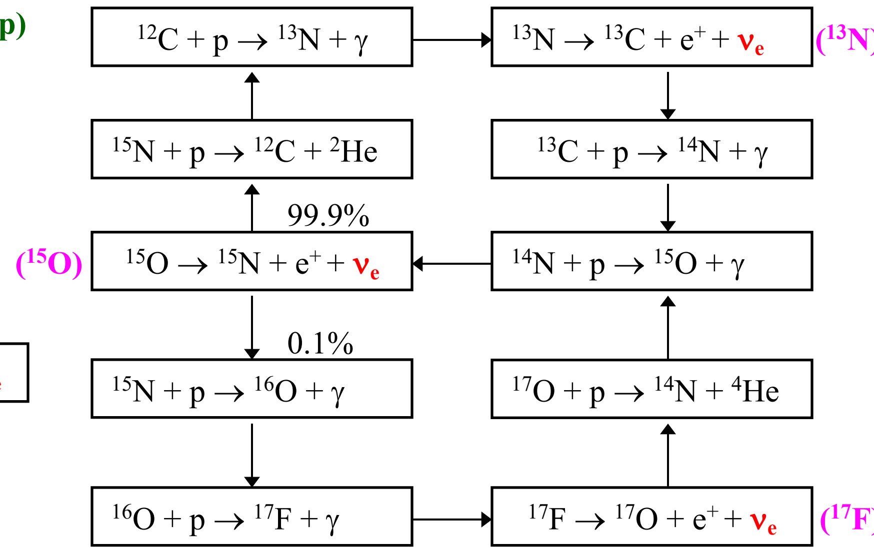
The solar ν are produced in the nuclear reactions in the solar core:

$$4p \rightarrow \alpha + 2e^+ + 2\nu_e + 26.7\ \text{MeV}$$

pp **chain**



CNO **cycle**



$$\Phi_\nu = \frac{2L_{sun}}{25\text{MeV}}\frac{1}{4\pi(1\text{AU})^2} = 7\cdot 10^{10}\ \text{sec}^{-1}\ \text{cm}^{-2}$$

Pioneers: Ray Davis and John Bahcall, starting in '60's

The fusion in stars and the origin of the elements

Note that:

First reaction of the chain is regulated by a weak interaction (β decay)

It has an extremely small σ with respect to the ones of the subsequent reactions:

$\sigma_{pp} \approx 10^{-51}$ cm^2, to be compared with:

$\sigma \approx 10^{-36}$ cm$^2 \div 10^{-33}$ cm^2

This forbids that the star quickly burn out all their fuel

The slowest reaction regulates the transformation velocity of the chain

T inside the Sun $\approx 10^7$ K corresponding to a $E_{kinetic}$ of order of keV

The fusion reactions proceed through tunnel effect

In stellar plasma particles have a Maxwellian-Boltzmann velocity distribution

The cross section of the interaction depends on the relative velocity of the interacting nuclei

Consider a stellar gas with

N_A particles per volume unit of type A and

N_B particles per volume unit of type B

The cross section σ depends only on the relative velocity

$\mathbf{v} = \mathbf{v_A} - \mathbf{v_B}$ → can be considered either particles of type A or B as projectiles or as targets

If we choose nuclei of type A that are moving with velocity v as projectiles, the nuclei B must be considered at rest

- The bullets see an effective area of reaction: $F = \sigma(v)N_B$.
- The flux J incident is instead given by: $J = N_A v$
- The number of reactions per unit volume and unit time, r (rate), will then be:

$$r = FJ = N_B N_A \sigma(v) v$$

If the speed of the particles have a distribution $\varphi(v)$, it is necessary to perform the convolution product of the function $\sigma(v)v$.

$$r = N_A N_B \int_0^\infty \varphi(v)\sigma(v) v\,dv = N_A N_B < \sigma v >$$

The function $\varphi(v)$ is described by the Maxwell-Boltzmann distribution:

$$\varphi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)$$

The function φ(v) can be re-written in terms of energy:

$$\varphi(E) \propto E \exp\left(-\frac{E}{kT}\right)$$

✓ At low energy, with E<<kT, the function φ(E) increases almost linearly with E and reaches a maximum at E=kT

✓ At high energy, with E≫kT, the function decreases exponentially and asymptotically goes to zero

If the temperature is measured in unit 10^6 K (T_6), the numerical value of kT in keV is given by:

$$kT=0.0862T_6 \text{ keV}$$

- At T_{env} the value of kT (2.6×10^{-5} keV) is small
- At the T internal to the Sun (T_6=15)   kT=1.3 keV
- In a supernova (T_6=5000) kT=430 keV

Virial Theorem Applied to the Sun

Virial Theorem $\langle E_{\text{kin}} \rangle = -\frac{1}{2} \langle E_{\text{grav}} \rangle$

Approximate Sun as a homogeneous sphere with

Mass $M_{\text{sun}} = 1.99 \times 10^{33}\,\text{g}$

Radius $R_{\text{sun}} = 6.96 \times 10^{10}\,\text{cm}$

Gravitational potential energy of a proton near center of the sphere

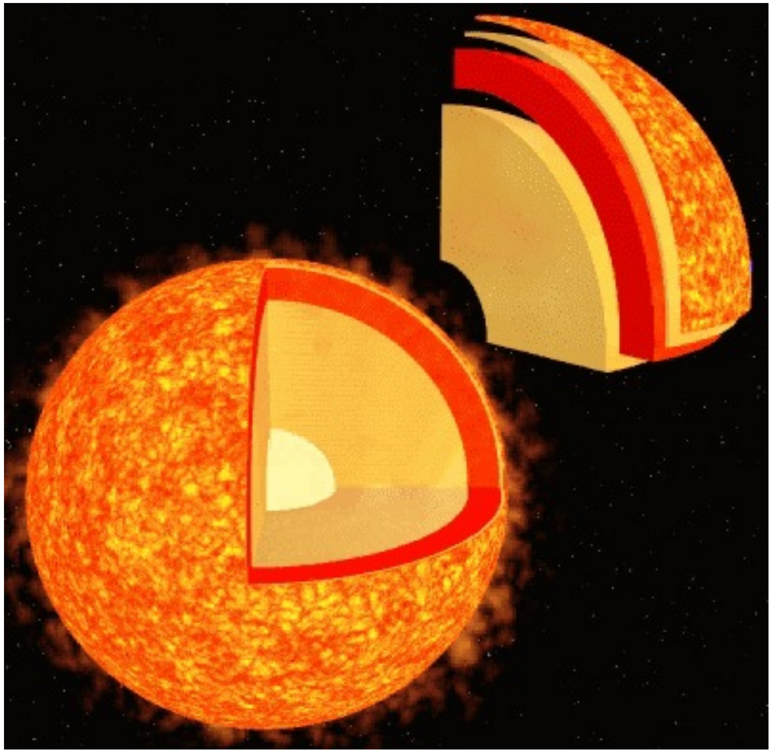
$$\langle E_{\text{grav}} \rangle = -\frac{3}{2} \frac{G_N M_{\text{sun}} m_p}{R_{\text{sun}}} = -3.2\ \text{keV}$$

Thermal velocity distribution

$$\langle E_{\text{kin}} \rangle = \frac{3}{2} k_{\text{B}} T = -\frac{1}{2} \langle E_{\text{grav}} \rangle$$

Estimated temperature

$$T = 1.1\ \text{keV}$$



Central temperature from standard solar models:

T_c=1.56 ×10^7 K = 1.34 keV

For the nuclear reactions in the stellar media both the velocities of the interacting nuclei are described by the Maxwell-Boltzmann distribution:

$$\varphi(v_A) = 4\pi v_A^2 \left(\frac{m_A}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m v_A^2}{2kT}\right)$$

$$\varphi(v_B) = 4\pi v_B^2 \left(\frac{m_B}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m v_B^2}{2kT}\right)$$

Rate of reaction for particle pairs:

$$<\sigma v> = \int_0^\infty \int_0^\infty \varphi(v_A)\varphi(v_B)\sigma(v) v \, dv_A \, dv_B$$

v_A and v_B are linked to the relative velocity, v, and to the velocity of the center of mass, V, through the relations:

$$\mathbf{v} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{V} = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B}$$

Using the reduced mass $\mu = \frac{m_A m_B}{m_A + m_B}$

and the total mass $M = m_A + m_B$

→ the rate can be written in terms of the variables v e V:

$$<\sigma v> = \int_0^\infty \int_0^\infty \varphi(V)\varphi(v)\sigma(v) v \, dV \, dv = \int_0^\infty \varphi(V) dV \int_0^\infty \varphi(v)\sigma(v) v \, dv$$

Recalling that the distribution functions are normalized, it remains: $<\sigma v> = \int_0^\infty \varphi(v)\sigma(v) v \, dv$

recalling that $E = 1/2 \, \mu v^2$, one gets:

$$<\sigma v> = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{kT}\right) dE$$

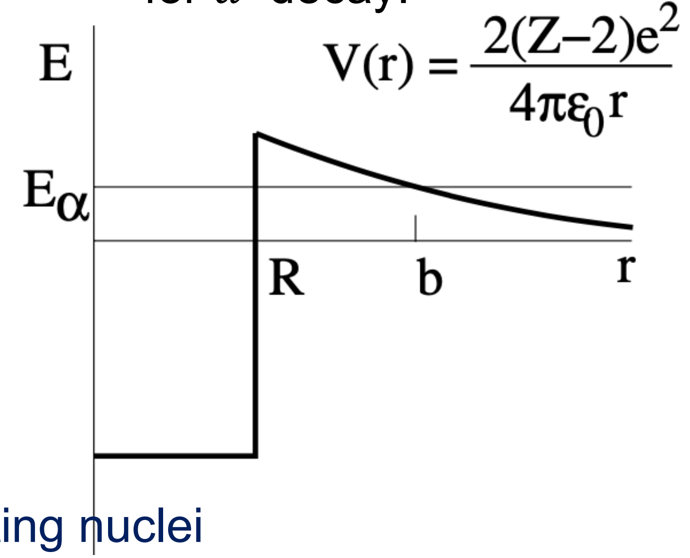
The cross section σ(E)

- The nuclei which interact are charged and, therefore, have a Coulomb repulsion.
- In order for the nuclei to have strong interactions, they must approach one another at a distance of the order of the nuclear forces, i.e. the radius of the nuclei.
- One must cross an electrostatic potential barrier. The barrier has a height given by

$$\frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 a} = \frac{Z_1 Z_2 \alpha\hbar c}{a} = 1.4\,\mathrm{MeV} \times Z_1 Z_2 \frac{1\,\mathrm{fm}}{a}$$

where a is the distance within which the attractive nuclear forces become larger than the Coulomb force.

for α–decay:



- If E is the energy of the particle impinging on this barrier, the probability to cross the barrier by quantum tunneling is proportional to the Gamow factor (as for α–decay):

$$P \sim \exp\left[-2\int_a^b \sqrt{\frac{2m(V(r)-E)}{\hbar^2}}\,\mathrm{d}r\right]$$

where *m* is the reduced mass of the two interacting nuclei and *b* is the classical turning point defined by *V(b)=E*

$$a \ll b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 E} = \frac{Z_1 Z_2 \alpha\hbar c}{E} = 143\,\mathrm{fm} \times Z_1 Z_2 \frac{10\,\mathrm{keV}}{E}$$

and therefore the radius a can be taken equal to zero in good approximation. This leads to the following expression due to Gamow (in 1934) for the tunnelling probability:

$$P \sim exp\left(\frac{-2\pi Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar v}\right) = exp\left(-\sqrt{E_B/E}\right)$$

Other formalism:

$$P(E) = \exp(-2\pi\eta),$$

where v is the relative velocity and $E = \frac{1}{2}mv^2$ is the center-of-mass kinetic energy (m reduced mass). The barrier is characterized by the parameter:

$$E_B = 2\pi^2 Z_1^2\, Z_2^2 \alpha^2 mc^2 = 1052\ keV \times Z_1^2\, Z_2^2 \frac{mc^2}{1\ GeV}$$

The cross section σ(E) can be factorized as:

$$\sigma(E) = \frac{1}{E} S(E) P(E)$$

- **P(E)** is probability of penetration of the Coulomb barrier;
- **S(E)**, named “astrophysical factor”, contains the dynamics of the interaction
- the term **1/E** takes into account the quantum nature of the interaction

$$\frac{1}{E} \propto \pi\lambda^2;$$

Since the astrophysical factor, S(E), is slightly dependent on energy, the cross section can be written as:

$$\sigma(E) = \frac{1}{E} S(E) exp\left(-\sqrt{E_B/E}\right) \approx \frac{1}{E} exp\left(-bE^{-1/2}\right)$$

Where the b parameter is linked to η and is:

$$b = (2\mu)^{1/2} \pi e^2 Z_1 Z_2 / \hbar \,.$$

and $b^2 = E_B$

The interaction rate will be:

$$<\sigma v> = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE$$

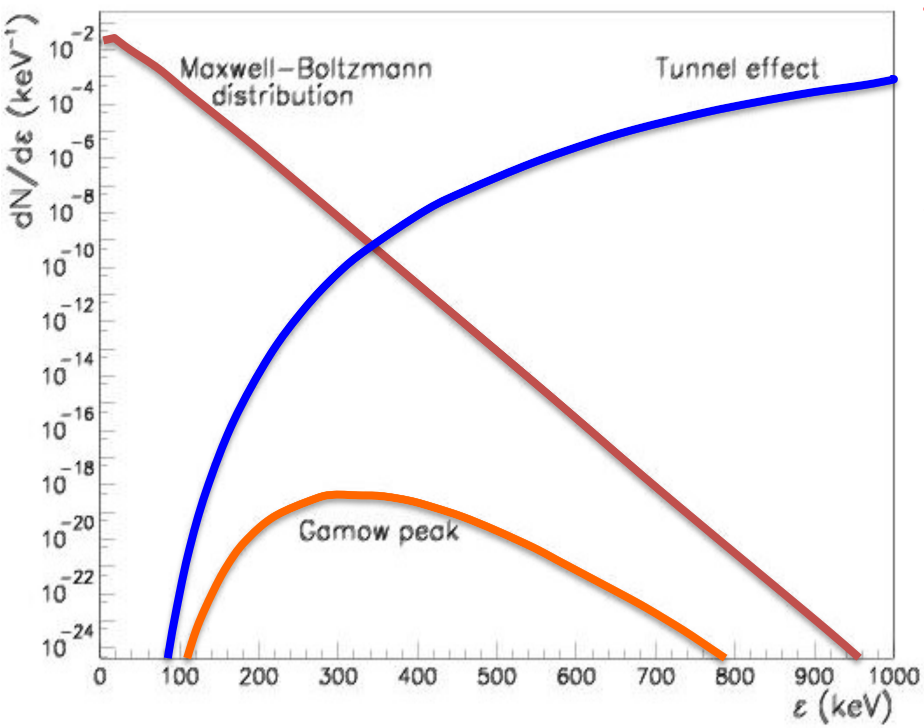
Since the astrophysical factor S(E) – in absence of resonant phenomena – is almost independent on the energy, the behaviour with energy of the reaction rate is given by the product of two functionis:

- exp(- E/kT), which decreases when energy increasing

- $exp(-(E_B/E)^{1/2})$ which instead increases with energy

- **Because of tunnel effect**: only the most energetic particles (on the tail of the Maxwell-Boltzmann distribution) contribute to nuclear reactions
- Nuclear reactions occur only in the neighbourhood of a well-defined energy E_G corresponding to the maximum of the integrand function (obtained by equating the first derivative to 0) given by:

$$E_G = \left(\frac{bkT}{2}\right)^{2/3} = 1.22 \cdot \left(Z_1^2 Z_2^2 \mu T_6^2\right)^{1/3} \mathrm{keV}$$



The Gamow peak is the product of the Maxwell-Boltzmann distribution with the probability of tunnelling of the nuclei through their Coulomb barrier.

- this is the energy region where the reaction is more likely to occur
- at higher energies the number of particles becomes insignificant
- at lower energies tunneling through the Coulomb barrier makes the reaction unlikely.

Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

$$p \propto E^{-1/2} e^{-2\pi\eta}$$

With Sommerfeld parameter

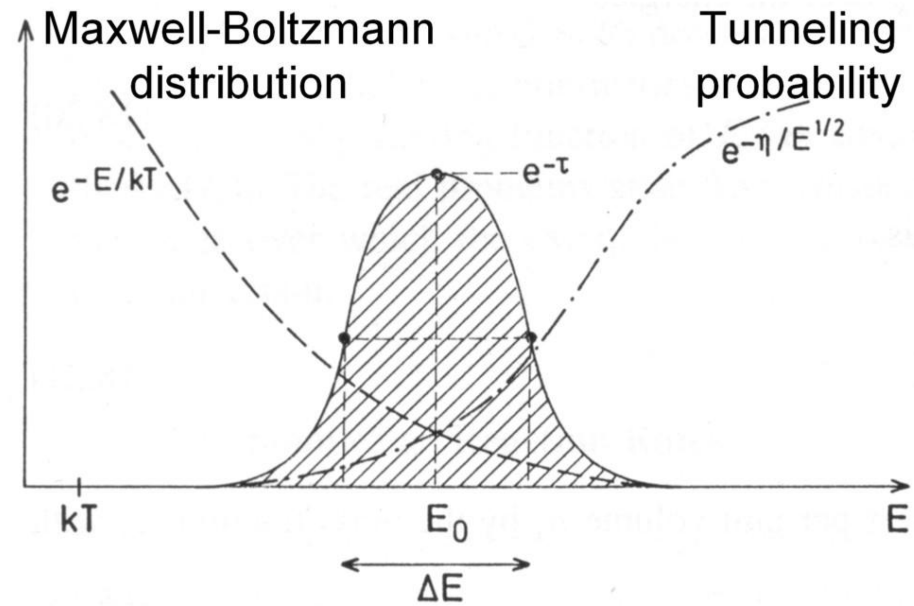
$$\eta = \left(\frac{m}{2E}\right)^{1/2} Z_1 Z_2 e^2$$

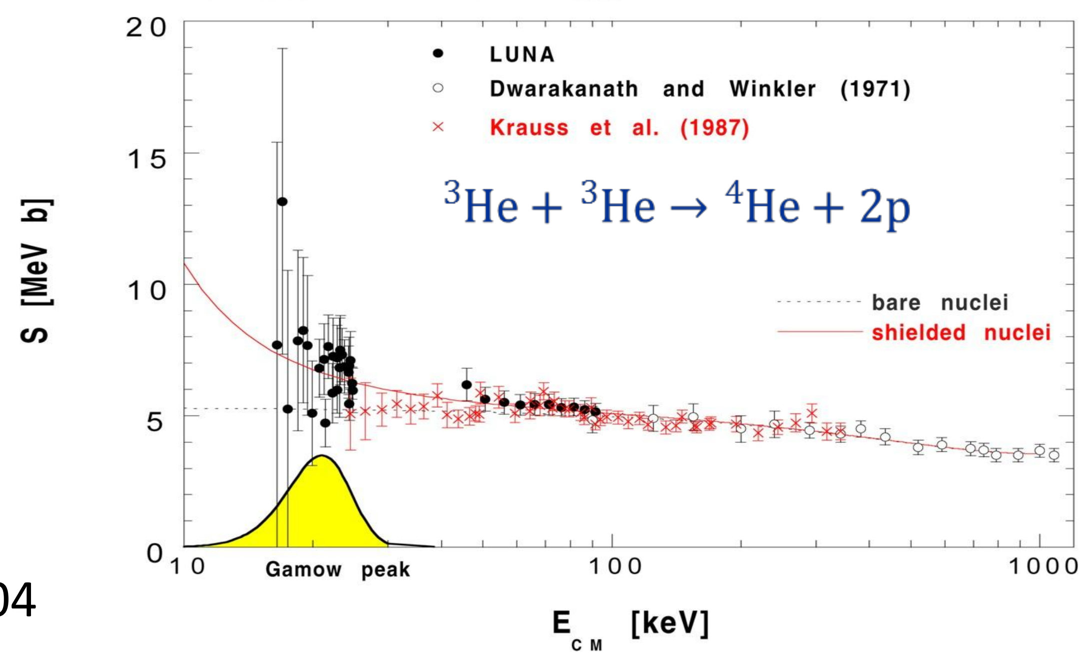
Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\eta(E)}$$

Note that while the cross-section varies by more than 10 orders of magnitude between E = 20keV and 1MeV, the factor S(E) varies only by a factor ~ 2.

LUNA Collaboration, nucl-ex/9902004





In a star like the Sun where $T_6 = 15$, the value of E_G for some reactions is as follows:

p + p	E_G=5.9 keV
p + ^{14}N	E_G=26.5 keV
α + ^{12}C	E_G=56 keV
^{16}O + ^{16}O	E_G=237 keV

Considering that $kT = 1.3$ keV (at $T_6 = 15$), is evident that in the Sun – apart from the relative abundance of nuclei reagents – reactions as α + ^{12}C, or ^{16}O + ^{16}O, or others cannot occur because of the high Coulomb barrier.

- ✓ In fact, these reactions take place in the hottest stars, during their evolutionary phases subsequent to the combustion of H
- ✓ The fusion is the mechanism of formation of the elements, at least until iron
- ✓ The stars represent the cosmic furnaces in which these processes take place: in the later stages of the life of the stars the central T increases and it is possible to form elements to Z gradually increasing.

The interaction rate is proportional to:

$$<\sigma v> = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \frac{b}{E^{1/2}}\right) dE$$

with $b^2 = E_B$. The exponential peaks around $E_G = E_B^{1/3}(kT/2)^{2/3}$

- As long as S(E) has no resonances only the narrow region around E_G contributes significantly to the integral.
- only the value $S(E_G)$ is relevant so it can be taken out of the integral.
- a Taylor expansion of the argument of the exponential in the region E_G:

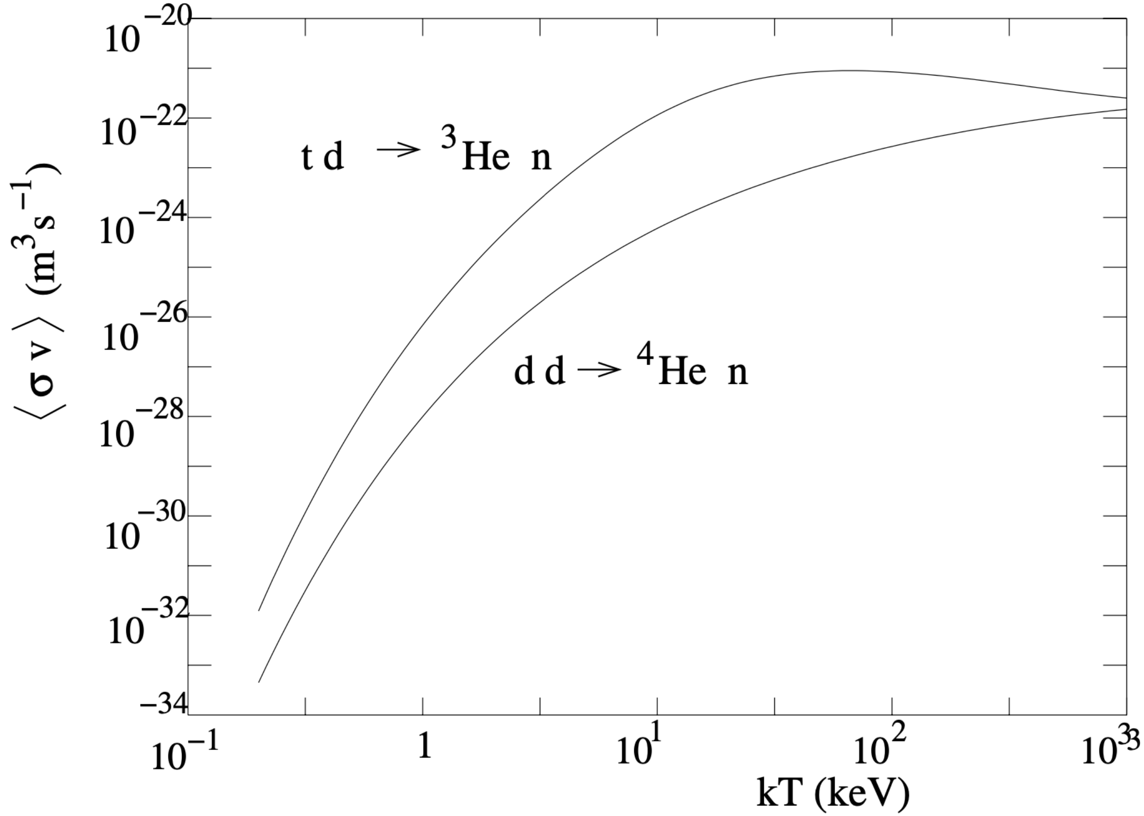
$$\sqrt{E_B/E} + E/kT \sim \frac{3}{2}\left(\frac{E_B}{kT/2}\right)^{1/3} + \frac{1}{2}\frac{(E-E_G)^2}{\Delta_E^2}$$

- where the width of the Gamow peak is $\Delta_E = \frac{2}{\sqrt{3}} E_G \left(\frac{kT}{E_G}\right)^{1/2} = \frac{2}{\sqrt{3}} E_B^{1/6} \left(\frac{kT}{2}\right)^{5/6}$
- note that the Gamow peak is relatively narrow: $\Delta_E/E_G \sim (kT/E_B)^{1/6}$.

$$\langle \sigma v \rangle \;=\; \frac{8\pi}{\sqrt{\mu}} (kT)^{-3/2} S(E_G) \exp\left[-(3/2)\left(\frac{E_B}{kT/2}\right)^{1/3}\right] \times \int \exp\left(\frac{(E-E_G)^2}{2\Delta_E^2}\right) \mathrm{d}E$$

solving the integral:

$$\langle \sigma v \rangle \;=\; \frac{8\pi}{\sqrt{\mu}} (kT)^{-2/3} E_B^{1/6} S(E_G) \exp\left[-(3/2)\left(\frac{E_B}{kT/2}\right)^{1/3}\right]$$



- ✓ The fusion reactions in stars
- ✓ Description, rates, cross sections
- ✓ The origin of the elements
- ❑ The life of a star
- ❑ The last stage of a star
- ❑ The formation of the heavier elements

A star is born

Formation of a star:

it begins when a portion of interstellar gas cools down to the point that it becomes unstable and begins to collapse due to the prevalence of gravitational attraction (E_G) on the motion of thermal motion of the atoms of the gas (E_T), i.e. when:

$$E_G = \int_0^R -G\frac{M(r)}{r}\rho(r)\cdot 4\pi r^2 dr \approx -\overline{\left(G\frac{M(r)}{r}\right)}\cdot M$$

$$E_T = \int_0^R \frac{3}{2}\frac{kT}{m}\rho(r)\cdot 4\pi r^2 dr \approx \overline{\left(\frac{3}{2}\frac{kT}{m}\right)}\cdot M$$

with: M,R,T=mass, radius, T of the portion of gas
m=mean molecular weight of the atoms that constitute it

$$E_G + E_T \leq 0$$

At first, the heat resulting from the contraction is rapidly radiated outside (high transparency of gas), gradually the gas becomes denser, its opacity increases, and part of the energy may be withheld.

The gaseous cloud stabilizes and begins to form a star when the fraction of the absorbed thermal energy (E_T) is 1/2 that gravitational (E_G)

The evolutionary path from the initial collapse to the equilibrium condition takes place in a range of 30 ÷ 100 yr: at this point the star becomes visible.

The star radiates from its surface; it must contract simultaneously enough to account for the energy emitted and that turned into internal thermal agitation:

➡ this stage of stellar evolution is named Kelvin-Helmholtz

In a contracting star the conversion of gravitational energy into internal energy causes a progressive increasing of the temperature

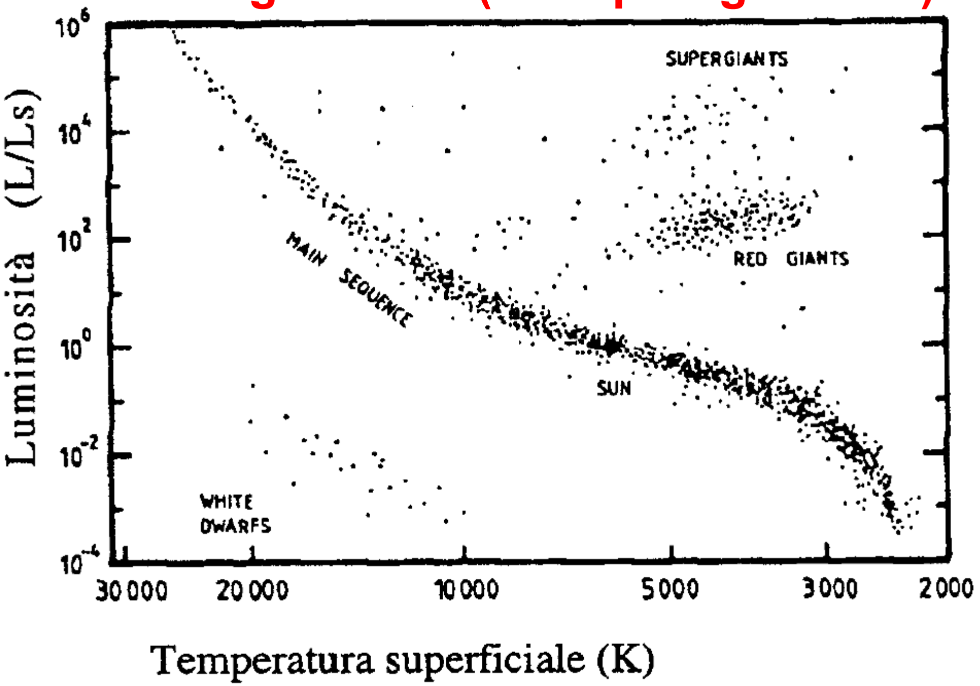
- at the end at the centre of the star $T \sim 10^7$ K
- starting the nuclear reactions between p to form Helium

Amount of energy released in these processes >> of that released from the gravitational contraction:

the star will reach a new long-term stage, in which the gravitational contraction will be blocked, nuclear energy produced in the core of the star will flow outside of the surface and for a long period, there will be no significant variations in size, shape and temperature for the star

Diagram H-R (Hertzprung-Russel)

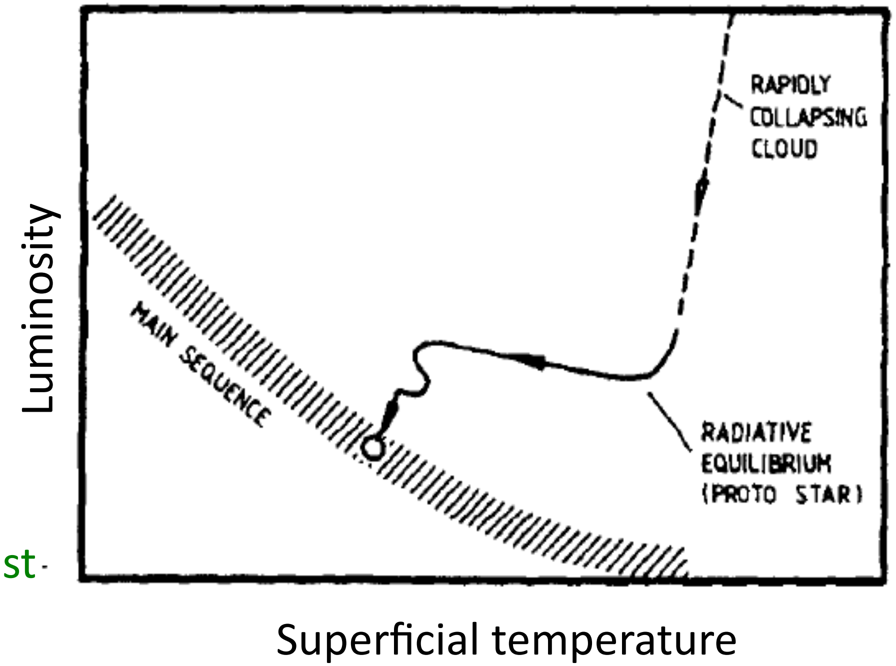
The set of stars that are at this stage constitute the so-called main sequence of the Hertzprung-Russell diagram (brightness of stars vs. surface temperature)



Evolution of the star

- History of a star in an HR diagram, we see that: from the stage of Kelvin-Helmholtz, increasing T the star moves with small changes in brightness, $\mathcal{L}$, horizontally up to reach its position in the main sequence
- $\mathcal{L}$ of a star closely related to its mass, M
- A star with $M > 10 \times M_{Sun}$ typically has $\mathcal{L}$ 10^4 largest

→ to offset this enormous outflow of energy it must convert H into He quickly as the Sun.



✓ In the Sun the H "fuel" will last at least for ~10 billion years

✓ In massive stars it will be exhausted in a few 100's of thousands of years – time well shorter than the estimated age of the Galaxy (14 billion yr)

✓ the history of the Universe has already met several generations of hot and massive stars, and these stars have given rise to the production of the heavier elements and to their relative abundance

✓ The star remains in its original position on the main sequence until nuclear energy generated inside is sufficient to support its $\mathcal{L}$

→ the gravitational contraction resumes until it triggers the combustion of H in an outer shell.

$E_{produced} > E_{emitted\ by\ the\ surface}$

→ T increases

→ pressure increases and restores balance through a hydrostatic expansion, accompanied by a cooling of the outer layers.

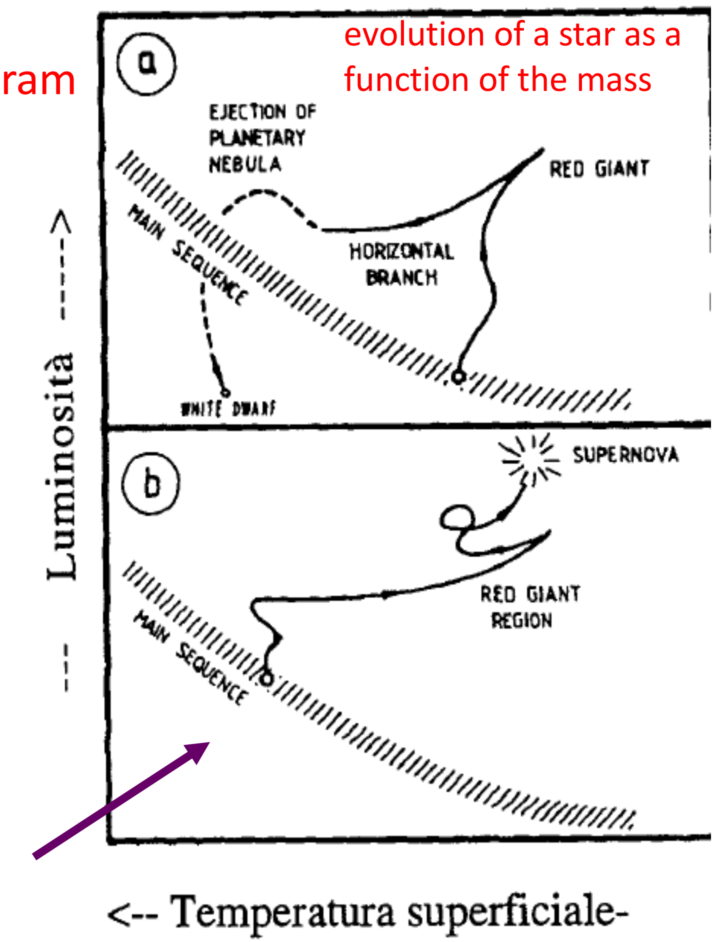
The star, irradiating a greater amount of E, becomes brighter, and more red because of the cooling of the surface

→ It moves toward the top region to the right of H-R diagram becoming a red giant

Following evolutionary path **depends on the mass of star**

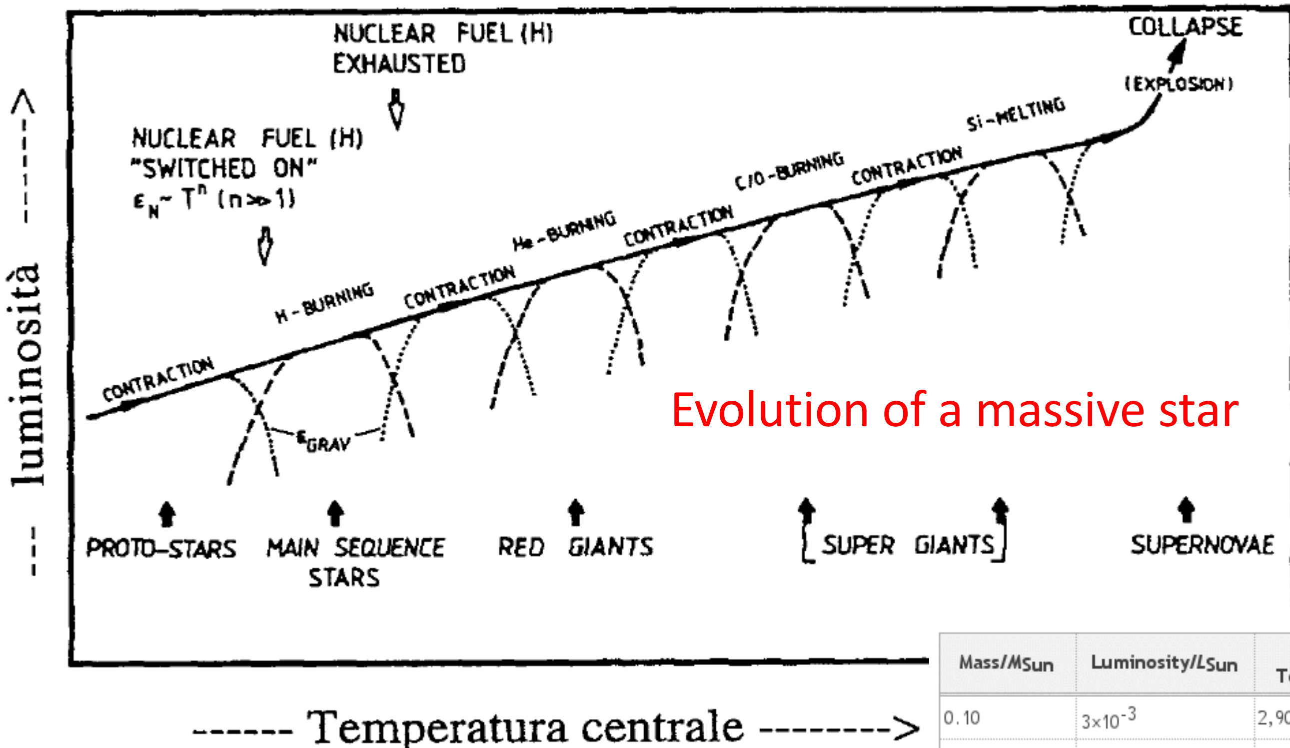
For $M_{star} \approx M_{Sun}$

- ✓ the contraction occurring after the exhaustion of the He is not sufficient to cause an increase of T suitable to trigger the reaction of C and O
- ✓ the star, through a phase of great instability with fluctuations and radiated, moves toward the bottom region in left of the diagram H-R
- ✓ Reached the stage of white dwarf, the star - now of planetary dimensions - ceases to contract due to the strong pressure exerted by the degenerate electron gas
- ✓ progressive cooling will bring it to become, after hundreds of millions of years, a black dwarf that is a dead star



M >> M_{sun} involve an evolution much more rapid and spectacular:

Gravitational contractions and reactions between heavier and heavier elements determine central T increasingly higher, up to billions of degrees



This process cannot continue indefinitely:

✓ every nuclear combustion produces less E than the previous one

✓ when the core of the star becomes composed of materials of the Fe group (for which the binding energy is maximum) thermonuclear reactions stop.

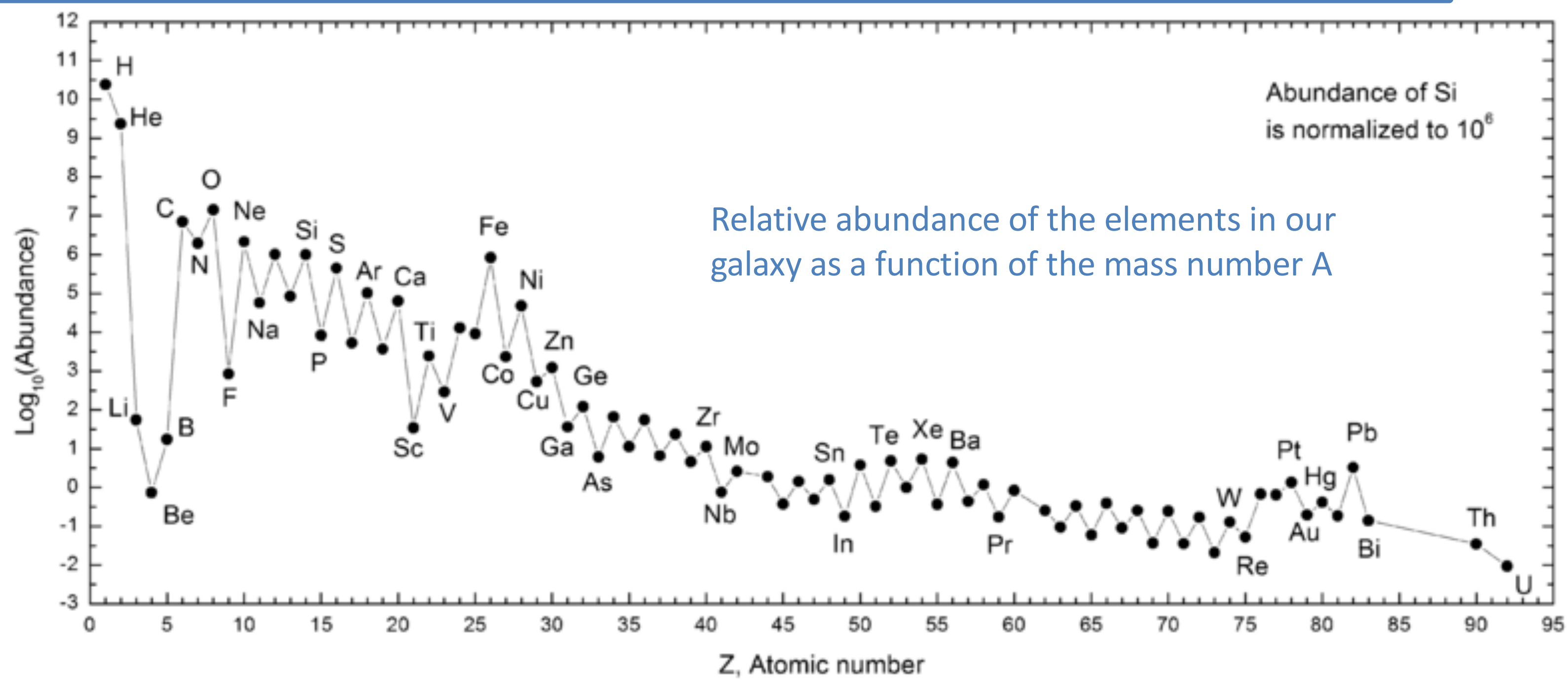
Mass/M_{Sun}	Luminosity/L_{Sun}	Effective Temperature (K)	Radius/R_{Sun}	Main sequence lifespan (yrs)
0.10	3×10^{-3}	2,900	0.16	2×10^{12}
0.50	0.03	3,800	0.6	2×10^{11}
0.75	0.3	5,000	0.8	3×10^{10}
1.0	1	6,000	1.0	1×10^{10}
1.5	5	7,000	1.4	2×10^{9}
3	60	11,000	2.5	2×10^{8}
5	600	17,000	3.8	7×10^{7}
10	10,000	22,000	5.6	2×10^{7}
15	17,000	28,000	6.8	1×10^{7}
25	80,000	35,000	8.7	7×10^{6}
60	790,000	44,500	15	3.4×10^{6}

The gravitational contraction makes the star unstable, and it collapses:

photodisintegrations, electron captures and ν coming out the star cause losses of E that can be compensated only through an extremely quick gravitational collapse of the Fe nucleus that - at the end - gives rise to an enormous explosion.

This is the phenomenon of supernovae, like the one that exploded in the Taurus Constellation in 1054 remaining visible by day and night for 23 days and during the night for a total of 21 months (→ CRAB Nebula)

The elements formed during the previous stages are well dispersed in interplanetary space and constitute the current Universe.



The presence of heavy materials (note 12 orders of magnitude of the scale of ordinates) is a sign that the Universe has known several generations of stars.

But how did heavier elements form?

When all the H is burned and turned into He, the star collapses on itself and the gravitational energy is converted into heat energy.

The T of the central area increases and starts the combustion cycle of He.

The key reaction is the formation of carbon, through the reaction:

$$\alpha + \alpha + \alpha \rightarrow {}^{12}C$$

(triple-α process) which occurs in two steps. Firstly,

$$\alpha + \alpha \leftrightarrow {}^{8}Be$$

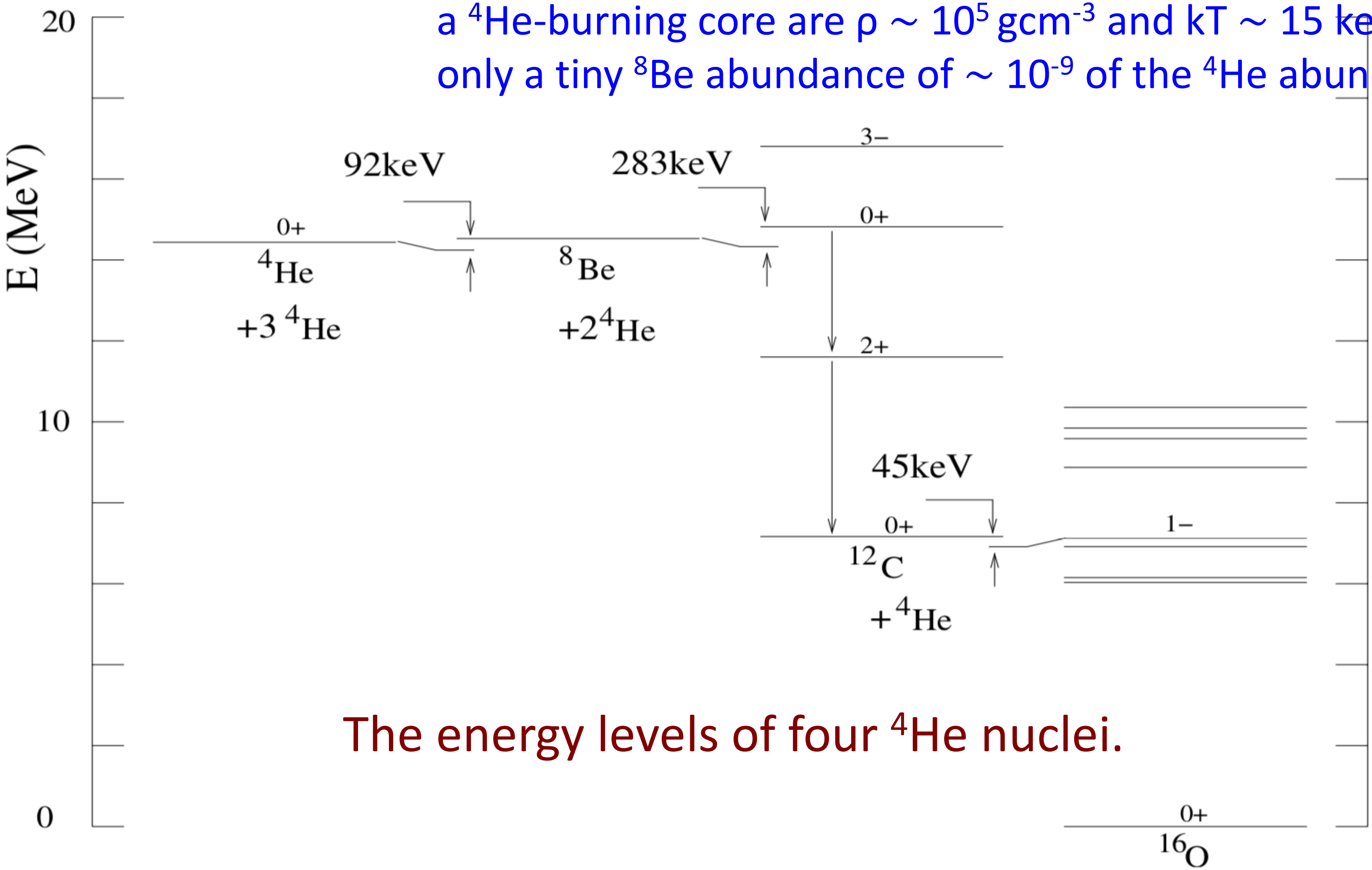
2 α particles join to give an ^{8}Be nucleus in the fundamental state which dissociates again in two α, with half-life equal to $\tau = 2 \times 10^{-16}$ s (Q=92 keV), **long enough** to allow a subsequent fusion reaction with a third α before dissociation, through the reaction:

$$^{8}Be + \alpha \rightarrow {}^{12}C + \gamma \qquad (Q = 7.366 MeV)$$

In a He burning star, a thermal equilibrium abundance of ^{8}Be is built up given by:

$$\frac{n_{8\mathrm{Be}}}{n_{4\mathrm{He}}} = \frac{n_{4\mathrm{He}}}{(mkT)^{3/2}/(4\pi^2\hbar^3)} e^{-92\,\mathrm{keV}/kT} \approx 10^{-9}$$

where m is the ^{4}He-^{4}He reduced mass. The typical density in a ^{4}He-burning core are $\rho \sim 10^5$ gcm^{-3} and kT ~ 15 keV, so only a tiny ^{8}Be abundance of ~ 10^{-9} of the ^{4}He abundance.



The energy levels of four ^{4}He nuclei.

$$ {}^4\mathrm{He}\ {}^8\mathrm{Be} \rightarrow {}^{12}\mathrm{C}\ \gamma \qquad Q = 7.366\,\mathrm{MeV} $$

- ✓ Because of the very small quantity of ^{8}Be, this would normally lead to a very **small production rate of ^{12}C**.
- ✓ However, the rate can be greatly increased if ^{12}C has an **excited state** near the Gamow energy for the reaction, $E_G \sim 200$ keV for $kT \sim 15$ keV.
- ✓ This leads Hoyle to predict the **existence** of such a state and subsequent measurements lead to its discovery.
- ✓ This **0^+ excited state** of ^{12}C is 7654 keV above the ^{12}C ground state and 283 keV above ^{4}He-^{8}Be.
- ✓ It decays mostly via α decay, returning the original ^{8}Be, but also has a $\sim 10^{-3}$ branching ratio to the ground state of ^{12}C:

$$ \begin{aligned} {}^4\mathrm{He}\ {}^8\mathrm{Be} &\rightarrow {}^{12}\mathrm{C}^* & Q &= -283\,\mathrm{keV} \\ {}^{12}\mathrm{C}^* &\rightarrow {}^8\mathrm{Be}\ {}^4\mathrm{He} & \Gamma &= 8.3\,\mathrm{eV} \\ {}^{12}\mathrm{C}^* &\rightarrow {}^{12}\mathrm{C}\ \gamma\gamma & \Gamma_\gamma &= 3 \times 10^{-3}\,\mathrm{eV} \end{aligned} $$

triple-α process

- ✓ The energy liberated by the triple-α process can generate the star's luminosity when the central temperature reaches kT ~ 10 keV, i.e. T ~ 10^8K.
- ✓ As the ^{4}He in the core is depleted, ^{12}C burning is initiated via the non-resonant reaction

$$^4\mathrm{He}\ ^{12}\mathrm{C} \rightarrow\ ^{16}\mathrm{O}\ \gamma \qquad Q = 7.162\,\mathrm{MeV}$$

- ✓ This reaction competes favourably with the triple-α process once the ^{4}He is depleted because its rate is linear in the concentration of ^{4}He while the rate of the triple-α process is proportional to the third power of the ^{4}He concentration.
- ✓ The helium-burning stage thus generates a mixture of ^{12}C and ^{16}O.

A peculiar characteristic of the triple-α process is that its result depends critically on the details of the remarkable energy alignments of the 0^+ states of ^{8}Be, ^{12}C and the 1^- state of ^{16}O:

$$\begin{aligned}
^4\mathrm{He}\ ^4\mathrm{He} &\rightarrow\ ^8\mathrm{Be} & Q_4 &= -0.092\,\mathrm{MeV}\,, \\
^4\mathrm{He}\ ^8\mathrm{Be} &\rightarrow\ ^{12}\mathrm{C}^* & Q_8 &= -0.283\,\mathrm{MeV}\,, \\
^4\mathrm{He}\ ^{12}\mathrm{C} &\rightarrow\ ^{16}\mathrm{O}^*\ \gamma & Q_{12} &= +0.045\,\mathrm{MeV}\,.
\end{aligned}$$

- ✓ The formed nucleus, ^{12}C, is stable.
- ✓ New reactions can occur:

$$
\begin{aligned}
{}^{12}\mathrm{C}\ {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg}\ \gamma & Q &= 13.93\,\mathrm{MeV} \\
&\rightarrow {}^{23}\mathrm{Na}\ \mathrm{p} & Q &= 2.24\,\mathrm{MeV} \\
&\rightarrow {}^{20}\mathrm{Ne}\ {}^{4}\mathrm{He} & Q &= 4.62\,\mathrm{MeV}
\end{aligned}
$$

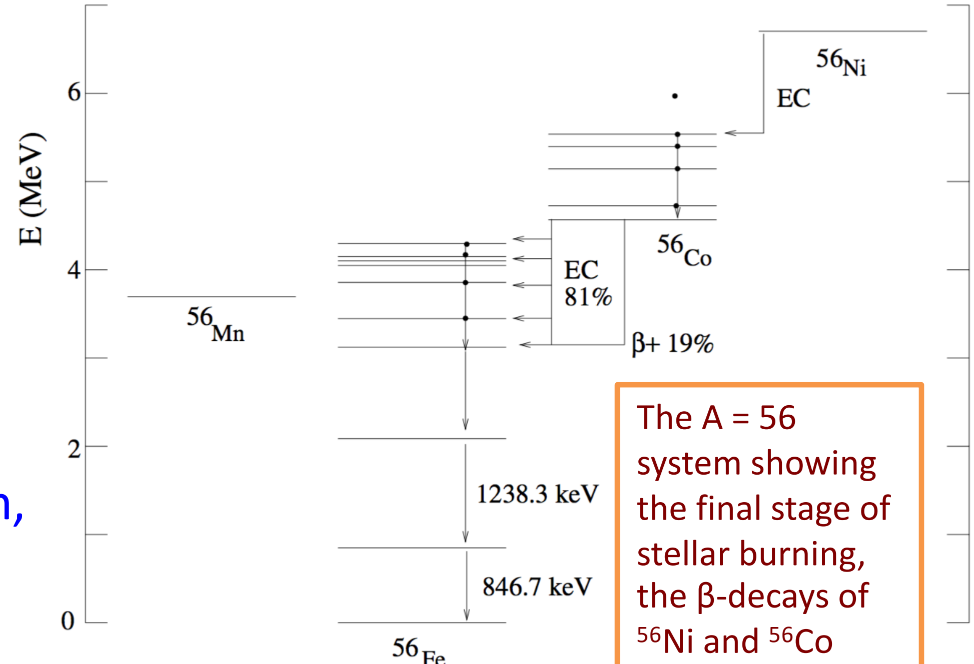
These three reactions can be considered to be a single reaction consisting of the formation of a "compound nucleus," i.e. an excited state of ^{24}Mg which then decays by photon, proton, or α emission

$$^{12}C + \alpha \rightarrow {}^{16}O + \gamma$$

$$^{16}O + \alpha \rightarrow {}^{20}Ne + \gamma$$

$$^{20}Ne + \alpha \rightarrow {}^{24}Mg + \gamma$$

- ✓ Because of the increasing Coulomb barrier, these reactions need an increasing of T to occur.
- ✓ Elements heavier than ^{56}Fe do not form, because for A=60 the B/A curve has reached its maximum.



The A = 56 system showing the final stage of stellar burning, the β-decays of ^{56}Ni and ^{56}Co

- ✓ For A>60 the Coulomb barrier becomes so high that it is no longer energetically favourable for a nucleus to capture another nucleus, but rather to capture a n gaining precisely the binding energy of the n.
- ✓ **→ The s-, r- and p-processes**

The s-, r- and p-processes

- ✓ The modern theory of the nucleosynthesis of **elements beyond the iron peak** was spelled out in a seminal paper by Burbidge, Burbidge, Fowler and Hoyle 1957
- ✓ They realized the importance of **neutron captures** in the production of heavy elements.
- ✓ The problem with neutron captures is that **very few neutrons** are present in stars.
- ✓ Neutron capture has the **advantage** not having the Coulomb barrier associated with proton captures.
- ✓ Its importance is immediately suggested by the fact that for isobars with fixed A > 56 with more than one β-stable, **the most neutron-rich isobar is the most abundant**.
- ✓ Important **neutron-producing** exothermic reactions are (α,n) reactions on the relatively rare nuclei ^{13}C and ^{22}Ne

$$^{4}\mathrm{He}\ ^{13}\mathrm{C} \rightarrow \mathrm{n}\ ^{16}\mathrm{O} \qquad Q = 3.00\,\mathrm{MeV}$$

$$^{4}\mathrm{He}\ ^{22}\mathrm{Ne} \rightarrow \mathrm{n}\ ^{25}\mathrm{Mg} \qquad Q = 0.30\,\mathrm{MeV}$$

- ✓ The slow build-up of heavy nuclei by capture of such neutrons is called the **"s-process"** for "slow" neutron capture.
- ✓ Neutrons would also be expected to be present in large numbers in explosive events like supernovae or neutron-star collisions.
- ✓ This type of nucleosynthesis is called the **"r-process"** for "rapid" neutron capture.
- ✓ The existence of both the s- and r- process is **necessary** to explain the observed abundances.

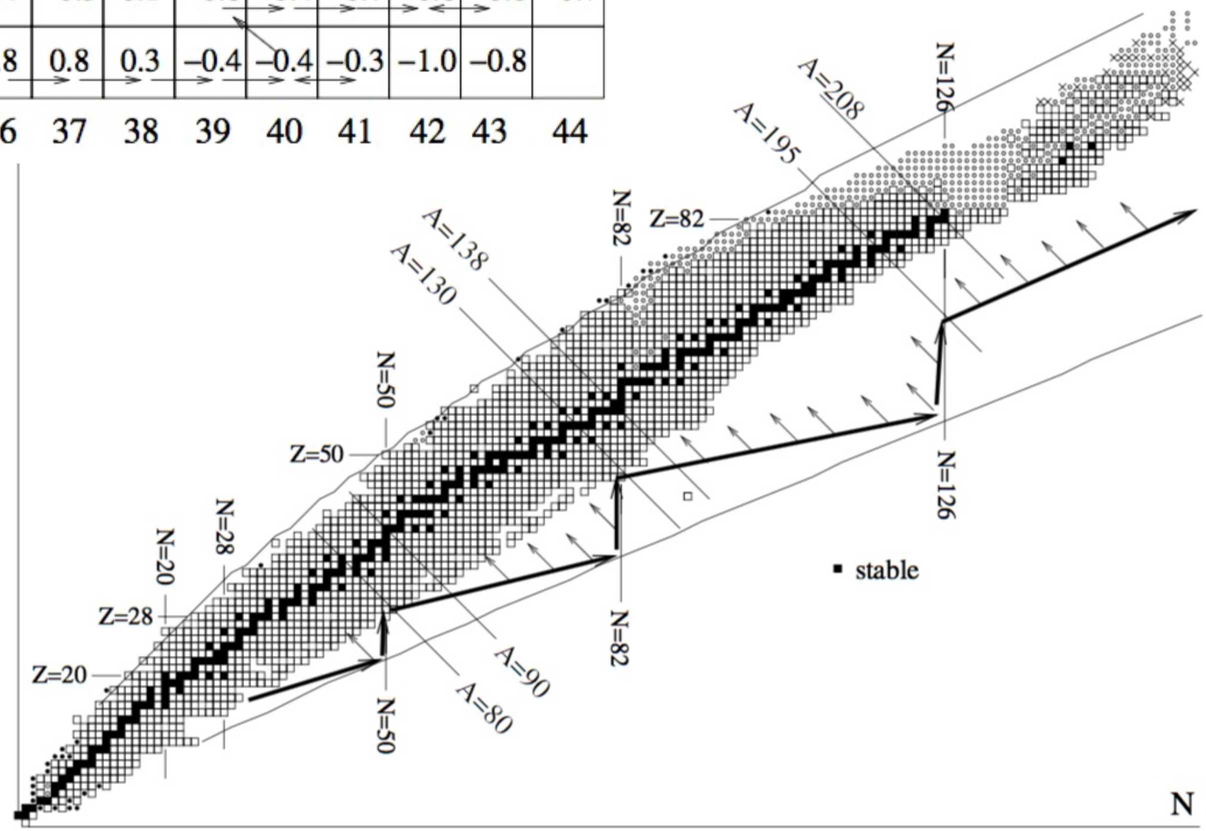
The r- and s-processes

	N=30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
Ge_{32}		−1.0	1.8	1.5	3.9	3.1	7.4	5.1		6.0				3.7	
Ga_{31}	−0.8	−0.9	1.5	2.2	3.0	4.5	5.5	3.6		3.1		4.7	4.2	2.7	2.1
Zn_{30}	2.2	1.9	4.5	3.4		7.3				3.5		2.2	5.2	1.4	2.0
Cu_{29}	1.9	3.2	4.1	2.8		4.7		2.5	5.3	1.5	2.2	0.7	1.3	0.8	0.6
Ni_{28}		12.4				9.5		4.0	5.3	1.3	1.3	1.1		0.3	0.3
Co_{27}	7.4	6.8		8.2	3.8	2.0	1.4	−0.5	0.1	−0.6	−0.4	−0.7	−0.6	−0.8	−0.7
Fe_{26}				6.6	13.7	2.6	1.8	0.8	0.3	−0.4	−0.4	−0.3	−1.0	−0.8	

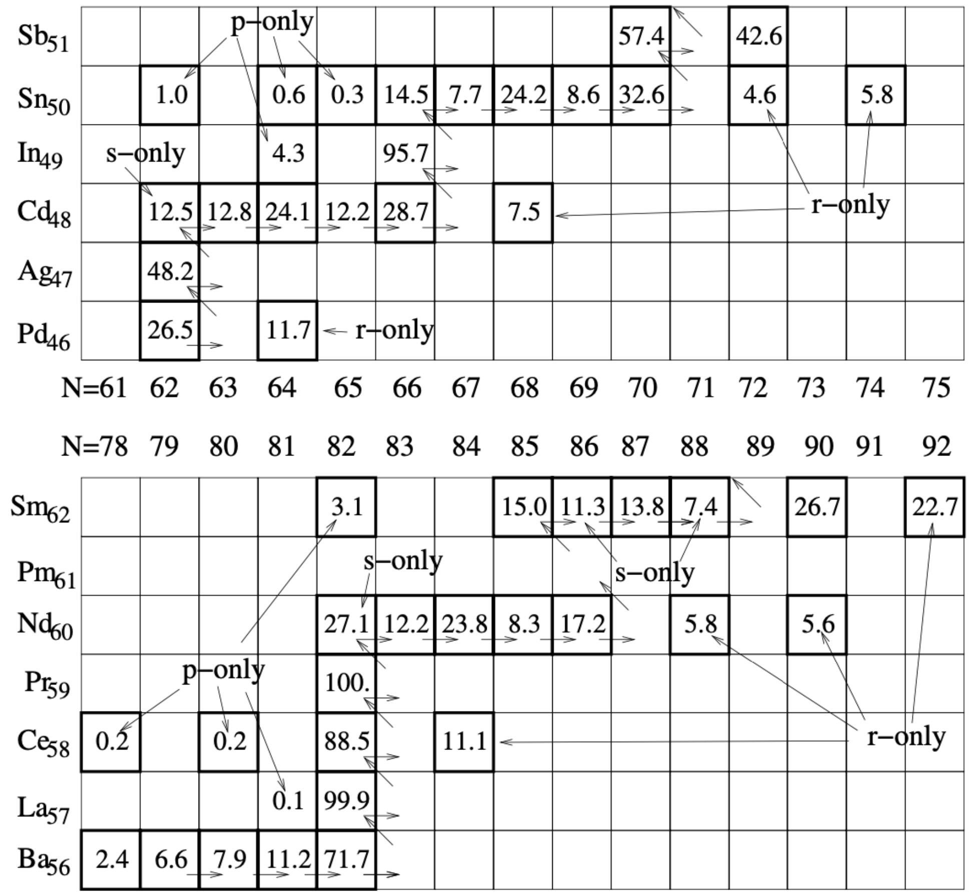
Nucleosynthesis by neutron capture starting at ^{56}Fe.

- The decimal logarithm of the half-life in seconds is shown for β-unstable nuclei.
- The black-contour boxes are stable nuclei.

→ neutron capture
↖ β-decay



- Other examples of r- s-processes.
- Nuclei below the s-process path are labelled "r-only" and can only be produced by rapid neutron capture in, for instance, supernovae.
- Nuclei that are shielded from the r-process by a r-only nucleus are labelled "s-only."
- Nuclei above the s-process path have low isotopic abundances. They can only be produced by the "p-process," due, for instance, to (p, γ) or (γ, n) reactions.



- Warning: differently from previous slide, the numbers are the solar-system isotopic abundances of the stable nuclei
- The empty boxes are unstable nuclei

- ✓ Following the capture of a neutron, to adjust the Z/A ratio typically a β^- decay follows and the final atomic number increases by one
- ✓ The **s-process** is clearly incapable of producing elements heavier than ^{209}Bi
- ✓ ⇨ This brings us to the ^{209}Bi nucleus
- ✓ Heavier elements do not easily form through neutron capture because ^{210}Bi is unstable for α decay:

$$^{209}_{83}Bi + n \rightarrow {}^{210}_{83}Bi \xrightarrow[\alpha]{} {}^{206}_{81}Tl$$

- ✓ The β-stable elements for A = 210, 211 are the short-lived α-emitters ^{210}Po ($T_{1/2}$ = 138.376 day) and ^{211}Bi ($T_{1/2}$ = 2.14m). The existence of natural Uranium and Thorium therefore necessitates the existence of the **r-process**.

- ✓ For the formation of heavier elements is essential to be in the presence of high flows of neutrons, so as to compensate for the decay of ^{210}Bi with its very high speed of creation. **"r-process"**
- ✓ These flows of neutrons are present during the explosions of stars: the explosions occur when the stars, exhausted nuclear fuel (i.e. reached the value A = 60), collapse on themselves.
- ✓ There are also a number of **proton-rich nuclei** that can be produced neither by the s- nor the r-process.
- ✓ By definition, these nuclei are created by the **"p-process"**
- ✓ All these nuclei have very low solar-system abundances indicating that the p-process **is less important** than the s- and r-processes.
- ✓ Originally, it was thought that these nuclei would be created by proton capture, but more recent work has indicated that **(γ, n) reactions (photo-ejection of a neutron)** in explosive environments may be the dominant process.

The stellar collapse

What happens when the fusion process ends?

- White Dwarf
- Black hole
- Neutron Star
- Whole disassembling

Life of a star

- Light stars ($M_{star} \lesssim 4M_{\odot}$)
  - the star consume M up to become a white dwarf

    A white dwarf can also lead to an explosive phenomena if it slowly gain mass gravitationally capturing it from a companion or from some near mass

    - when a critical M is reached, the thermonuclear reactions start again; they trigger a violent explosion, as in the H bomb, and the star is torn apart
    - a supernova of Type 1a was born

    Because these explosions usually occur in specific mass and density, they are very similar

- Massive stars ($M_{star} \gtrsim 4M_{\odot}$)
  - it can become a supernova which can give rise to:

    a black hole

    a neutron star

    be fully disassembled

The Death of Stars

$M_{star} < 1\ M_{sun}$	Slow gravitational contraction **Brown Dwarfs**
$1\ M_{sun}$ to $\sim 5\ M_{sun}$	Mild core collapse $\rho \sim 10^7$ g/cm^3,R~7000km **White Dwarfs**
$\sim 5\ M_{sun}$ to $15\ M_{sun}$	Fast core collapse $\rho \sim 3 \times 10^{14}$ g/cm^3, R ~ 20 km **Neutron Stars**
$M_{star} > 15\ M_{sun}$	Very fast core collapse $\rho > 10^{16}$ g/cm^3, R~4km **Black Holes**

Importance of Mass

- The fate of a star is linked to its mass when it nears the end of its life.
- This depends upon
  - Its initial mass
  - How much mass it loses along the way.

→ Chandrasekhar mass limit

White Dwarfs

- For M_{core} < 1.4 M_{sun}, the core is stable.
- A white dwarf forms.
  – Size of the earth but mass of the sun!
- As the star cools we might expect it to get smaller and smaller. It doesn't!
- What stops core collapse? The Pauli Exclusion Principle:
  – No two electrons can be at the same place at the same time with the same energy.
- The strong repulsion caused by the Exclusion Principle is called **Electron Degeneracy Pressure**

Degenerate stars

- A collection of particles must radiate photons and contract, with the contraction pausing whenever nuclear reactions are ignited to provide the photon luminosity
- This process must stop when the collection reaches its quantum-mechanical "ground state."
- We want to estimate the ground-state energy of a collection of electrons and nucleons bound by gravitation.
- The maximum phase-space density allowed for fermions is two particles per $(\Delta p \Delta x)^3 = (2\pi\hbar)^3$. All the fermions are in the Fermi sphere:

$$\frac{N}{V \times (4\pi/3)p_{\mathrm{F}}^3} = \frac{2}{(2\pi\hbar)^3} \longrightarrow \frac{4\pi p_{\mathrm{F}}^3}{3} = N\frac{(2\pi\hbar)^3}{2}\frac{1}{4\pi R^3/3}$$

- The mean momentum squared is connected to the Fermi energy and is independent on the particle mass. The mean kinetic energy, $p^2/2m$, will be dominated by the lighter particles (electrons):

$$\langle p^2 \rangle = \frac{\int_0^{p_{\mathrm{F}}} p^2 p^2 \mathrm{d}p}{\int_0^{p_{\mathrm{F}}} p^2 \mathrm{d}p} = (3/5)p_{\mathrm{F}}^2$$

- The energy of the star is then

$$E \sim N_{\mathrm{e}}\sqrt{\langle p^2 \rangle c^2 + m_{\mathrm{e}}^2 c^4} - (3/5)\frac{GM^2}{R}$$

Using $M = N_b m_p$, we get:

$$E \sim N_{\rm e}\sqrt{(3/5)p_{\rm F}^2 c^2 + m_{\rm e}^2 c^4} - \frac{3Gm_{\rm p}^2 N_{\rm b}^2}{5N_{\rm e}^{1/3}}\frac{p_{\rm F}}{2\pi\hbar}\left(\frac{32\pi^2}{9}\right)^{1/3}$$

- N_e N_b are the numbers of electrons and baryons
- Taking the derivative of E with respect to the momentum we find that the minimum energy occurs for $\frac{p_F}{\sqrt{p_F^2+m_e^2c^2}} = \left(\frac{N_b}{N_c}\right)^{2/3}$ a part from some factors
- where the "critical" number of baryons is: $N_{\rm c} = \left(\frac{1}{\alpha_G}\right)^{3/2}\left(\frac{N_{\rm e}}{N_{\rm b}}\right)^2 = 1.82 \times 10^{57}\left(\frac{N_{\rm e}}{N_{\rm b}}\right)^2$

where $\alpha_G = Gm_{\rm p}^2/\hbar c$. This corresponds to a total mass

$$M_{\rm c} = m_{\rm p}N_{\rm c} = 3.04 \times 10^{30}\ {\rm kg}\left(\frac{N_{\rm e}}{N_{\rm b}}\right)^2 = 1.52 M_\odot\left(\frac{N_{\rm e}}{N_{\rm b}}\right)^2$$

The fate of a star depends on whether its mass is greater than or less than the Chandrasekhar mass. The correct value is:

$$M_{\rm Ch} \sim 5M_\odot\left(\frac{N_{\rm e}}{N_{\rm b}}\right)^2 \sim 1.25 M_\odot \quad \text{for } N_{\rm e}/N_{\rm b} = 1/2\ .$$

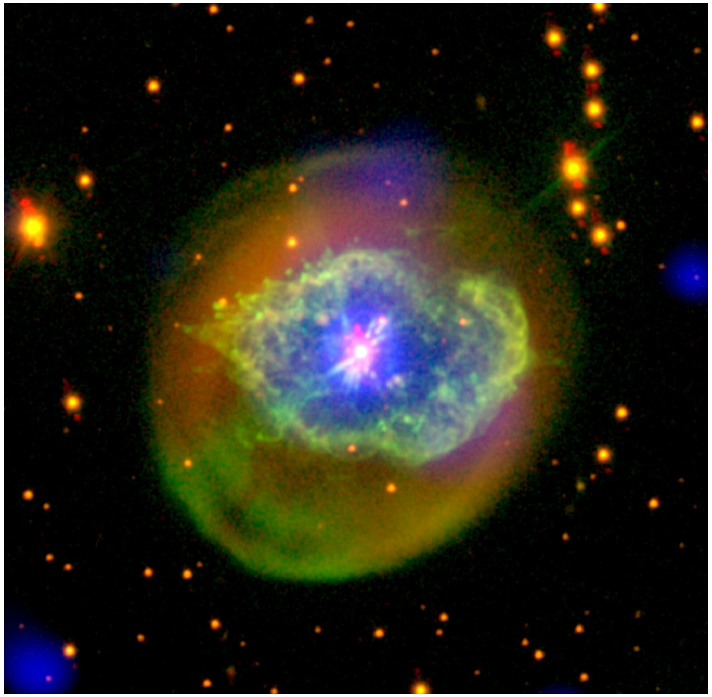
Stars with a number of baryons less than N_c have a well-defined ground state.
The radius of the star in the ground state can be calculated by the previous formulae

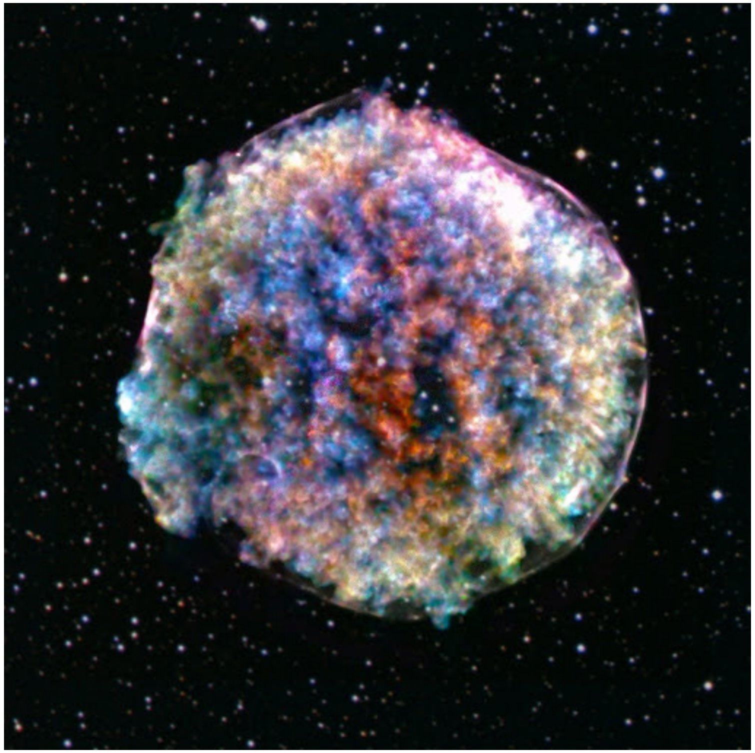
$$R_{\rm gs} \sim \frac{\hbar}{m_{\rm e}c} N_{\rm e}^{1/3} \left(\frac{N_{\rm c}}{N_{\rm b}}\right)^{2/3} \left(1 - \frac{N_{\rm b}}{N_{\rm c}}\right)^{1/2}$$

- For the Sun, $N_b = 1.2 \times 10^{57}$ and $N_e/N_b = 0.7$ which gives $R_{gs} \sim 10^4$ km, implying that the Sun is far from its ground state.
- 10^4 km is a typical radius for **white dwarfs**
- For a very light stars, $N_b \rightarrow 0$, $R_{gs} \rightarrow \infty$ such a star will contract to its ground state before becoming hot enough to ignite nuclear reactions. Detailed models suggest that this happens for stars with $M < 0.07 M_\odot$. **Planets** are examples of such objects.

Stars explosion

- Mild Explosion → Planetary Nebula
  – Ejection of the outer layers of the red giant.
- Strong Explosion → Nova
  – Eruptions in a binary star system
- Catastrophic Explosion → Supernova
  – Blasting away of the outer parts of a star





Remnant of the Star

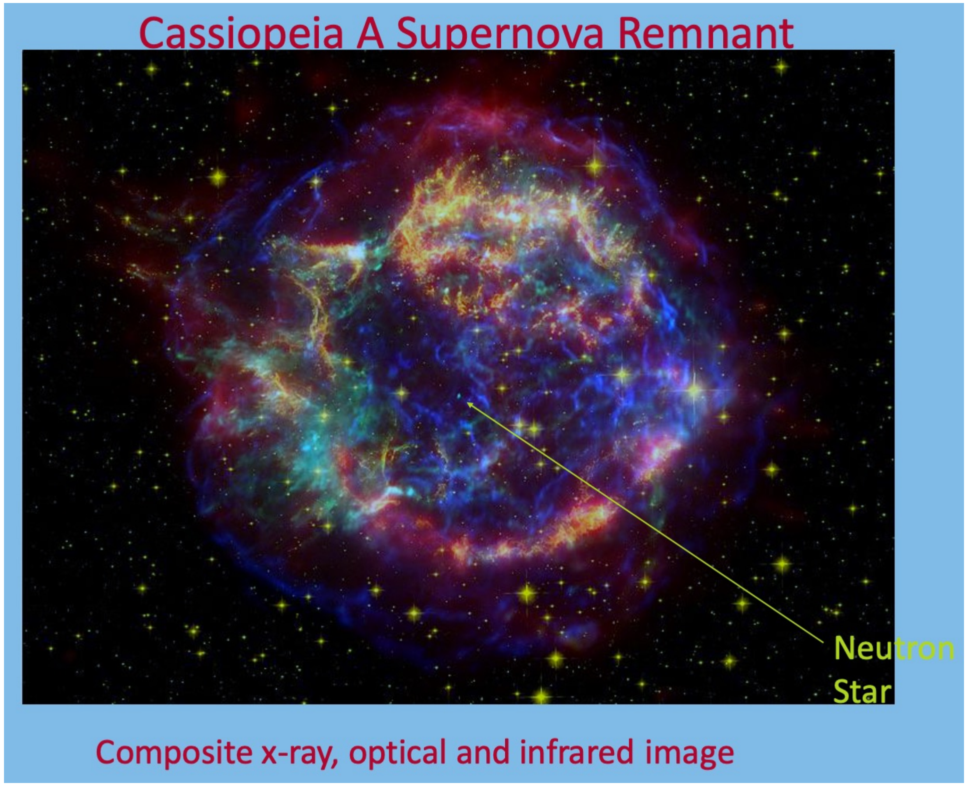
- White dwarfs
  – Light up planetary nebulae for a while
  – Eventually cool and fade away. They become too faint to see.
- Pulsars → cold Neutron Stars
  – A big nucleus in the sky
- Black Holes → singularity

Supernova

- Massive stars reaching the end of their life can explode violently.
- The interior of the star contracts very rapidly, and the core bounce causes an explosion.
- For M_{core} > Chandrasekhar mass (about 1.25 solar mass) → **Core Collapse**
- During the Red Giant phase, iron is produced in the core.
- Iron won't "burn", so the core contracts and the temperature rises to billions of degrees.
- If the iron core becomes too dense, the electrons get high enough energy to penetrate atomic nuclei
- That is, the Fermi energy rises until the majority of the electrons have energies above the threshold for electron capture: $\mathbf{e^-(A, Z) \rightarrow (A, Z-1)\nu_e}$
- Proton and electrons combine into neutron and neutrinos in a process called "Neutron Drip": $p^+ + e^- \rightarrow n + \nu_e$
- The "disappearance" of the electrons → **no more electron degeneracy pressure**

Supernova (cont'd)

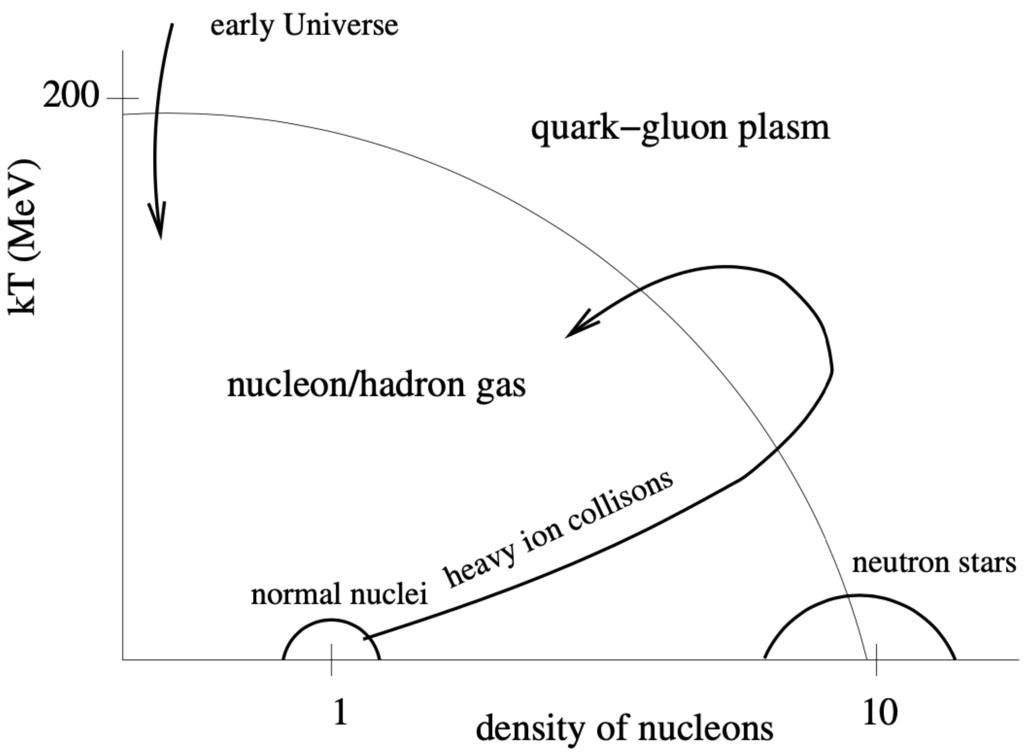
- As the protons are transformed to neutrons by this reaction (**neutronization**), the temperature rises to the point where nuclei are dissociated to create **a star made mostly of neutrons.**
- The core collapses catastrophically.
- ν' s escape carrying away the energy.
- The neutrons fall toward the center reaching speeds ~0.1-0.2 c.
- The collapse occurs over ~1 second.



- The **Pauli Exclusion Principle for neutrons** takes effect → the falling matter stops instantly
- Many of the neutrons BOUNCE and fly outward (like billiard balls).
- They sweep material up with them as they fly outward.
- And we have a very CATASTROPHIC explosion.

Supernova (cont'd)

- We must then consider the ground state of a gravitationally bound collection of neutrons. To estimate its parameters, we can repeat the previous analysis replacing m_e with m_n and N_e/N_b with unity.
- This last replacement means that the critical number of baryons for a neutron star ($N_c = 1.82\times10^{57}$) is larger than the critical number for a white dwarf ($N_c = 1.82\times10^{57}\left(\frac{N_e}{N_b}\right)^2$) → mass limit of Tolman, Oppenheimer, Volkoff
- The core with mass in between can not be white dwarf but will reach a stable state as a neutron star.
- The radius of the neutron star is: $R_{\mathrm{gs}} \sim \dfrac{\hbar}{m_{\mathrm{n}}c} N_{\mathrm{b}}^{1/3}$ If: $\frac{N_b}{N_c}\sim0.8 \rightarrow \left(\frac{N_c}{N_b}\right)^{2/3}\left(1-\frac{N_b}{N_c}\right)^{1/2} \approx 0.5$
- For a solar mass this gives $R_{gs} \sim 3$ km, comparable with the observed sizes of **neutron stars**. It corresponds to a density about 10 times greater than that of normal nuclei.
- At this point, in general, the compression ceases because the nucleons ensure the pressure required to stop the collapse
- However, if $M_{star} > 25 \times M_{sun}$, the gas of nucleons is not enough to provide a sufficient pressure and the gravitational collapse continues up to form a **black hole**



- The expected phase diagram for nuclear matter showing the nuclear state as a function of temperature and baryon density (minus the antibaryon density).
- At high temperature and density, quarks and gluons act as free particles in a **quark–gluon plasma**.
- At low temperature and density the quarks and gluons combine to form hadrons and nucleons.

- At vanishing temperature, the transition corresponds to a density about 10 times that of normal nuclei, i.e. nucleons in contact.
- Neutron stars are believed to have densities near this point.
- At low density, the transition is at $\mathrm{kT} \sim m_\pi c^2 \sim 200$ MeV.
- Such a **phase transition** is believed to have occurred in the early universe.
- High–energy (>100 GeV/nucleon) heavy ion collisions are believed to sometimes create quark–gluon plasmas that quickly cool back to the nucleon–hadron phase.

Black holes

- The black holes contract until they approximate R ≈ 3 Km (Schwarzschild radius) and a $\rho > 10^{16}$ g/cm^3
- **The $R_{Schwarzschild}$ represents the "dimension" of a black hole**
- In a simple way (although not fully correct) it can be derived setting the escape velocity from a black hole equal to the light velocity

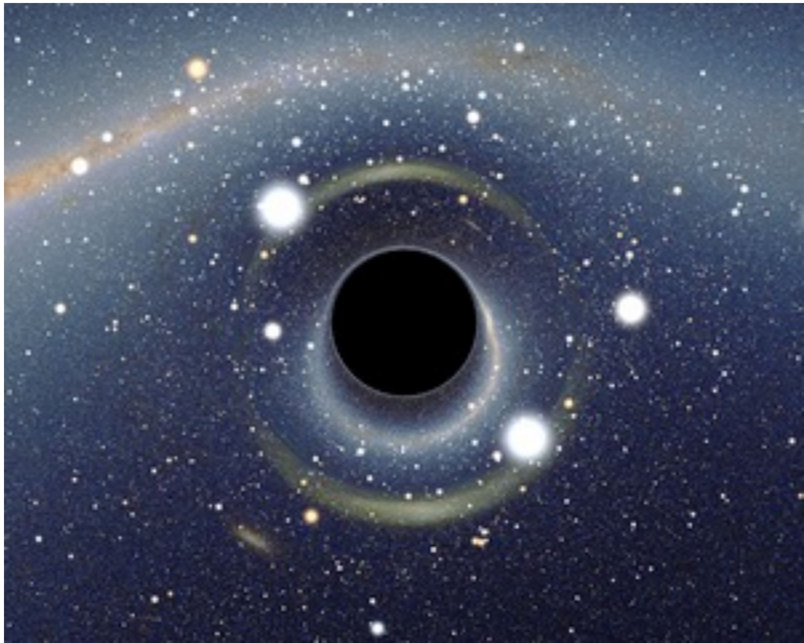
$$v_{escape} = \sqrt{\frac{2GM}{r_{Schwarzschild}}} = c$$

G = gravitational constant

***M* = mass of the black hole**

one gets:

$$r_{Schwarzschild} = \frac{2GM}{c^2}$$



Simulated view of a black hole

Supernova

- Enormous amount of energy (≈ 3×10^{53} erg) is released over a very short time.
- Mainly radiated in form of neutrinos in ≈ 10 s
- The neutron stars have R ≈ 10 km and a $\rho_{central}$ ≈ 10^{14} g/cm^3
- The “star” brightens tremendously.
- During a supernova, a star may shine as brightly as an entire galaxy.

So what’s left?

- The core becomes a super dense object, either a
  - Neutron Star: M_{core} < few M_{sun}
  - Black Hole: M_{core} > few M_{sun}
- The rest of the star is blown away, becoming a Supernova Remnant.

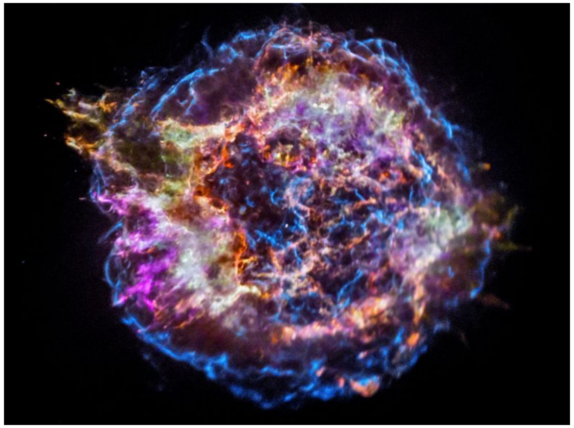
- The rate of Supernovae is ~ 1 SN / Galaxy / 50 years
- But there hasn’t been one seen in our galaxy in over 390 years!

- For a supernova with M_v = -19.
- At 0.25 pc (0.8 lyr) from us it would appear as bright as the Sun.
- At 160 pc (520 lyr) from us it would appear as bright as a full moon.

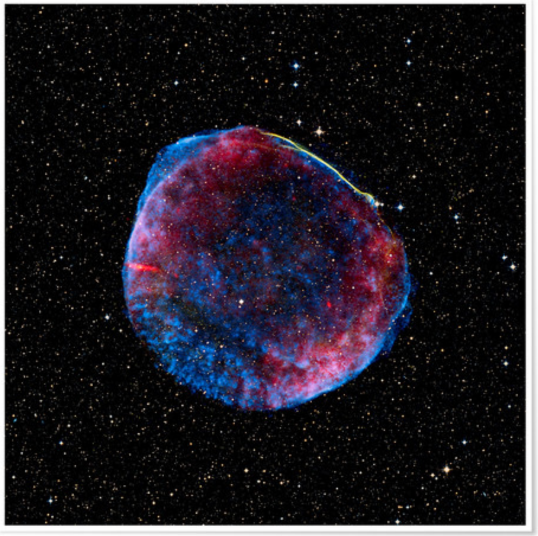
Historical (Naked Eye) Supernovae

Date (A.D.)	Constellation	Apparent Mag./Dist	Where Observed
1006	Lupus	-5 (> Venus) 3 kpc	Many Places
1054	Taurus (Crab Nebula)	-5 (> Venus) 2 kpc	China, SW America
1572	Cassiopeia (Tycho’s SN)	-4 (< Venus) 5 kpc	Many Places
1604	Ophiucus (Kepler’s SN)	-2 (> Sirius) 6 kpc	Many Places
1987	LMC	+3 (Avg. Star) 50 kpc	Southern Hemisphere

Supernova Remnants (SNR)



- Residual material ejected by the explosion.
- Expanding at large velocities initially.
- Sweeps up material around the star.
- The expanding shock wave generated by a supernova expands out into the surrounding interstellar medium, generating a supernova remnant which may be visible for many thousands of years.
- Very bright in the radio due to synchrotron radiation.
  - High energy electrons spiral around the magnetic fields of the SNR.
  - Emit lots of radio frequency photons.



Neutron Stars

- Neutron Star:
  - – Left over (stellar endpoint) from supernova
  - – A sea of neutrons
  - – A giant atomic nucleus in the sky!!

- Mass from 1.4 to ~3 M_{sun}
- Size ~10 km
- Density ~3 x 10^{14} g/cm^3
- Intense magnetic fields, rapidly rotating

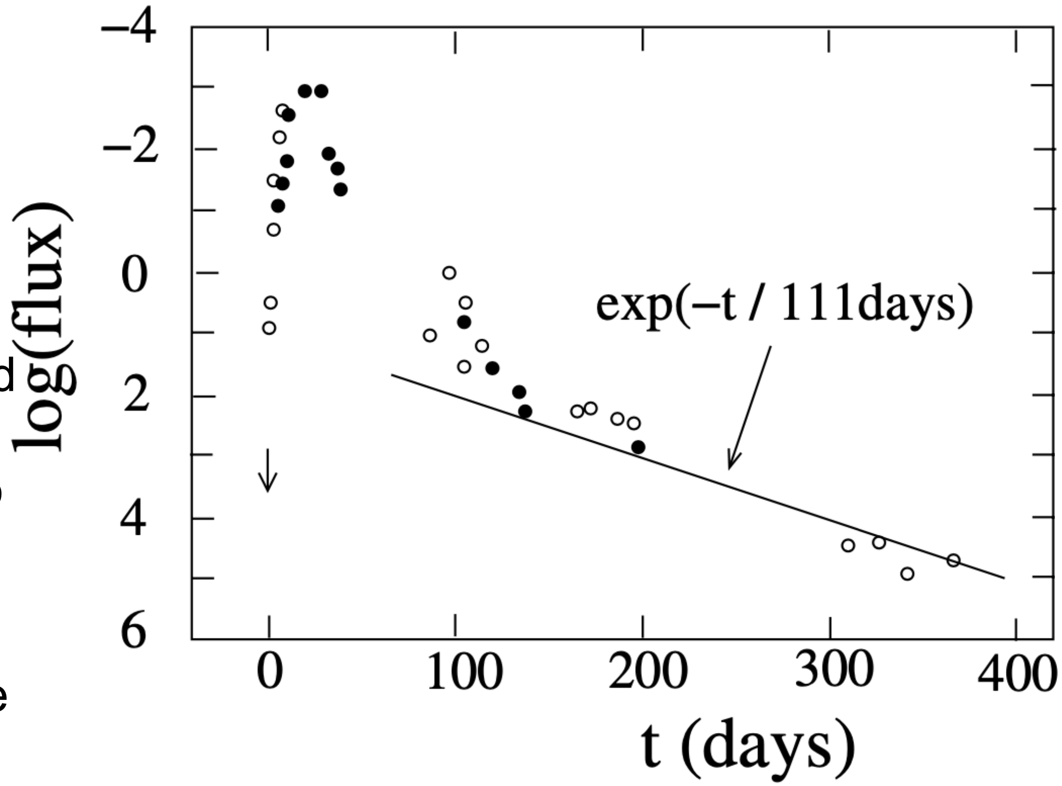
Types of Supernovae

- **Type I** – no hydrogen absorption lines
  - Ia – no hydrogen lines, no helium lines, but does have strong absorption line of ionized Silicon (Si II)
  - Ib – strong helium lines, still no hydrogen
  - Ic – no helium lines, still no hydrogen
- **Type II** – hydrogen absorption lines

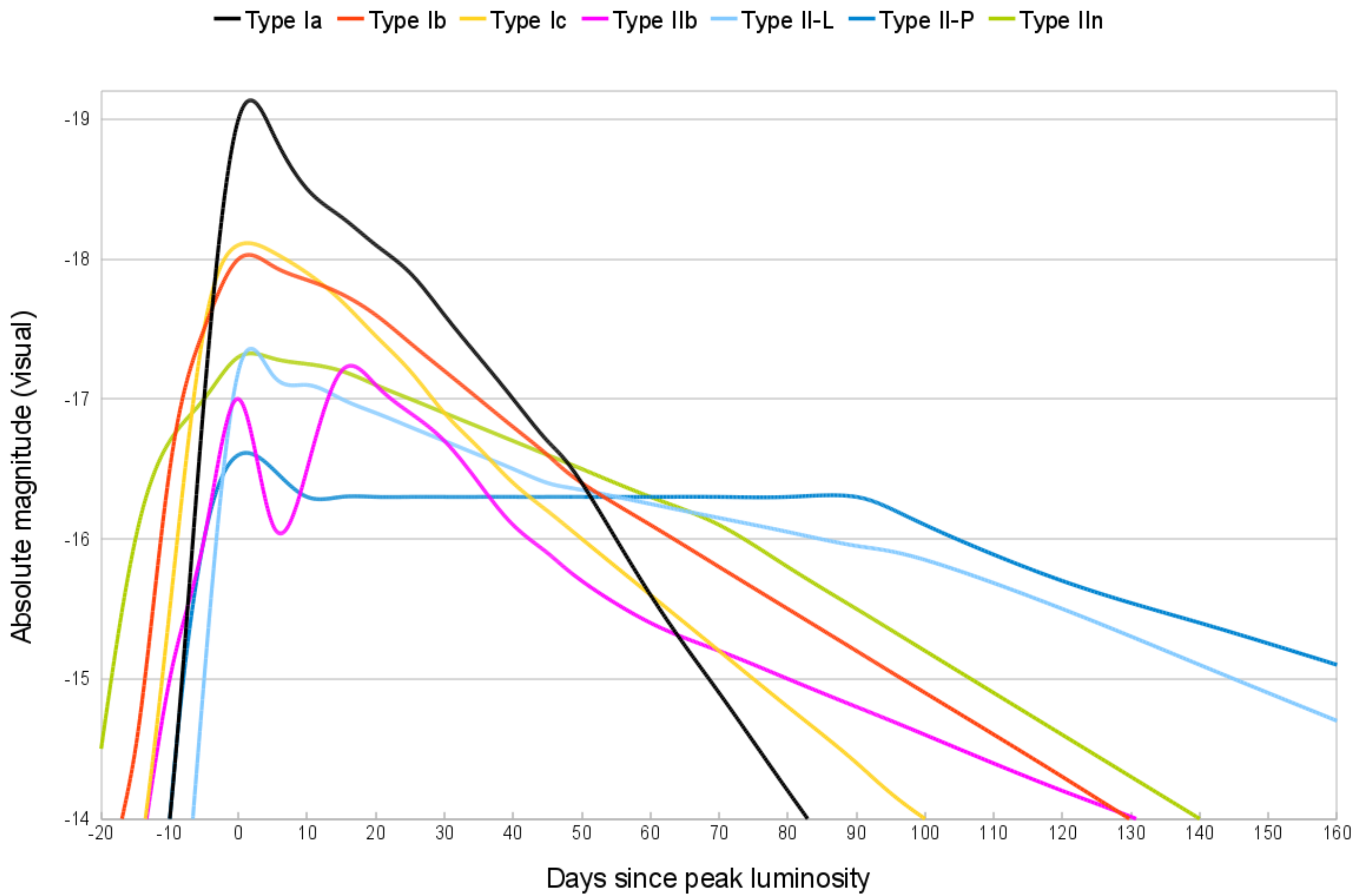
- Collapse of massive stars leads to type II and Ib, only difference is whether star sheds outer hydrogen layer before exploding

Kepler's supernova

- Open circles are European measurements and filled circles are Korean measurements.
- Astronomers at the time measured the evolution of the luminosity of the supernovae by comparing it to known stars and planets.
- It has been possible to determine the positions of planets at the time when they were observed, and, with the notebooks, to reconstruct the luminosity curves.



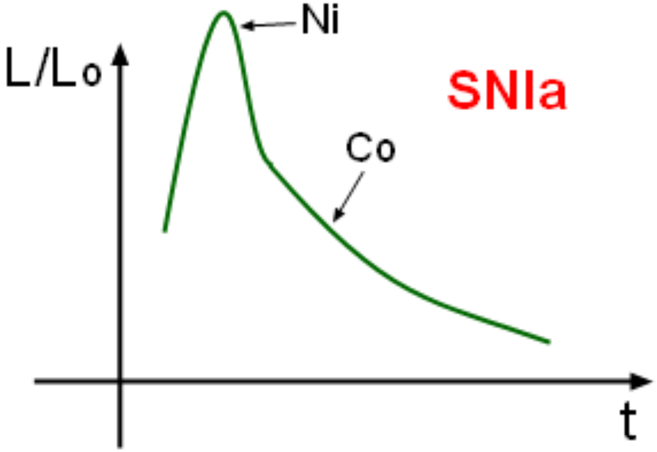
- The superimposed curve shows the rate of ^{56}Co decay using the laboratory-measured half life.
- The vertical scale gives the visual magnitude V of the star, proportional to the logarithm of the photon flux.
- V = 0 corresponds to a bright star, while V = 5 is the dimmest star that can be observed with the naked eye.


Type Ia
Type Ib
Type Ic
Type IIb
Type II-L
Type II-P
Type IIn
Absolute magnitude (visual)
-19
-18
-17
-16
-15
-14
-20
-10
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
Days since peak luminosity

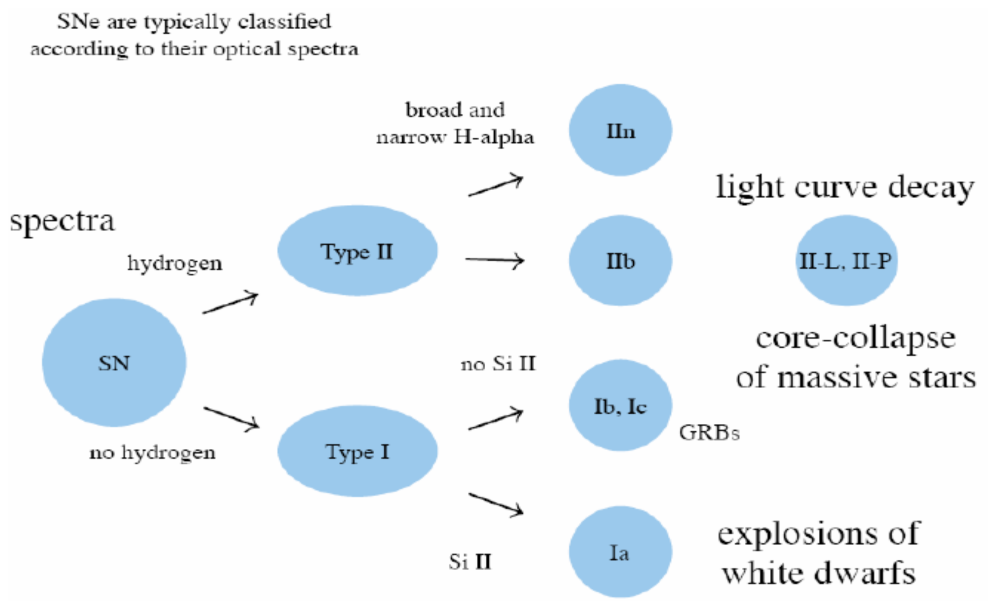
Supernova Type Ia

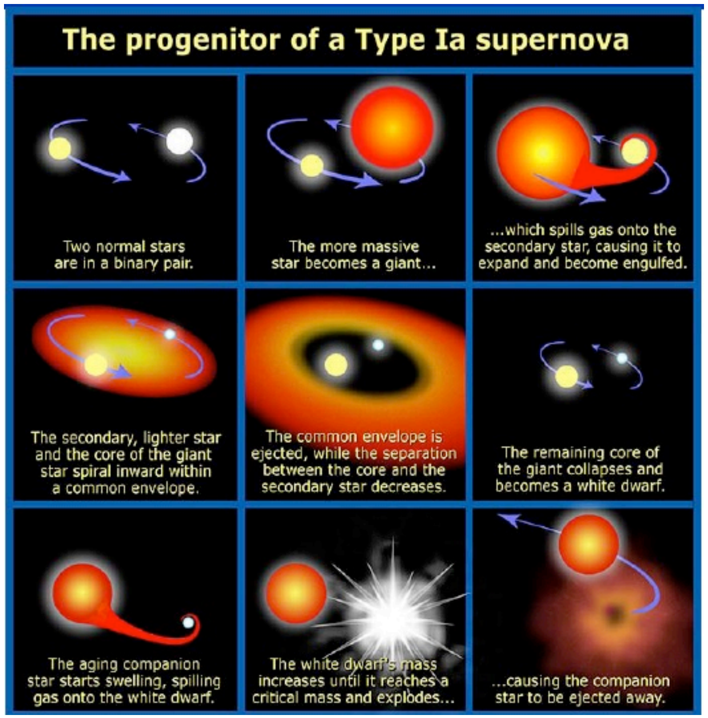
- When a slowly-rotating, carbon-oxygen white dwarf accretes matter from a companion, it can exceed the Chandrasekhar limit of about 1.38 solar masses, beyond which it would no longer be able to support its weight through electron degeneracy pressure and begin to collapse.
- The typical visual absolute magnitude of Type Ia supernovae is $M_v = -19.3$ (≈ 5 billion times brighter than the Sun), with little variation.

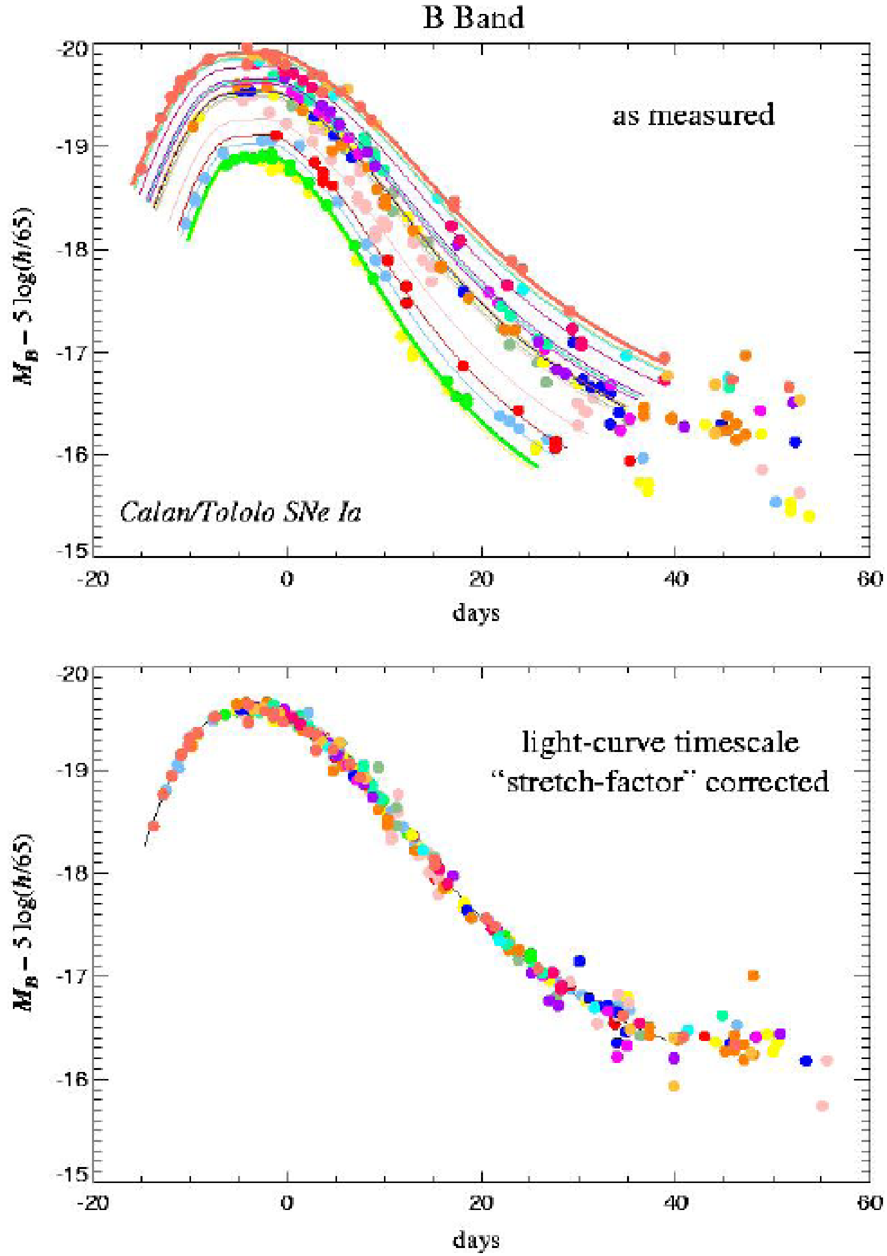
- **Light curve:** the plot of luminosity shows the characteristic light curve for a Type Ia SN.
- The peak is primarily due to the decay of Ni, while the later stage is powered by Co.
- Near maximum luminosity, the spectrum contains lines of intermediate-mass elements from oxygen to calcium; these are the main constituents of the outer layers of the star.
- Months after the explosion, when the outer layers are transparent, the spectrum is dominated by light emitted by materials near the core; most prominently isotopes close to the mass of iron.
- The radioactive decay of ^{56}Ni through ^{56}Co to ^{56}Fe produces high-energy photons which dominate the energy output of the ejecta at intermediate to late times.
- The similarity in the absolute luminosity profiles of nearly all known Type Ia supernovae has led to their use as a secondary standard candle in extragalactic astronomy.
- The cause of this uniformity in the luminosity curve is still an open question.



Supernovae Type 1A

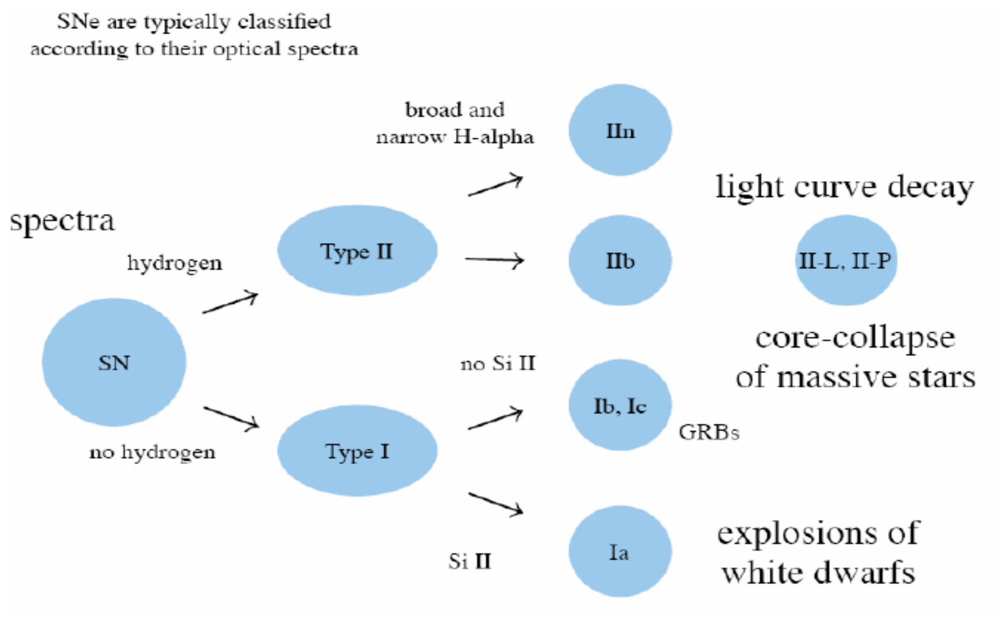


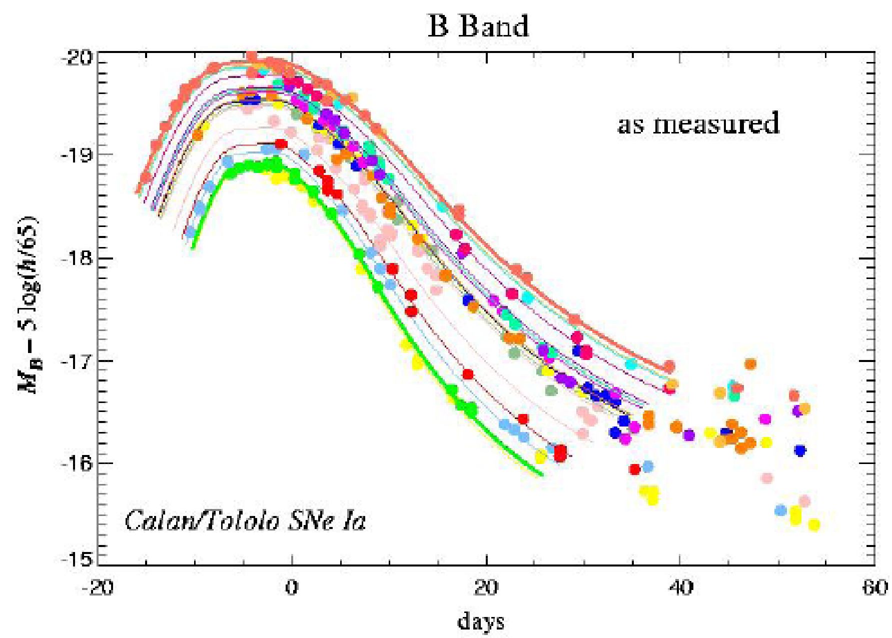


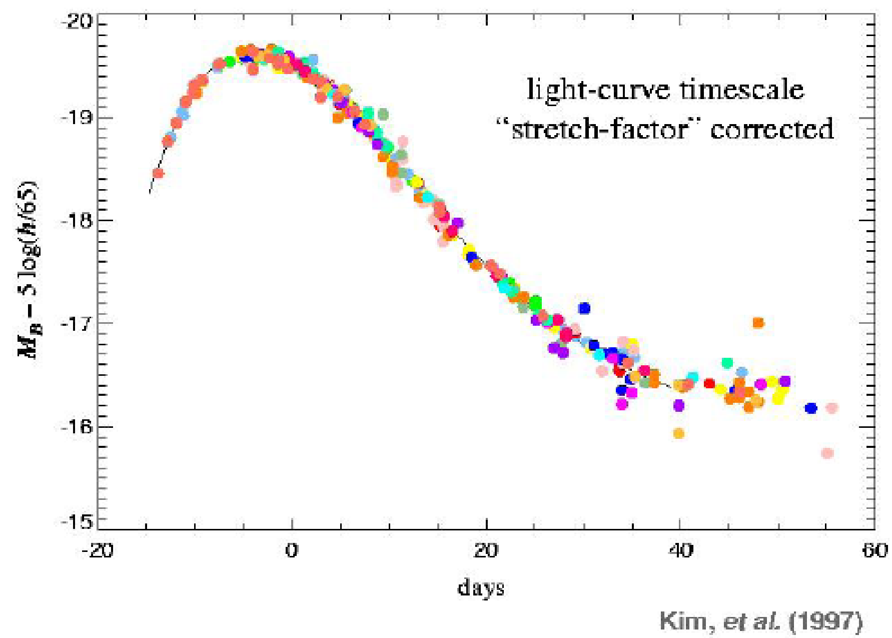


Kim, et al. (1997)

Supernovae Type 1A





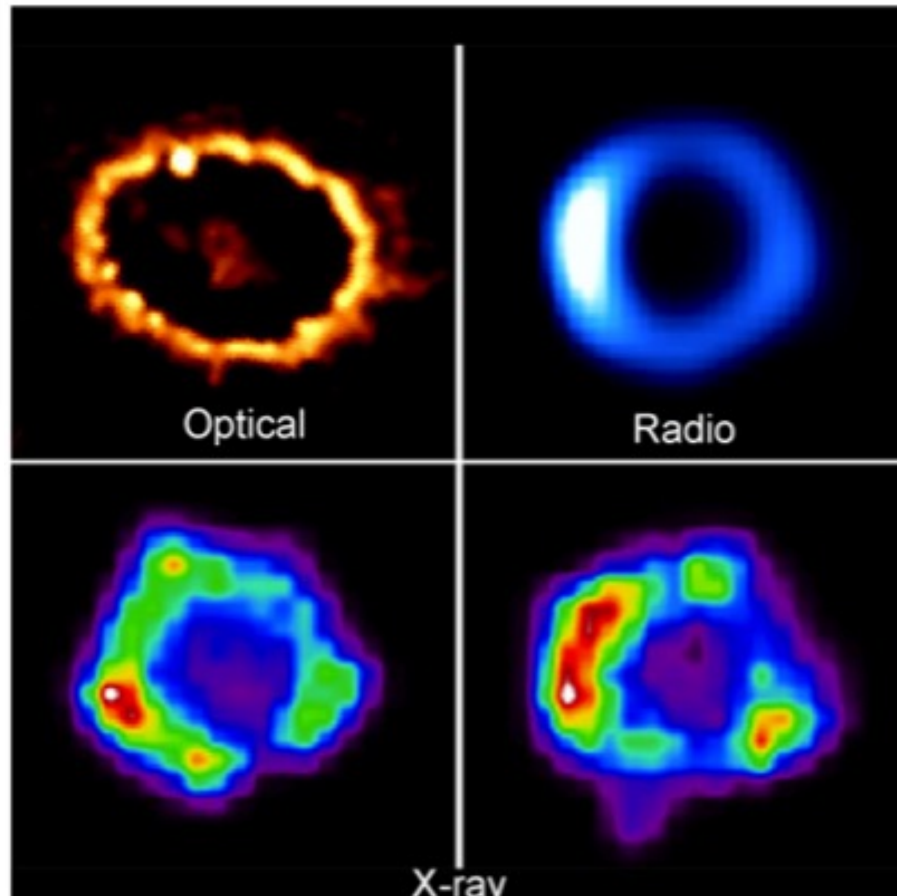


- SNe Ia **do not** all have exactly the same absolute magnitude, but absolute magnitude is **strongly correlated** with rate of decline (faster = fainter).
- Apply **"stretch factor"** to compensate for this
- Also need to correct for **spectral redshift** and interstellar **absorption**

→ **Standard candles**

An example - SN1987A

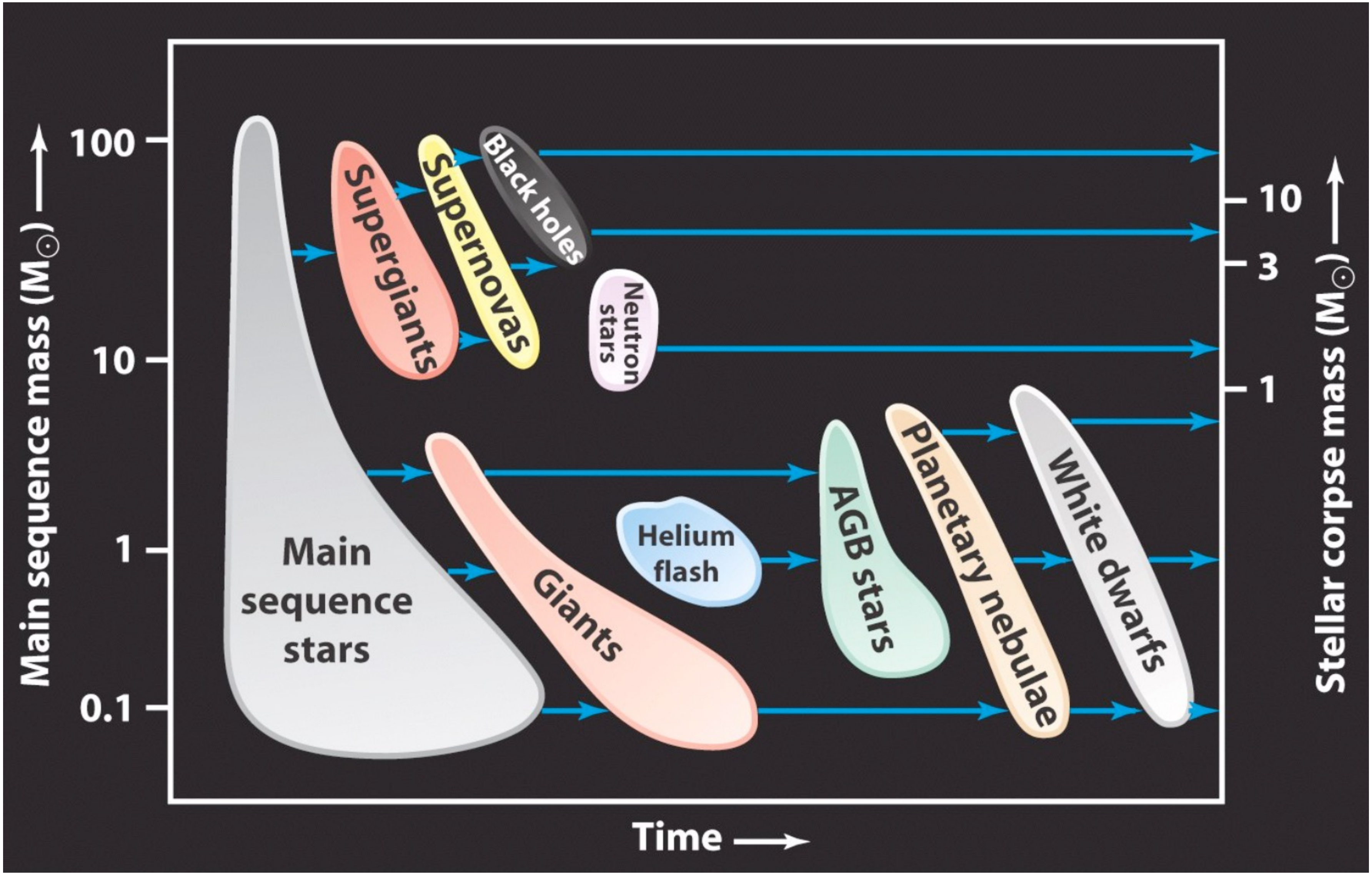
Supernova remnants



- SN1987A in the Large Magellanic Cloud is the closest supernova seen for over 300 years.
- Its peak luminosity of $\sim 10^8 L_\odot$ was rather low by typical SN standards.

Summary – Pathway of stellar evolution

asymptotic giant branch (AGB)



PULSAR

- The discovery was made in 1967: by Jocelyn Bell (graduate student), advisor: Antony Hewish (Nobel prize) → LGM (*Little Green Men*)

  point-like objects that emitted periodical radio signals. They were called Pulsar and - although they are rotating objects and not pulsating - the name still is used
- Pulsars are neutron stars.
- The period of the pulsar is associated with the rotational period of the neutron star; their slowdown is due to loss of rotational energy

e.g .:

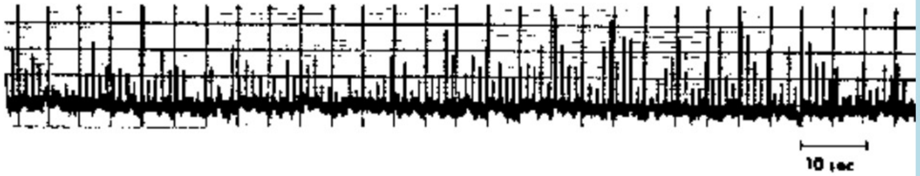
  The rotational energy lost by the pulsar Crab is of the same order of the total energy emitted by the Crab Nebula.

  The neutron star is the source of the great power of the Crab nebula.
- Pulsars also periodically emit light
- Increasing the knowledge, the pulsars reveal more and more properties of the neutron stars

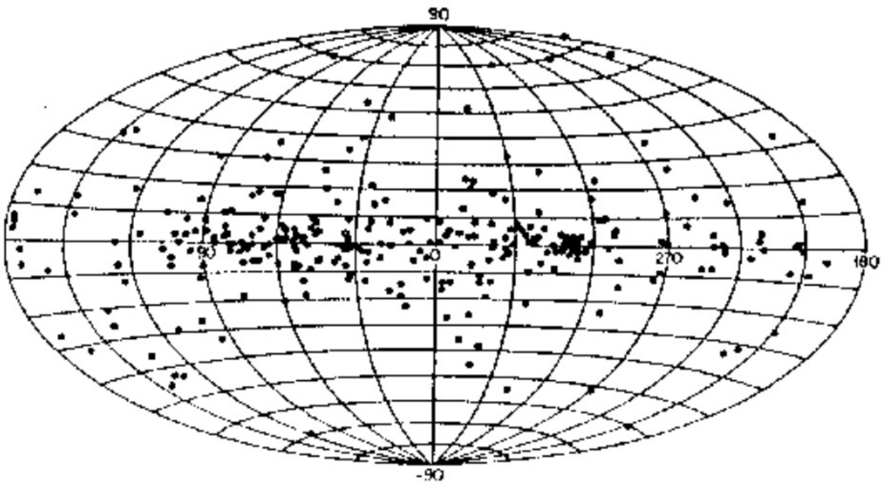
  → a laboratory, where $\rho > 10^{15}$ g/cm^3 are possible

Neutron star rotation

- Neutron stars initially spin very rapidly.
- Conservation of angular momentum! – mass x velocity x radius = constant
- Rotation period of Sun = 25 days
- Shrinking the Sun to 10 km would give a rotation period of much less than 1 second!



Pulsar Radio Record from Arecibo.

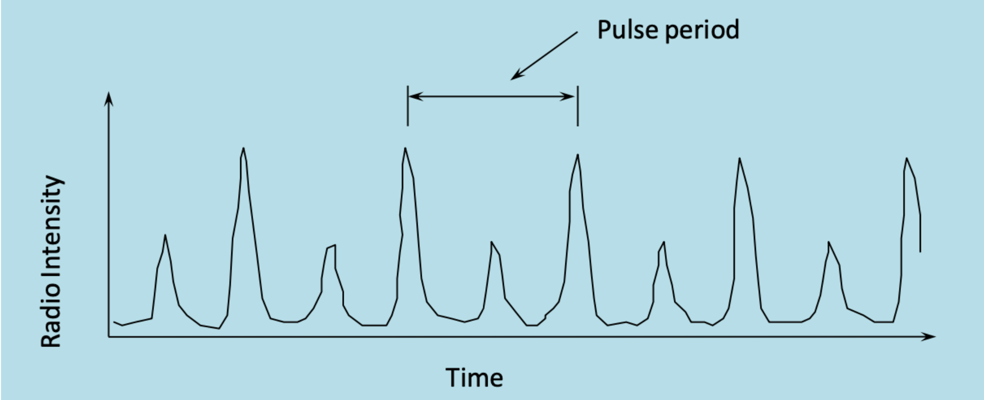


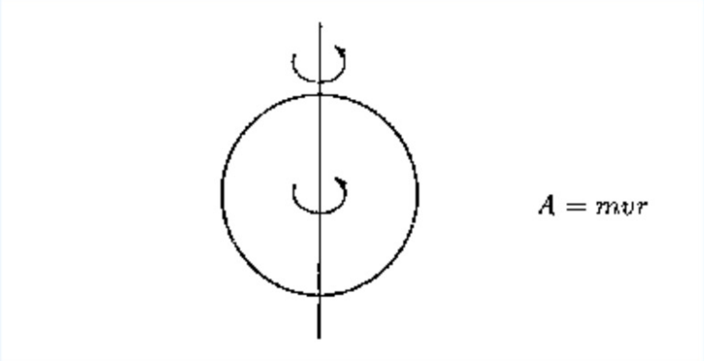
Distribution of Pulsars.

Conservation of Angular Momentum

mass x *speed* x *radius* = CONSTANT

As the radius of a star goes down, the speed must go up.



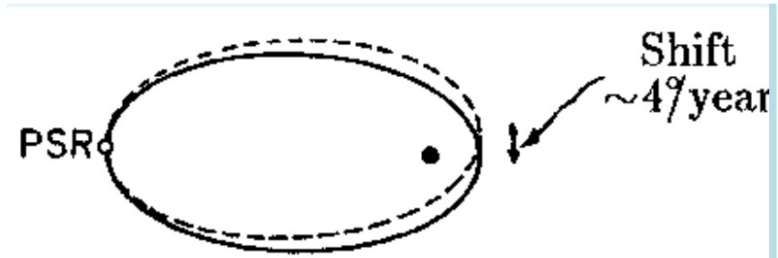


Pulsar Characteristics

- Rotating Neutron Stars
- Period 1 sec
- Size 20 km
- Density ~3 x 10^{14} g/cm^3
- Mass ~1 to 2 M_{sun}
- Surface Temperature ~10^6 K
- Mostly neutrons
- Afterwards millisecond pulsars were discovered

Binary Pulsars

The orbit processes rapidly according to Einstein's General Theory of Relativity



General Relativity prediction confirmed on 1974 by J.H. Taylor, Jr. and R. Hulse (Nobel prize winners 1993)